# Algebra and Trigonometry Student Solution Manual



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# Chapter 1

## Try It

**1.1**.

a. $\frac{11}{1}$

b. $\frac{3}{1}$

c. $-\frac{4}{1}$

**1.2**.

a. 4 (or 4.0), terminating;

b. $\overline{0.615384,}$ repeating;

c. –0.85, terminating

**1.3**.

a. rational and repeating;
b. rational and terminating;
c. irrational;
d. rational and repeating;
e. irrational

**1.4**.

a. positive, irrational; right
b. negative, rational; left
c. positive, rational; right
d. negative, irrational; left
e. positive, rational; right

**1.5**.

| | *N* | *W* | *I* | *Q* | *Q'* |
|---|---|---|---|---|---|
| a. $-\frac{35}{7}$ | | | X | X | |
| b. 0 | | X | X | X | |
| c. $\sqrt{169}$ | X | X | X | X | |
| d. $\sqrt{24}$ | | | | | X |
| e. 4.763763763... | | | | X | |

**1.6**.

a. 10
b. 2
c. 4.5
d. 25
e. 26

**1.7**.

a. 11, commutative property of multiplication, associative property of multiplication, inverse property of multiplication, identity property of multiplication;
b. 33, distributive property;
c. 26, distributive property;
d. $\frac{4}{9}$, commutative property of addition, associative property of addition, inverse property of addition, identity property of addition;
e. 0, distributive property, inverse property of addition, identity property of addition

**1.8**.

| | **Constants** | **Variables** |
|---|---|---|
| a. $2\pi r(r+h)$ | $2,\ \pi$ | $r,\ h$ |
| b. 2(L + W) | 2 | L, W |
| c. $4y^3+y$ | 4 | $y$ |

**1.9**.

a. 5;
b. 11;
c. 9;
d. 26

**1.10**.

a. 4;
b. 11;
c. $\frac{121}{3}\pi$;
d. 1728;
e. 3

**1.11**. 1,152 $cm^2$

**1.12**.

a. $-2y-2z$ or $-2(y+z)$;
b. $\frac{2}{t}-1$;
c. $3pq-4p+q$;
d. $7r-2s+6$

**1.13**.

$A=P(1+rt)$

**1.14**.

a. $k^{15}$
b. $\left(\frac{2}{y}\right)^5$
c. $t^{14}$

**1.15**.

a.

$s^7$

b.

$(-3)^5$

c.

$\left(ef^2\right)^2$

**1.16**.

a.

$(3y)^{24}$

b.

$t^{35}$

c.

$(-g)^{16}$

**1.17**.

a.

$1$

b.

$\frac{1}{2}$

c.

$1$

d.

$1$

**1.18**.

a.

$\frac{1}{(-3t)^6}$

b.

$\frac{1}{f^3}$

c.

$\frac{2}{5k^3}$

**1.19**.

a.

$t^{-5} = \frac{1}{t^5}$

b.

$\frac{1}{25}$

**1.20**.

a.

$g^{10}h^{15}$

b.

$125t^3$

c.

$-27y^{15}$

d.

$\frac{1}{a^{18}b^{21}}$

e.

$$\frac{r^{12}}{s^8}$$

**1.21**.

a.

$$\frac{b^{15}}{c^3}$$

b.

$$\frac{625}{u^{32}}$$

c.

$$\frac{-1}{w^{105}}$$

d.

$$\frac{q^{24}}{p^{32}}$$

e.

$$\frac{1}{c^{20}d^{12}}$$

**1.22**.

a.

$$\frac{v^6}{8u^3}$$

b.

$$\frac{1}{x^3}$$

c.

$$\frac{e^4}{f^4}$$

d.

$$\frac{27r}{s}$$

e.

$$1$$

f.

$$\frac{16h^{10}}{49}$$

**1.23**.

a.

$$\$1.52\times10^5$$

b.

$$7.158\times10^9$$

c.

$$\$8.55\times10^{13}$$

d.

$$3.34\times10^{-9}$$

e.

$$7.15\times10^{-8}$$

**1.24**.

a.

$$703,\ 000$$

b.

$$-816,\ 000,\ 000,\ 000$$

c.
$-0.000\,000\,000\,000\,39$

d.
$0.000008$

**1.25**.

a.
$-8.475 \times 10^{6}$

b.
$8 \times 10^{-8}$

c.
$2.976 \times 10^{13}$

d.
$-4.3 \times 10^{6}$

e.
$\approx 1.24 \times 10^{15}$

**1.26**. Number of cells:
$3 \times 10^{13}$; length of a cell:
$8 \times 10^{-6}$ m; total length:
$2.4 \times 10^{8}$ m or
$240,\ 000,\ 000$ m.

**1.27**.

a.
15

b.
3

c.
4

d.
17

**1.28**.
$5|x||y|\sqrt{2yz}$. Notice the absolute value signs around $x$ and $y$? That's because their value must be positive!

**1.29**.
$10|x|$

**1.30**.
$\frac{x\sqrt{2}}{3y^{2}}$. We do not need the absolute value signs for
$y^{2}$ because that term will always be nonnegative.

**1.31**.
$b^{4}\sqrt{3ab}$

**1.32**.
$13\sqrt{5}$

**1.33**.
$0$

**1.34**.
$6\sqrt{6}$

**1.35**.
$14-7\sqrt{3}$

**1.36**.

a.
$-6$

b.
$6$

c.

$88\sqrt[3]{9}$

**1.37**.

$\left(\sqrt{9}\right)^5 = 3^5 = 243$

**1.38**.

$x(5y)^{\frac{9}{2}}$

**1.39**.

$28x^{\frac{23}{15}}$

**1.40**. The degree is 6, the leading term is $-x^6$, and the leading coefficient is $-1$.

**1.41**.

$2x^3 + 7x^2 - 4x - 3$

**1.42**.

$-11x^3 - x^2 + 7x - 9$

**1.43**.

$3x^4 - 10x^3 - 8x^2 + 21x + 14$

**1.44**.

$3x^2 + 16x - 35$

**1.45**.

$16x^2 - 8x + 1$

**1.46**.

$4x^2 - 49$

**1.47**.

$6x^2 + 21xy - 29x - 7y + 9$

**1.48**.

$(b^2 - a)(x + 6)$

**1.49**.

$(x-6)(x-1)$

**1.50**. a.

$(2x + 3)(x + 3)$ b.

$(3x-1)(2x + 1)$

**1.51**.

$(7x-1)^2$

**1.52**.

$(9y + 10)(9y - 10)$

**1.53**.

$(6a + b)(36a^2 - 6ab + b^2)$

**1.54**.

$(10x - 1)\left(100x^2 + 10x + 1\right)$

**1.55**.

$(5a-1)^{-\frac{1}{4}}(17a-2)$

**1.56**.

$\frac{1}{x + 6}$

**1.57**.
$\frac{(x+5)(x+6)}{(x+2)(x+4)}$

**1.58**.
1

**1.59**.
$\frac{2(x-7)}{(x+5)(x-3)}$

**1.60**.
$\frac{x^2-y^2}{xy^2}$

## Section Exercises

**1**. irrational number. The square root of two does not terminate, and it does not repeat a pattern. It cannot be written as a quotient of two integers, so it is irrational.

**3**. The Associative Properties state that the sum or product of multiple numbers can be grouped differently without affecting the result. This is because the same operation is performed (either addition or subtraction), so the terms can be re-ordered.

**5**.
$-6$

**7**.
$-2$

**9**.
$-9$

**11**. 9

**13**. 4

**15**. 4

**17**. 0

**19**. 9

**21**. 25

**23**.
$-6$

**25**. 17

**27**. 4

**29**.
$-4$

**31**.
$-6$

**33**.
$\pm 1$

**35**. 2

**37**. 2

**39**.
$-14y-11$

**41**.
$-4b+1$

**43**.
$43z-3$

**45**.
$9y+45$

**47**.
$-6b+6$

**49**.
$\frac{16x}{3}$

**51**.
$9x$

**53**.
$\frac{1}{2}(40-10)+5$

**55**. irrational number

**57**.
$g+400-2(600)=1200$

**59**. inverse property of addition

**61**. 68.4

**63**. true

**65**. irrational

**67**. rational

**69**. No, the two expressions are not the same. An exponent tells how many times you multiply the base. So $2^3$ is the same as $2\times2\times2,$ which is 8.

$3^2$ is the same as $3\times3,$ which is 9.

**71**. It is a method of writing very small and very large numbers.

**73**. 81

**75**. 243

**77**.
$\frac{1}{16}$

**79**.
$\frac{1}{11}$

**81**. 1

**83**.
$4^9$

**85**.
$12^{40}$

**87**.
$\frac{1}{7^9}$

**89**.
$3.14\times10^{-5}$

**91**. 16,000,000,000

**93**.
$a^4$

**95**.
$b^6c^8$

**97**.
$ab^2d^3$

**99**.
$m^4$

**101**.
$\frac{q^5}{p^6}$

**103**.
$\frac{y^{21}}{x^{14}}$

**105**.
25

**107**.
$72a^2$
**109**.
$\frac{c^3}{b^9}$
**111**.
$\frac{y}{81z^6}$
**113**. 0.00135 m
**115**.
$1.0995\times10^{12}$
**117**. 0.00000000003397 in.
**119**. 12,230,590,464
$m^{66}$
**121**.
$\frac{a^{14}}{1296}$
**123**.
$\frac{n}{a^9c}$
**125**.
$\frac{1}{a^6b^6c^6}$
**127**. 0.00000000000000000000000000000000662606957
**128**. When there is no index, it is assumed to be 2 or the square root. The expression would only be equal to the radicand if the index were 1.
**130**. The principal square root is the nonnegative root of the number.
**132**. 16
**134**. 10
**136**. 14
**138**.
$7\sqrt{2}$
**140**.
$\frac{9\sqrt{5}}{5}$
**142**. 25
**144**.
$\sqrt{2}$
**146**.
$2\sqrt{6}$
**148**.
$5\sqrt{6}$
**150**.
$6\sqrt{35}$
**152**.
$\frac{2}{15}$
**154**.
$\frac{6\sqrt{10}}{19}$
**156**.
$-\frac{1+\sqrt{17}}{2}$
**158**.
$7\sqrt[3]{2}$

**160**.
$15\sqrt{5}$
**162**.
$20x^2$
**164**.
$7\sqrt{p}$
**166**.
$18m^2\sqrt{m}$
**168**.
$2b\sqrt{a}$
**170**.
$\frac{15x}{7}$
**172**.
$5y^4\sqrt{2}$
**174**.
$\frac{4\sqrt{7d}}{7d}$
**176**.
$\frac{2\sqrt{2}+2\sqrt{6x}}{1-3x}$
**178**.
$-w\sqrt{2w}$
**180**.
$\frac{3\sqrt{x}-\sqrt{3x}}{2}$
**182**.
$5n^5\sqrt{5}$
**184**.
$\frac{9\sqrt{m}}{19m}$
**186**.
$\frac{2}{3d}$
**188**.
$\frac{3\sqrt[4]{2x^2}}{2}$
**190**.
$6z\sqrt[3]{2}$
**192**. 500 feet
**194**.
$\frac{-5\sqrt{2}-6}{7}$
**196**.
$\frac{\sqrt{mnc}}{a^9 cmn}$
**198**.
$\frac{2\sqrt{2}x+\sqrt{2}}{4}$
**200**.
$\frac{\sqrt{3}}{3}$

**201**. The statement is true. In standard form, the polynomial with the highest value exponent is placed first and is the leading term. The degree of a polynomial is the value of the highest exponent, which in standard form is also the exponent of the leading term.
**203**. Use the distributive property, multiply, combine like terms, and simplify.

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**205**. 2
**207**. 8
**209**. 2
**211**.
$4x^2 + 3x + 19$
**213**.
$3w^2 + 30w + 21$
**215**.
$11b^4 - 9b^3 + 12b^2 - 7b + 8$
**217**.
$24x^2 - 4x - 8$
**219**.
$24b^4 - 48b^2 + 24$
**221**.
$99v^2 - 202v + 99$
**223**.
$8n^3 - 4n^2 + 72n - 36$
**225**.
$9y^2 - 42y + 49$
**227**.
$16p^2 + 72p + 81$
**229**.
$9y^2 - 36y + 36$
**231**.
$16c^2 - 1$
**233**.
$225n^2 - 36$
**235**.
$-16m^2 + 16$
**237**.
$121q^2 - 100$
**239**.
$16t^4 + 4t^3 - 32t^2 - t + 7$
**241**.
$y^3 - 6y^2 - y + 18$
**243**.
$3p^3 - p^2 - 12p + 10$
**245**.
$a^2 - b^2$
**247**.
$16t^2 - 40tu + 25u^2$
**249**.
$4t^2 + x^2 + 4t - 5tx - x$
**251**.
$24r^2 + 22rd - 7d^2$
**253**.
$32x^2 - 4x - 3\ \text{m}^2$
**255**.
$32t^3 - 100t^2 + 40t + 38$

**257**.
$a^4 + 4a^3c - 16ac^3 - 16c^4$

**258**. The terms of a polynomial do not have to have a common factor for the entire polynomial to be factorable. For example, $4x^2$ and $-9y^2$ don't have a common factor, but the whole polynomial is still factorable: $4x^2 - 9y^2 = (2x + 3y)(2x - 3y)$.

**260**. Divide the $x$ term into the sum of two terms, factor each portion of the expression separately, and then factor out the GCF of the entire expression.

**262**.
$7m$

**264**.
$10m^3$

**266**.
$y$

**268**.
$(2a - 3)(a + 6)$

**270**.
$(3n - 11)(2n + 1)$

**272**.
$(p + 1)(2p - 7)$

**274**.
$(5h + 3)(2h - 3)$

**276**.
$(9d - 1)(d - 8)$

**278**.
$(12t + 13)(t - 1)$

**280**.
$(4x + 10)(4x - 10)$

**282**.
$(11p + 13)(11p - 13)$

**284**.
$(19d + 9)(19d - 9)$

**286**.
$(12b + 5c)(12b - 5c)$

**288**.
$(7n + 12)^2$

**290**.
$(15y + 4)^2$

**292**.
$(5p - 12)^2$

**294**.
$(x + 6)\left(x^2 - 6x + 36\right)$

**296**.
$(5a + 7)\left(25a^2 - 35a + 49\right)$

**298**.
$(4x - 5)\left(16x^2 + 20x + 25\right)$

**300**.
$(5r + 12s)\left(25r^2 - 60rs + 144s^2\right)$

**302**.

$(2c+3)^{-\frac{1}{4}}(-7c-15)$

**304**.

$(x+2)^{-\frac{2}{5}}(19x+10)$

**306**.

$(2z-9)^{-\frac{3}{2}}(27z-99)$

**308**.

$(14x-3)(7x+9)$

**310**.

$(3x+5)(3x-5)$

**312**.

$(2x+5)^2(2x-5)^2$

**314**.

$\left(4z^2+49a^2\right)(2z+7a)(2z-7a)$

**316**.

$\frac{1}{(4x+9)(4x-9)(2x+3)}$

**317**. You can factor the numerator and denominator to see if any of the terms can cancel one another out.

**319**. True. Multiplication and division do not require finding the LCD because the denominators can be combined through those operations, whereas addition and subtraction require like terms.

**321**.

$\frac{y+5}{y+6}$

**323**.

$3b+3$

**325**.

$\frac{x+4}{2x+2}$

**327**.

$\frac{a+3}{a-3}$

**329**.

$\frac{3n-8}{7n-3}$

**331**.

$\frac{c-6}{c+6}$

**333**.

$1$

**335**.

$\frac{d^2-25}{25d^2-1}$

**337**.

$\frac{t+5}{t+3}$

**339**.

$\frac{6x-5}{6x+5}$

**341**.

$\frac{p+6}{4p+3}$

**343**.
$\frac{2d+9}{d+11}$

**345**.
$\frac{12b+5}{3b-1}$

**347**.
$\frac{4y-1}{y+4}$

**349**.
$\frac{10x+4y}{xy}$

**351**.
$\frac{9a-7}{a^2-2a-3}$

**353**.
$\frac{2y^2-y+9}{y^2-y-2}$

**355**.
$\frac{5z^2+z+5}{z^2-z-2}$

**357**.
$\frac{x+2xy+y}{x+xy+y+1}$

**359**.
$\frac{2b+7a}{ab^2}$

**361**.
$\frac{18+ab}{4b}$

**363**.
$a-b$

**365**.
$\frac{3c^2+3c-2}{2c^2+5c+2}$

**367**.
$\frac{15x+7}{x-1}$

**369**.
$\frac{x+9}{x-9}$

**371**.
$\frac{1}{y+2}$

**373**.
4

## Review Exercises

**374**.
$-5$

**376**. 53

**378**.
$y=24$

**380**.
$32m$

**382**. whole
**384**. irrational
**386**.
$16$
**388**.
$a^6$
**390**.
$\frac{x^3}{32y^3}$
**392**.
$a$
**394**.
$1.634 \times 10^7$
**396**. 14
**398**.
$5\sqrt{3}$
**400**.
$\frac{4\sqrt{2}}{5}$
**402**.
$\frac{7\sqrt{2}}{50}$
**404**.
$10\sqrt{3}$
**406**.
$-3$
**408**.
$3x^3 + 4x^2 + 6$
**410**.
$5x^2 - x + 3$
**412**.
$k^2 - 3k - 18$
**414**.
$x^3 + x^2 + x + 1$
**416**.
$3a^2 + 5ab - 2b^2$
**418**.
$9p$
**420**.
$4a^2$
**422**.
$(4a - 3)(2a + 9)$
**424**.
$(x + 5)^2$
**426**.
$(2h - 3k)^2$
**428**.
$(p + 6)\left(p^2 - 6p + 36\right)$
**430**.
$(4q - 3p)\left(16q^2 + 12pq + 9p^2\right)$

**432**.
$(p+3)^{\frac{1}{3}}(-5p-24)$
**434**.
$\frac{x+3}{x-4}$
**436**.
$\frac{1}{2}$
**438**.
$\frac{m+2}{m-3}$
**440**.
$\frac{6x+10y}{xy}$
**442**.
$\frac{1}{6}$

## Practice Test

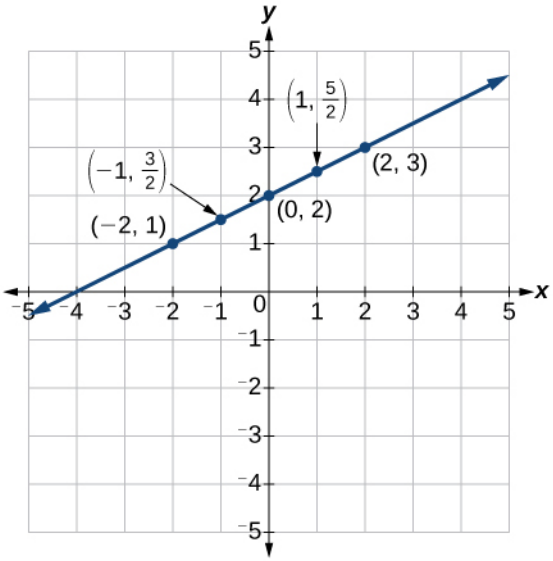
**444**. rational
**446**.
$x=12$
**448**. 3,141,500
**450**.
16
**452**. 9
**454**.
$2x$
**456**. 21
**458**.
$\frac{3\sqrt{x}}{4}$
**460**.
$21\sqrt{6}$
**462**.
$13q^3-4q^2-5q$
**464**.
$n^3-6n^2+12n-8$
**466**.
$(4x+9)(4x-9)$
**468**.
$(3c-11)\left(9c^2+33c+121\right)$
**470**.
$\frac{4z-3}{2z-1}$
**472**.
$\frac{3a+2b}{3b}$

## Chapter 2

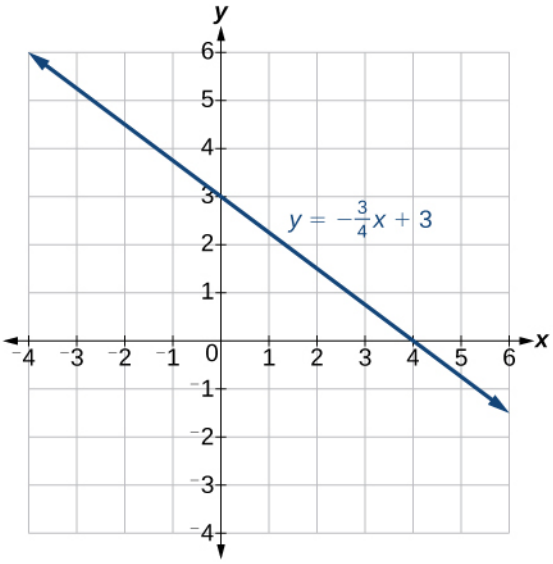
## Try It

**2.1**.

| $x$ | $y = \frac{1}{2}x + 2$ | $(x, y)$ |
|---|---|---|
| $-2$ | $y = \frac{1}{2}(-2) + 2 = 1$ | $(-2, 1)$ |
| $-1$ | $y = \frac{1}{2}(-1) + 2 = \frac{3}{2}$ | $\left(-1, \frac{3}{2}\right)$ |
| $0$ | $y = \frac{1}{2}(0) + 2 = 2$ | $(0, 2)$ |
| $1$ | $y = \frac{1}{2}(1) + 2 = \frac{5}{2}$ | $\left(1, \frac{5}{2}\right)$ |
| $2$ | $y = \frac{1}{2}(2) + 2 = 3$ | $(2, 3)$ |




**2.2**. $x$-intercept is $(4, 0)$; $y$-intercept is $(0, 3)$.




**2.3**.
$\sqrt{125} = 5\sqrt{5}$

**2.4**.
$\left(-5, \frac{5}{2}\right)$

**2.5**.
$x = -5$

**2.6**.
$x = -3$

**2.7**.
$x = \frac{10}{3}$

**2.8**.
$x = 1$

**2.9**.
$x = -\frac{7}{17}$. Excluded values are $x = -\frac{1}{2}$ and $x = -\frac{1}{3}$.

**2.10**.
$x = \frac{1}{3}$

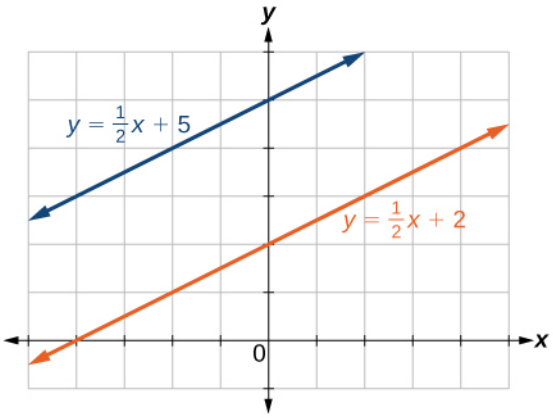
**2.11**.
$m = -\frac{2}{3}$

**2.12**.
$y = 4x - 3$

**2.13**.
$x + 3y = 2$

**2.14**. Horizontal line:
$y = 2$

**2.15**. Parallel lines: equations are written in slope-intercept form.




**2.16**.
$y = 5x + 3$

**2.17**. 11 and 25

**2.18**.
$C = 2.5x + 3,\ 650$

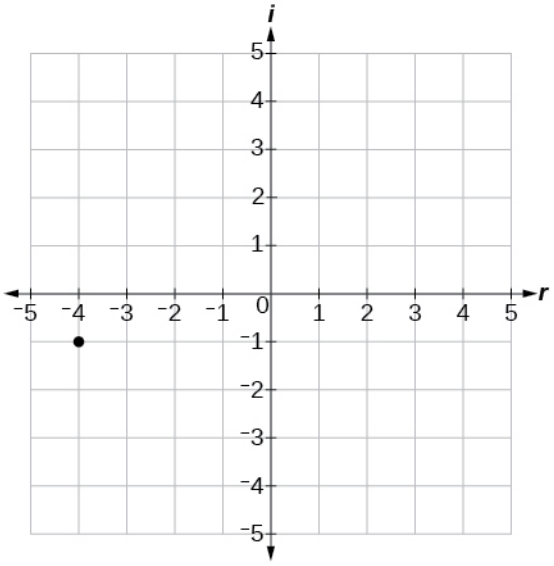
**2.19**. 45 mi/h

**2.20**.
$L = 37$ cm,
$W = 18$ cm

**2.21**. 250 $\text{ft}^2$

**2.22**.
$\sqrt{-24} = 0 + 2i\sqrt{6}$

**2.23**.




**2.24**.
$(3-4i) - (2 + 5i) = 1-9i$

**2.25**.
$\frac{5}{2} - i$

**2.26**.
$18 + i$

**2.27**.
$-3-4i$

**2.28**.
$-1$

**2.29**.
$(x-6)(x+1)=0;\ x=6,\ x=-1$

**2.30**.
$(x-7)(x+3)=0,$

$x=7,$

$x=-3.$

**2.31**.
$(x+5)(x-5)=0,$

$x=-5,$

$x=5.$

**2.32**.
$(3x+2)(4x+1)=0,$

$x=-\frac{2}{3},$

$x=-\frac{1}{4}$

**2.33**.
$x=0,\ x=-10,\ x=-1$

**2.34**.
$x=4\pm\sqrt{5}$

**2.35**.
$x=3\pm\sqrt{22}$

**2.36**.
$x=-\frac{2}{3},$

$x=\frac{1}{3}$

**2.37**.
$5$ units

**2.38**.
$\frac{1}{4}$

**2.39**.
$25$

**2.40**.
$\{-1\}$

**2.41**.
$x=0,$

$x=\frac{1}{2},$

$x=-\frac{1}{2}$

**2.42**.
$x=1;$ extraneous solution

$x=-\frac{2}{9}$

**2.43**.
$x=-2;$ extraneous solution

$x=-1$

**2.44**.
$x=-1,$

$x=\frac{3}{2}$

**2.45**.
$x = -3, 3, -i, i$

**2.46**.
$x = 2, x = 12$

**2.47**.
$x = -1,$
$x = 0$ is not a solution.

**2.48**.
$[-3, 5]$

**2.49**.
$(-\infty, -2) \cup [3, \infty)$

**2.50**.
$x < 1$

**2.51**.
$x \geq -5$

**2.52**.
$(2, \infty)$

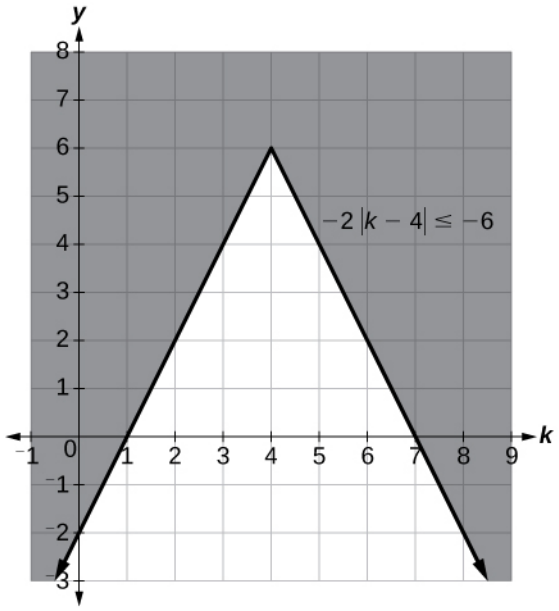
**2.53**.
$\left[-\frac{3}{14}, \infty\right)$

**2.54**.
$6 < x \leq 9$ or $(6, 9]$

**2.55**.
$\left(-\frac{1}{8}, \frac{1}{2}\right)$

**2.56**.
$|x-2| \leq 3$

**2.57**.
$k \leq 1$ or
$k \geq 7;$ in interval notation, this would be
$(-\infty, 1] \cup [7, \infty)$.




## Section Exercises

**1**. Answers may vary. Yes. It is possible for a point to be on the *x*-axis or on the *y*-axis and therefore is considered to NOT be in one of the quadrants.

**3**. The *y*-intercept is the point where the graph crosses the *y*-axis.

**5**. The $x$-intercept is $(2,\ 0)$ and the $y$-intercept is $(0,\ 6)$.

**7**. The $x$-intercept is $(2,\ 0)$ and the $y$-intercept is $(0,\ -3)$.

**9**. The $x$-intercept is $(3,\ 0)$ and the $y$-intercept is $\left(0,\ \frac{9}{8}\right)$.

**11**.
$y = 4 - 2x$

**13**.
$y = \frac{5 - 2x}{3}$

**15**.
$y = 2x - \frac{4}{5}$

**17**.
$d = \sqrt{74}$

**19**.
$d = \sqrt{36} = 6$

**21**.
$d \approx 62.97$

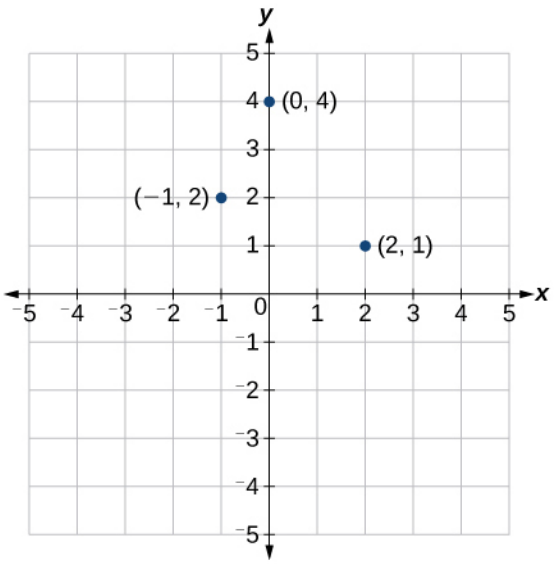
**23**.
$\left(3,\ \frac{-3}{2}\right)$

**25**.
$(2,\ -1)$

**27**.
$(0,\ 0)$

**29**.
$y = 0$

**31**.

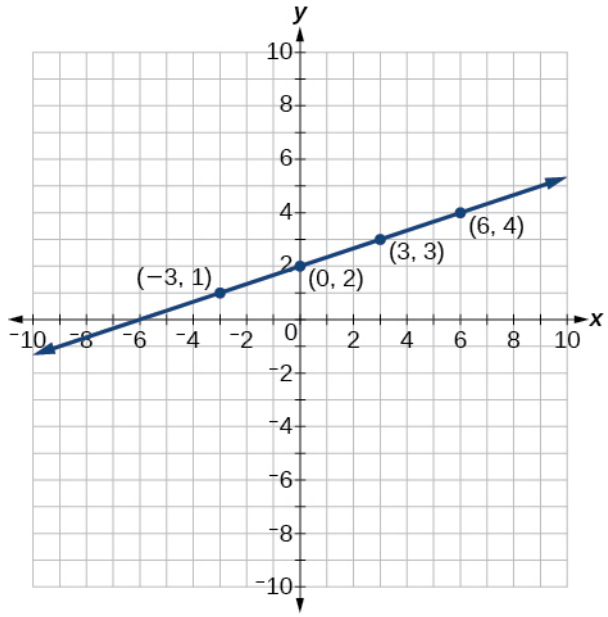



not collinear

**33**.
$(-3, 2), (1, 3), (4, 0)$

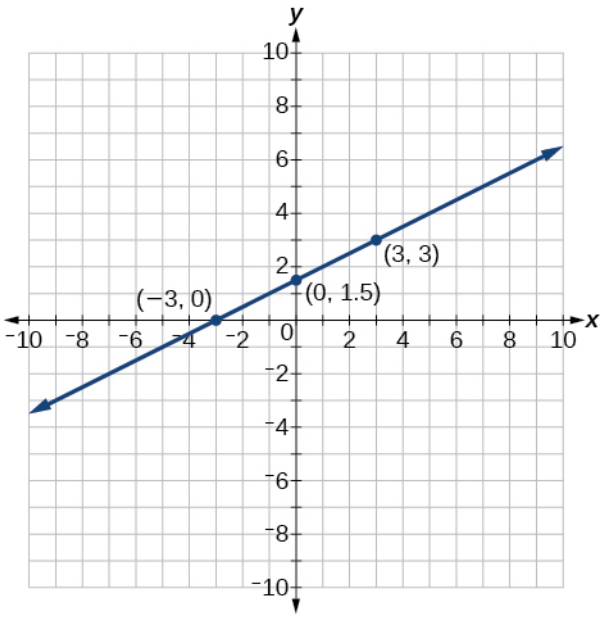
**35**.

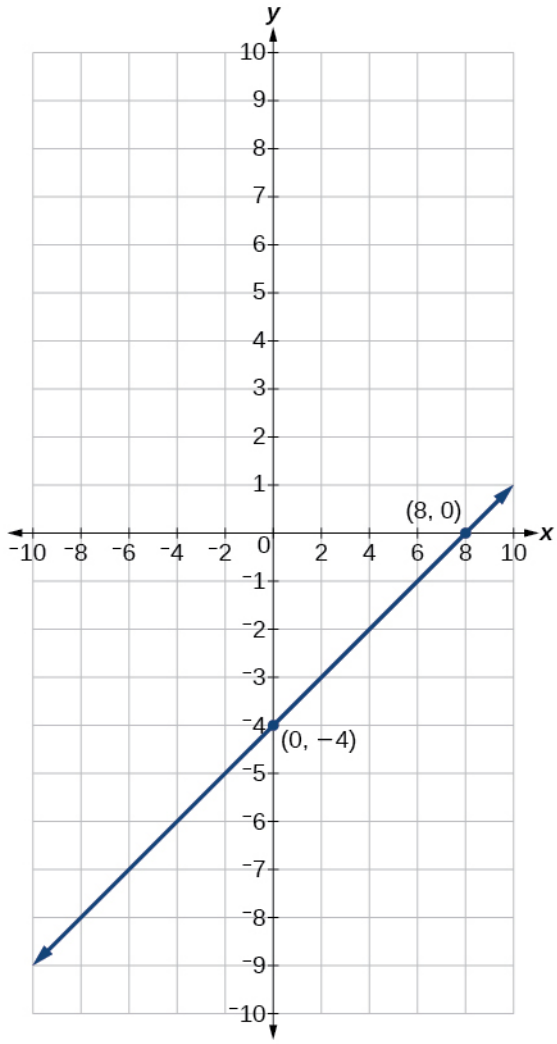
| $x$ | $y$ |
|---|---|
| $-3$ | 1 |
| 0 | 2 |
| 3 | 3 |
| 6 | 4 |



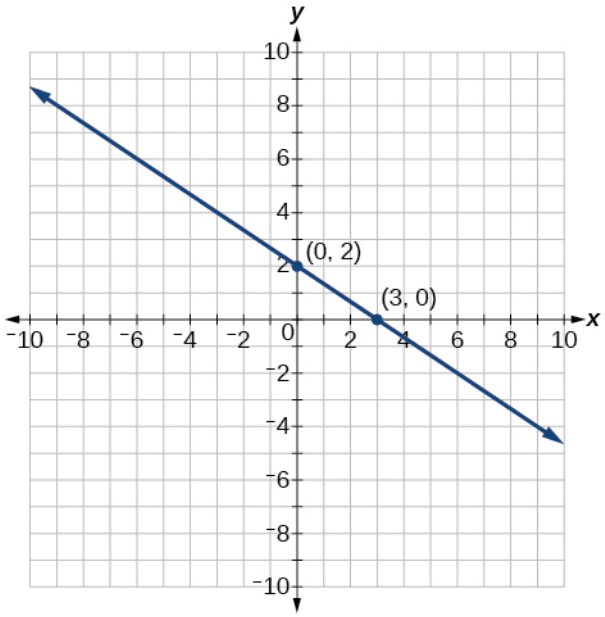

**37**.

| $x$ | $y$ |
|---|---|
| $-3$ | 0 |
| 0 | 1.5 |
| 3 | 3 |




**39**.




**41**.

**43**.
$d = 8.246$

**45**.
$d = 5$

**47**.
$(-3, 4)$

**49**.
$x = 0 \qquad y = -2$

**51**.
$x = 0.75 \quad y = 0$

**53**.
$x = -1.667 \quad y = 0$

**55**.
$15 - 11.2 = 3.8$ mi shorter

**57**.
6.042

**59**. Midpoint of each diagonal is the same point $(2, 2)$. Note this is a characteristic of rectangles, but not other quadrilaterals.

**61**.
37 mi

**63**. 54 ft

**65**. It means they have the same slope.

**67**. The exponent of the $x$ variable is 1. It is called a first-degree equation.

**69**. If we insert either value into the equation, they make an expression in the equation undefined (zero in the denominator).

**71**.
$x = 2$

**73**.
$x = \frac{2}{7}$

**75**.
$x = 6$

**77**.
$x = 3$

**79**.
$x = -14$

**81**.
$x \neq -4;$
$x = -3$

**83**.
$x \neq 1;$ when we solve this we get
$x = 1,$ which is excluded, therefore NO solution

**85**.
$x \neq 0;$
$x = \frac{-5}{2}$

**87**.
$y = \frac{-4}{5}x + \frac{14}{5}$

**89**.
$y = \frac{-3}{4}x + 2$

**91**.
$y = \frac{1}{2}x + \frac{5}{2}$

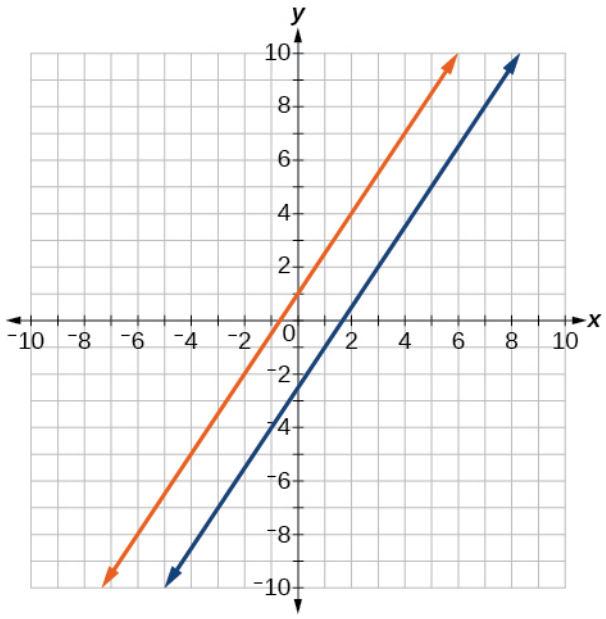
**93**.
$y = -3x - 5$

**95**.
$y = 7$

**97**.
$y = -4$

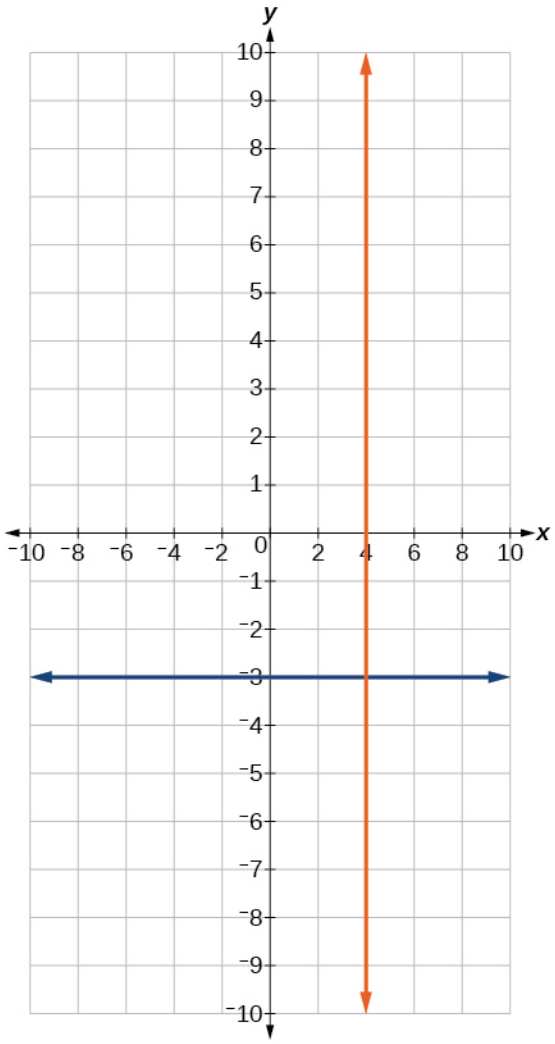
**99**.
$8x + 5y = 7$

**101**.




Parallel

**103**.




Perpendicular

**105**.

$m = \frac{-9}{7}$

**107**.

$m = \frac{3}{2}$

**109**.

$m_1 = \frac{-1}{3}, \quad m_2 = 3; \quad$ Perpendicular.

**111**.

$y = 0.245x - 45.662.$ Answers may vary.

$y_{\min} = -50, \ y_{\max} = -40$

**113**.

$y = -2.333x + 6.667.$ Answers may vary.

$y_{\min} = -10, \quad y_{\max} = 10$

**115**.

$y = \frac{-A}{B}x + \frac{C}{B}$

**117**.
The slope for $(-1,\ 1)$ to $(0,\ 4)$ is 3.
The slope for $(-1,\ 1)$ to $(2,\ 0)$ is $\frac{-1}{3}$.

The slope for $(2,\ 0)$ to $(3,\ 3)$ is 3.
The slope for $(0,\ 4)$ to $(3,\ 3)$ is $\frac{-1}{3}$.
Yes they are perpendicular.

**119**. 30 ft
**121**. $57.50
**123**. 220 mi
**124**. Answers may vary. Possible answers: We should define in words what our variable is representing. We should declare the variable. A heading.
**126**.
$2,\ 000 - x$
**128**.
$v + 10$
**130**. Ann:
$23$; Beth:
$46$
**132**.
$20 + 0.05m$
**134**. 300 min
**136**.
$90 + 40P$
**138**. 6 devices
**140**.
$50,\ 000 - x$
**142**. 4 h
**144**. She traveled for 2 h at 20 mi/h, or 40 miles.
**146**. $5,000 at 8% and $15,000 at 12%
**148**.
$B = 100 + .05x$
**150**. Plan A
**152**.
$R = 9$
**154**.
$r = \frac{4}{5}$ or 0.8
**156**.
$W = \frac{P - 2L}{2} = \frac{58 - 2(15)}{2} = 14$
**158**.
$f = \frac{pq}{p + q} = \frac{8(13)}{8 + 13} = \frac{104}{21}$
**160**.
$m = \frac{-5}{4}$
**162**.
$h = \frac{2A}{b_1 + b_2}$
**164**. length = 360 ft; width = 160 ft
**166**. 405 mi
**168**.
$A = 88 \text{ in.}^2$
**170**. 28.7

**172**.

$h = \frac{V}{\pi r^2}$

**174**.

$r = \sqrt{\frac{V}{\pi h}}$

**176**.

$C = 12\pi$

**178**. Add the real parts together and the imaginary parts together.

**180**. Possible answer:

$i$ times

$i$ equals 1, which is not imaginary.

**182**.

$-8 + 2i$

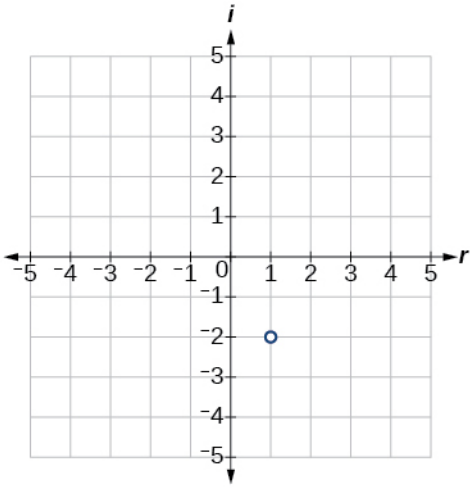
**184**.

$14 + 7i$

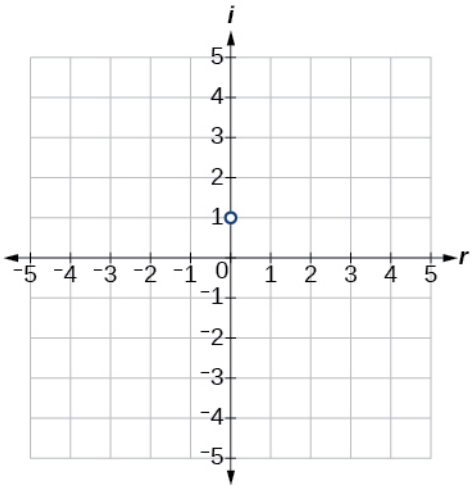
**186**.

$-\frac{23}{29} + \frac{15}{29}i$

**188**.




**190**.



**192**.
$8 - i$
**194**.
$-11 + 4i$
**196**.
$2-5i$
**198**.
$6 + 15i$
**200**.
$-16 + 32i$
**202**.
$-4-7i$
**204**. 25
**206**.
$2 - \frac{2}{3}i$
**208**.
$4 - 6i$
**210**.
$\frac{2}{5} + \frac{11}{5}i$
**212**.
$15i$
**214**.
$1 + i\sqrt{3}$
**216**.
1
**218**.
$-1$
**220**. 128i
**222**.
$\left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)^6 = -1$
**224**.
$3i$
**226**. 0
**228**.
$5-5i$
**230**.
$-2i$
**232**.
$\frac{9}{2} - \frac{9}{2}i$
**234**. It is a second-degree equation (the highest variable exponent is 2).
**236**. We want to take advantage of the zero property of multiplication in the fact that if
$a \cdot b = 0$ then it must follow that each factor separately offers a solution to the product being zero:
$a = 0$ *or* b $= 0$.
**238**. One, when no linear term is present (no *x* term), such as
$x^2 = 16$. Two, when the equation is already in the form
$(ax + b)^2 = d$.
**240**.
$x = 6$,
$x = 3$

**242**.
$x = \frac{-5}{2},$
$x = \frac{-1}{3}$
**244**.
$x = 5,$
$x = -5$
**246**.
$x = \frac{-3}{2},$
$x = \frac{3}{2}$
**248**.
$x = -2,$
**250**.
$x = 0,$
$x = \frac{-3}{7}$
**252**.
$x = -6,$
$x = 6$
**254**.
$x = 6,$
$x = -4$
**256**.
$x = 1,$
$x = -2$
**258**.
$x = -2,$
$x = 11$
**260**.
$x = 3 \pm \sqrt{22}$
**262**.
$z = \frac{2}{3},$
$z = -\frac{1}{2}$
**264**.
$x = \frac{3 \pm \sqrt{17}}{4}$
**266**. Not real
**268**. One rational
**270**. Two real; rational
**272**.
$x = \frac{-1 \pm \sqrt{17}}{2}$
**274**.
$x = \frac{5 \pm \sqrt{13}}{6}$
**276**.
$x = \frac{-1 \pm \sqrt{17}}{8}$
**278**.
$x \approx 0.131$ and
$x \approx 2.535$

**280**.
$x \approx -6.7$ and
$x \approx 1.7$

**282**.

$$\begin{aligned} ax^2+bx+c &= 0 \\ x^2+\frac{b}{a}x &= \frac{-c}{a} \\ x^2+\frac{b}{a}x+\frac{b^2}{4a^2} &= \frac{-c}{a}+\frac{b}{4a^2} \\ \left(x+\frac{b}{2a}\right)^2 &= \frac{b^2-4ac}{4a^2} \\ x+\frac{b}{2a} &= \pm\sqrt{\frac{b^2-4ac}{4a^2}} \\ x &= \frac{-b\pm\sqrt{b^2-4ac}}{2a} \end{aligned}$$

**284**.
$x(x+10)=119$; 7 ft. and 17 ft.

**286**. maximum at
$x=70$

**288**. The quadratic equation would be
$(100x-0.5x^2)-(60x+300)=300.$ The two values of
$x$ are 20 and 60.

**290**. 3 feet

**292**. This is not a solution to the radical equation, it is a value obtained from squaring both sides and thus changing the signs of an equation which has caused it not to be a solution in the original equation.

**294**. He or she is probably trying to enter negative 9, but taking the square root of
$-9$ is not a real number. The negative sign is in front of this, so your friend should be taking the square root of 9, cubing it, and then putting the negative sign in front, resulting in
$-27$.

**296**. A rational exponent is a fraction: the denominator of the fraction is the root or index number and the numerator is the power to which it is raised.

**298**.
$x=81$

**300**.
$x=17$

**302**.
$x=8, \quad x=27$

**304**.
$x=-2,\ 1,\ -1$

**306**.
$y=0, \quad \frac{3}{2}, \quad \frac{-3}{2}$

**308**.
$m=1,\ -1$

**310**.
$x=\frac{2}{5}$

**312**.
$x=32$

**314**.
$t=\frac{44}{3}$

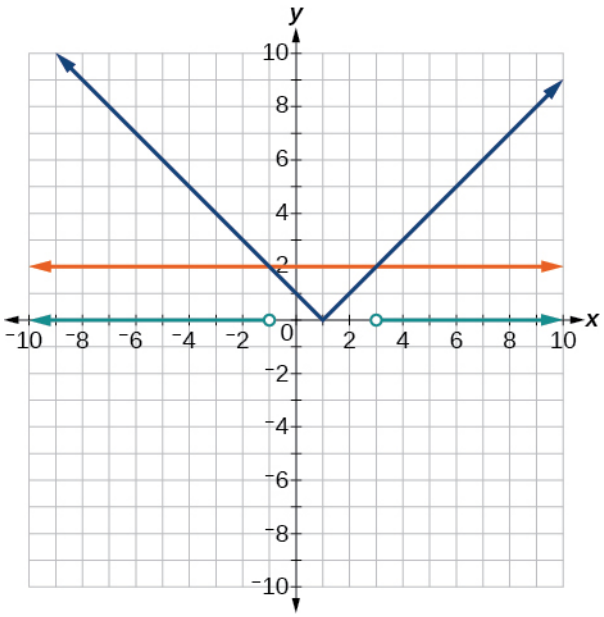
**316**.
$x=3$

**318**.
$x = -2$
**320**.
$x = 4, \frac{-4}{3}$
**322**.
$x = \frac{-5}{4}, \frac{7}{4}$
**324**.
$x = 3, -2$
**326**.
$x = -5$
**328**.
$x = 1, -1, 3, -3$
**330**.
$x = 2, -2$
**332**.
$x = 1, 5$
**334**. All real numbers
**336**.
$x = 4, 6, -6, -8$
**338**. 10 in.
**340**. 90 kg
**341**. When we divide both sides by a negative it changes the sign of both sides so the sense of the inequality sign changes.
**343**.
$(-\infty, \infty)$
**345**. We start by finding the *x*-intercept, or where the function = 0. Once we have that point, which is
$(3, 0),$ we graph to the right the straight line graph
$y = x - 3,$ and then when we draw it to the left we plot positive *y* values, taking the absolute value of them.
**347**.
$\left(-\infty, \frac{3}{4}\right]$
**349**.
$\left[\frac{-13}{2}, \infty\right)$
**351**.
$(-\infty, 3)$
**353**.
$\left(-\infty, -\frac{37}{3}\right]$
**355**. All real numbers
$(-\infty, \infty)$
**357**.
$\left(-\infty, \frac{-10}{3}\right) \cup (4, \infty)$
**359**.
$(-\infty, -4] \cup [8, +\infty)$
**361**. No solution
**363**.
$(-5, 11)$
**365**.
$[6, 12]$
**367**.
$[-10, 12]$

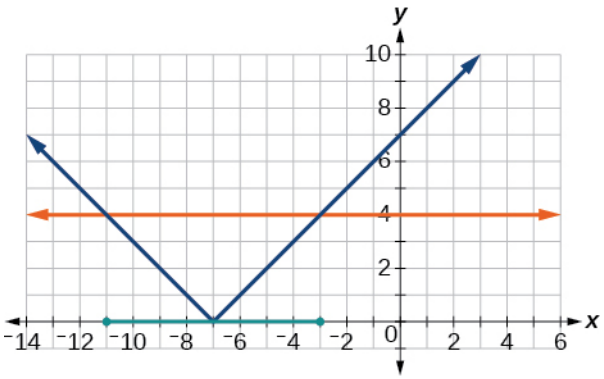
**369**.
$x > -6$ and $x > -2$ Take the intersection of two sets.
$x > -2, \quad (-2, +\infty)$

**371**.
$x < -3$ or $x \geq 1$ Take the union of the two sets.
$(-\infty, -3) \cup [1, \infty)$

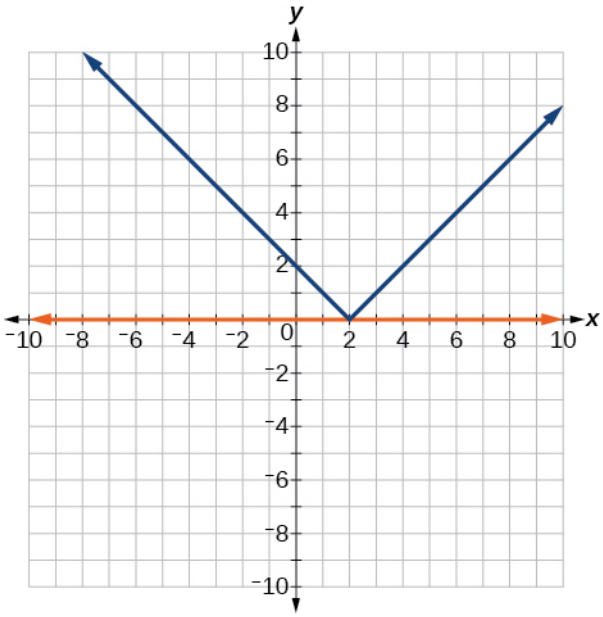
**373**.
$(-\infty, -1) \cup (3, \infty)$




**375**.
$[-11, -3]$

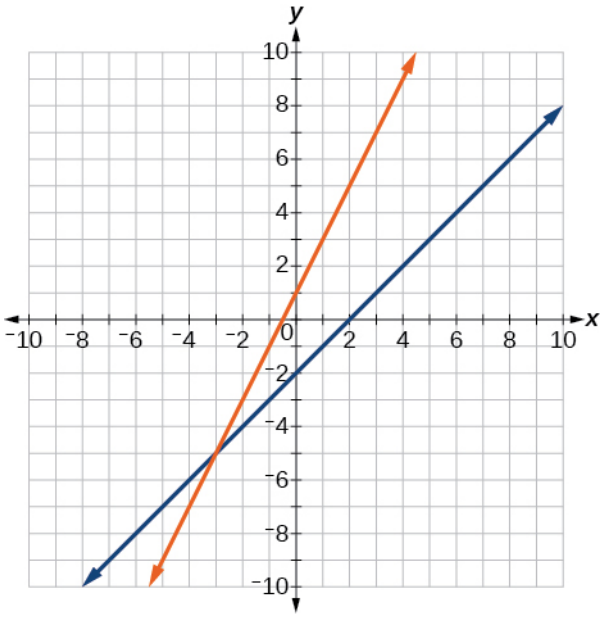


**377**. It is never less than zero. No solution.

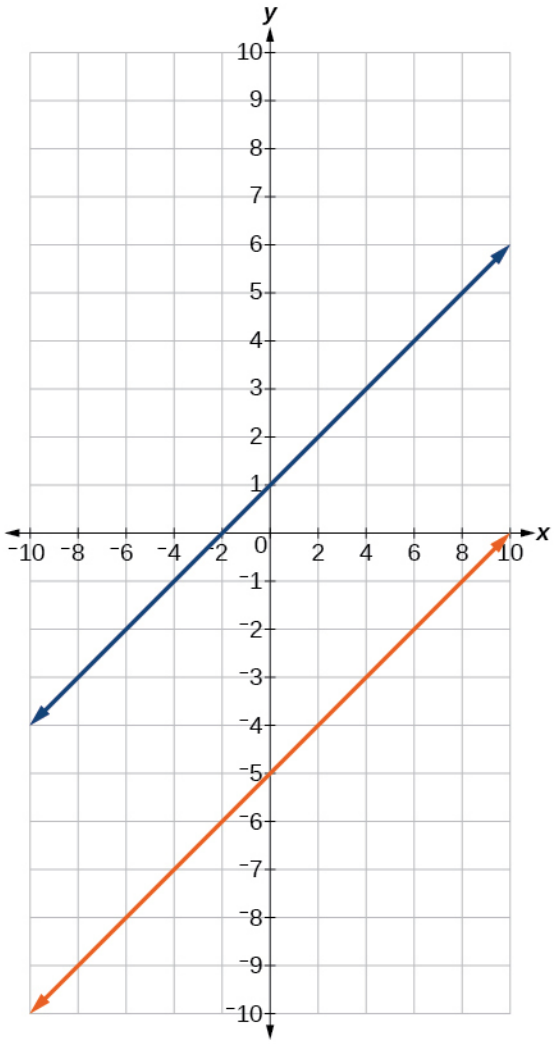



**379**. Where the blue line is above the orange line; point of intersection is $x = -3$.

$(-\infty, -3)$



**381**. Where the blue line is above the orange line; always. All real numbers.
$(-\infty, -\infty)$




**383**.
$(-1, 3)$

**385**.
$(-\infty, 4)$

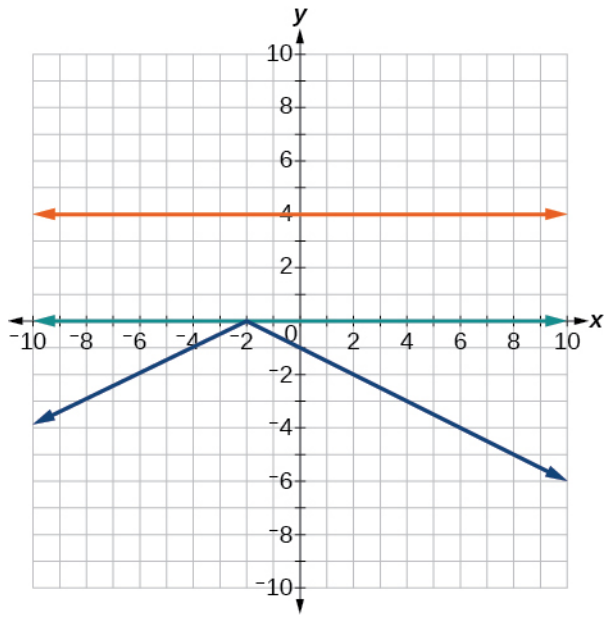
**387**.
$\{x|x < 6\}$

**389**.
$\{x|-3 \leq x < 5\}$

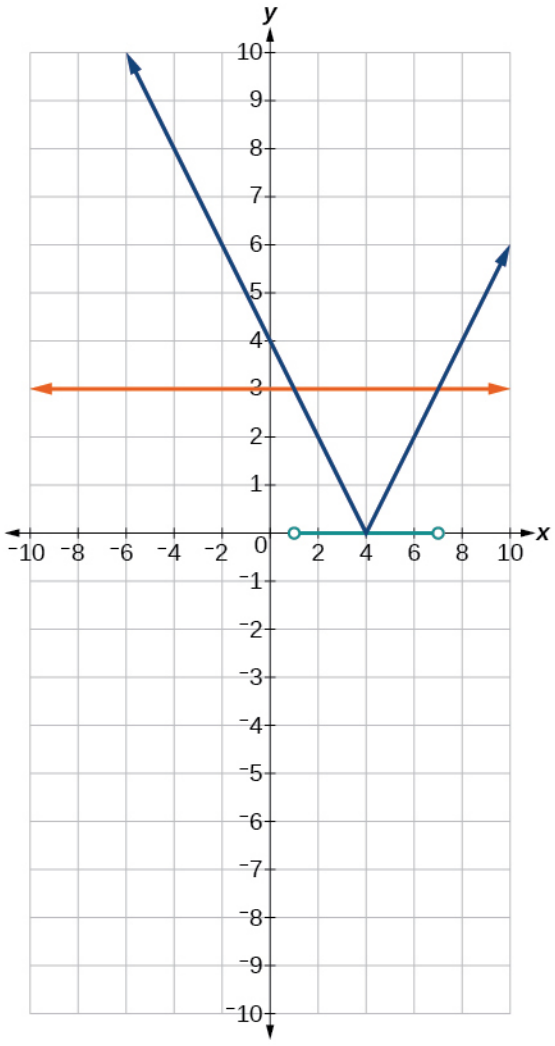
**391**.
$(-2, 1]$

**393**.
$(-\infty, 4]$

**395**. Where the blue is below the orange; always. All real numbers.
$(-\infty, +\infty)$.



**397**. Where the blue is below the orange;
$(1, 7)$.




**399**.
$x = 2, \frac{-4}{5}$

**401**.
$(-7, 5]$

**403**.
$80 \leq T \leq 120$
$1,\ 600 \leq 20T \leq 2,\ 400$
$[1,\ 600,\quad 2,\ 400]$

## Review Exercises

**405**. $x$-intercept:
$(3, 0)$; $y$-intercept:
$(0, -4)$

**407**.
$y = \frac{5}{3}x + 4$

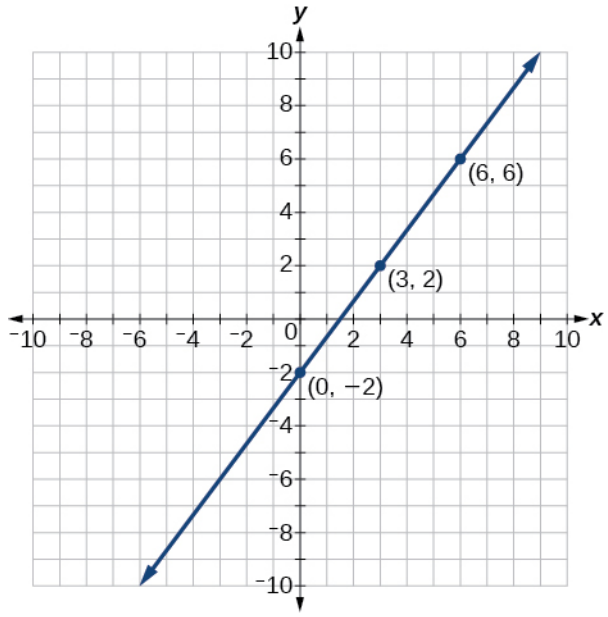
**409**.
$\sqrt{72} = 6\sqrt{2}$

**411**.
620.097

**413**. midpoint is
$\left(2, \frac{23}{2}\right)$

**415**.

| $x$ | $y$ |
|---|---|
| 0 | −2 |
| 3 | 2 |
| 6 | 6 |




**417**.
$x = 4$

**419**.
$x = \frac{12}{7}$

**421**. No solution

**423**.
$y = \frac{1}{6}x + \frac{4}{3}$

**425**.
$y = \frac{2}{3}x + 6$

**427**. females 17, males 56

**429**. 84 mi

**431**.
$x = \frac{-3}{4} \pm \frac{i\sqrt{47}}{4}$

**433**. horizontal component
$-2$; vertical component
$-1$

**435**.
$7 + 11i$

**437**.
$16i$

**439**.
$-16 - 30i$

**441**.
$-4 - i\sqrt{10}$

**443**.
$x = 7 - 3i$

**445**.
$x = -1, -5$

**447**.
$x = 0, \frac{9}{7}$

**449**.
$x = 10, -2$

**451**.
$x = \frac{-1 \pm \sqrt{5}}{4}$

**453**.
$x = \frac{2}{5}, \frac{-1}{3}$

**455**.
$x = 5 \pm 2\sqrt{7}$

**457**.
$x = 0, 256$

**459**.
$x = 0, \pm\sqrt{2}$

**461**.
$x = -2$

**463**.
$x = \frac{11}{2}, \frac{-17}{2}$

**465**.
$(-\infty, 4)$

**467**.
$\left[\frac{-10}{3}, 2\right]$

**469**. No solution

**471**.
$\left(-\frac{4}{3}, \frac{1}{5}\right)$

**473**. Where the blue is below the orange line; point of intersection is
$x = 3.5$.

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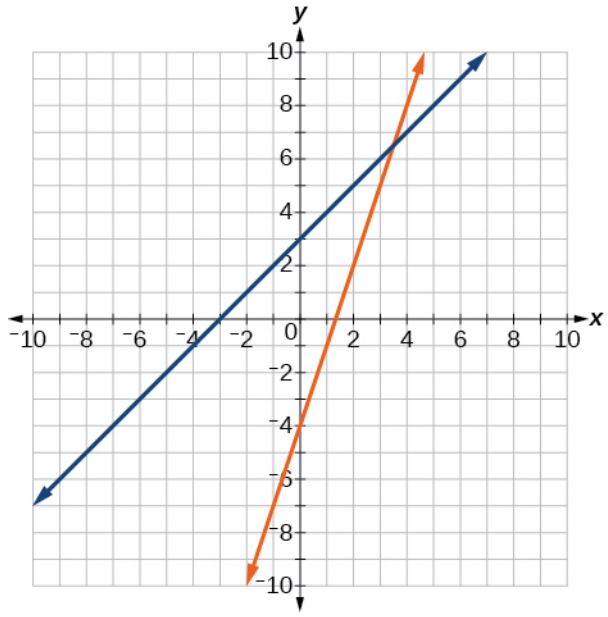
$(3.5, \infty)$

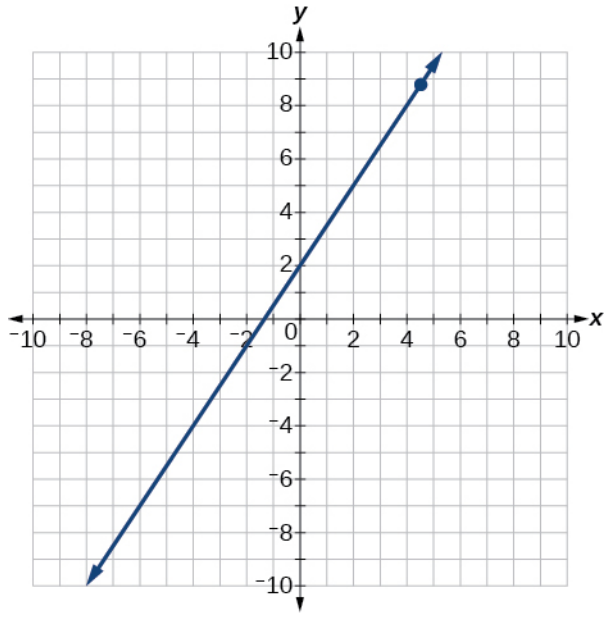



## Practice Test

**474.**

$y = \frac{3}{2}x + 2$

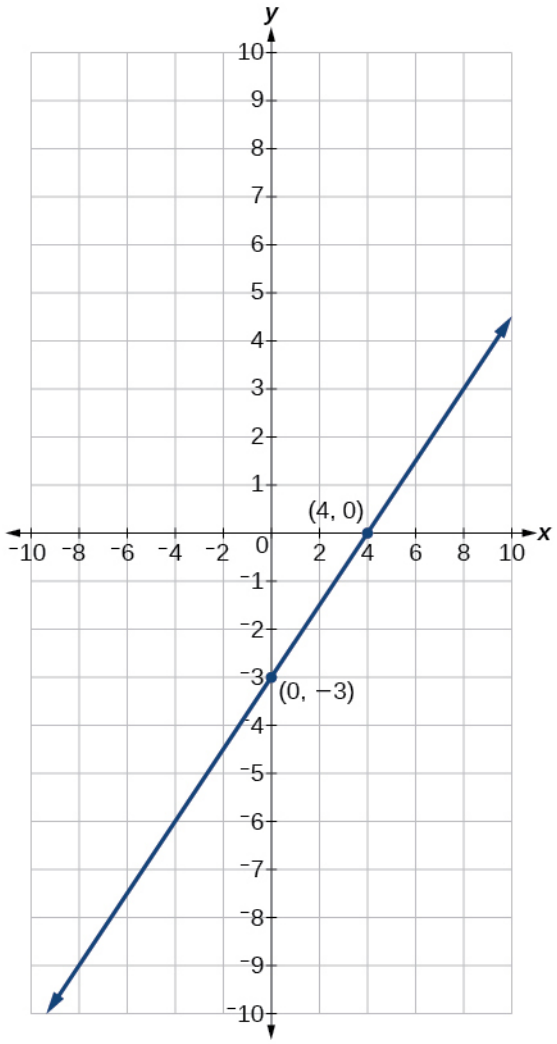
| *x* | *y* |
|---|---|
| 0 | 2 |
| 2 | 5 |
| 4 | 8 |




**475**.
$2x - 5y = 6$

**476**.
$(0, -3)$

(4, 0)




**477**.
$(-\infty, 9]$

**479**.
$x = -15$

**481**.
$x \neq -4, 2;$

$x = \frac{-5}{2}, 1$

**483**.
$x = \frac{3 \pm \sqrt{3}}{2}$

**485**.
$(-4, 1)$

**487**.
$y = \frac{-5}{9}x - \frac{2}{9}$

**489**.
$y = \frac{5}{2}x - 4$

**491**.
$14i$
**493**.
$\frac{5}{13} - \frac{14}{13}i$
**495**.
$x = 2, \frac{-4}{3}$
**497**.
$x = \frac{1}{2} \pm \frac{\sqrt{2}}{2}$
**499**.
4
**501**.
$x = \frac{1}{2}, 2, -2$

## Chapter 3

## Try It

**3.1**. a. yes; b. yes. (Note: If two players had been tied for, say, 4th place, then the name would not have been a function of rank.)
**3.2**.
$w = f(d)$
**3.3**. yes
**3.4**.
$g(5) = 1$
**3.5**.
$m = 8$
**3.6**.
$y = f(x) = \frac{\sqrt[3]{x}}{2}$
**3.7**.
$g(1) = 8$
**3.8**.
$x = 0$ or
$x = 2$
**3.9**. a. yes, because each bank account has a single balance at any given time; b. no, because several bank account numbers may have the same balance; c. no, because the same output may correspond to more than one input.
**3.10**.
a. Yes, letter grade is a function of percent grade;
b. No, it is not one-to-one. There are 100 different percent numbers we could get but only about five possible letter grades, so there cannot be only one percent number that corresponds to each letter grade.

**3.11**. yes
**3.12**. No, because it does not pass the horizontal line test.
**3.13**.
$\{-5, 0, 5, 10, 15\}$
**3.14**.
$(-\infty, \infty)$
**3.15**.
$\left(-\infty, \frac{1}{2}\right) \cup \left(\frac{1}{2}, \infty\right)$
**3.16**.
$\left[-\frac{5}{2}, \infty\right)$
**3.17**.
a. values that are less than or equal to –2, or values that are greater than or equal to –1 and less than 3;
b.
$\{x | x \leq -2 \text{ or } -1 \leq x < 3\}$ ;

c.

$(-\infty, -2] \cup [-1, 3)$

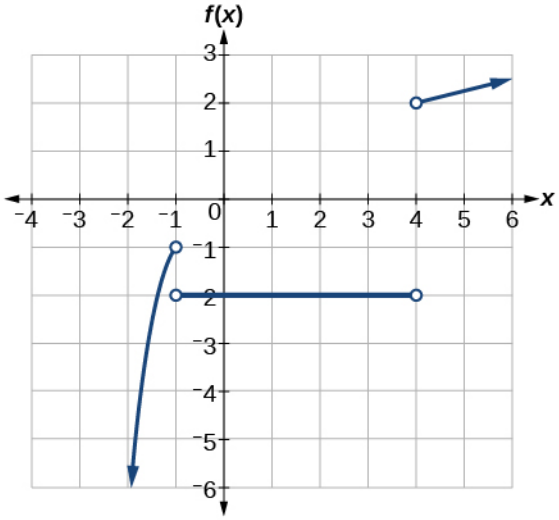
**3.18**. domain =[1950,2002] range = [47,000,000,89,000,000]

**3.19**. domain:

$(-\infty, 2]$; range:

$(-\infty, 0]$

**3.20**.




**3.21**.

$\frac{\$2.84 - \$2.31}{5 \text{ years}} = \frac{\$0.53}{5 \text{ years}} = \$0.106$ per year.

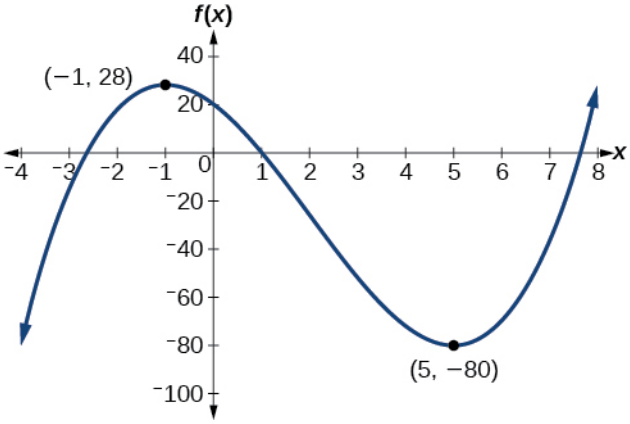
**3.22**.

$\frac{1}{2}$

**3.23**.

$a + 7$

**3.24**. The local maximum appears to occur at $(-1, 28)$, and the local minimum occurs at $(5, -80)$. The function is increasing on $(-\infty, -1) \cup (5, \infty)$ and decreasing on $(-1, 5)$.




**3.25**.

$(fg)(x) = f(x)g(x) = (x-1)\left(x^2-1\right) = x^3 - x^2 - x + 1$

$(f-g)(x) = f(x) - g(x) = (x-1) - \left(x^2-1\right) = x - x^2$

No, the functions are not the same.

**3.26**. A gravitational force is still a force, so
$a(G(r))$ makes sense as the acceleration of a planet at a distance $r$ from the Sun (due to gravity), but
$G(a(F))$ does not make sense.

**3.27**.
$f(g(1)) = f(3) = 3$ and
$g(f(4)) = g(1) = 3$

**3.28**.
$g(f(2)) = g(5) = 3$

**3.29**. a. 8; b. 20

**3.30**.
$[-4,\ 0) \cup (0,\ \infty)$

**3.31**. Possible answer:

$g(x) = \sqrt{4 + x^2}$

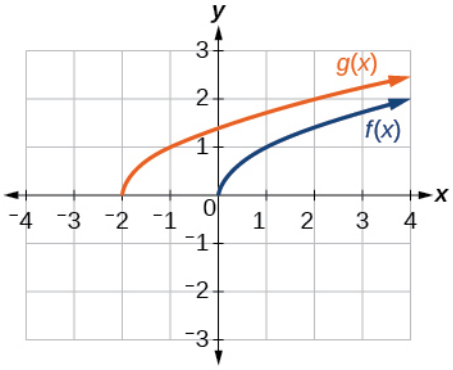
$h(x) = \frac{4}{3 - x}$

$f = h \circ g$

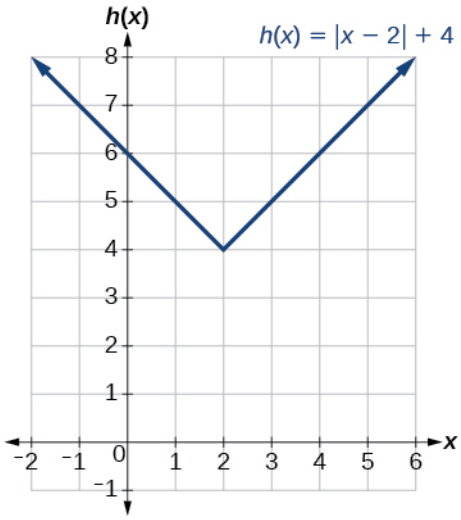
**3.32**. (3.76)

$b(t) = h(t) + 10 = -4.9t^2 + 30t + 10$

**3.33**. The graphs of
$f(x)$ and
$g(x)$ are shown below. The transformation is a horizontal shift. The function is shifted to the left by 2 units.




**3.34**.

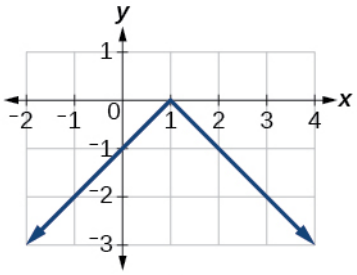



**3.35**.

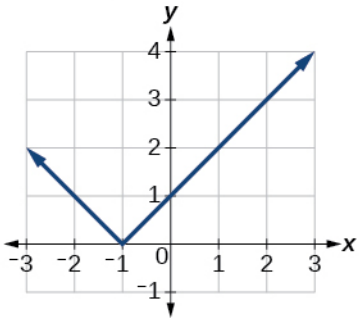
$g(x) = \frac{1}{x - 1} + 1$

**3.36**.

a.




b.




**3.37**.

a.

$g(x) = -f(x)$

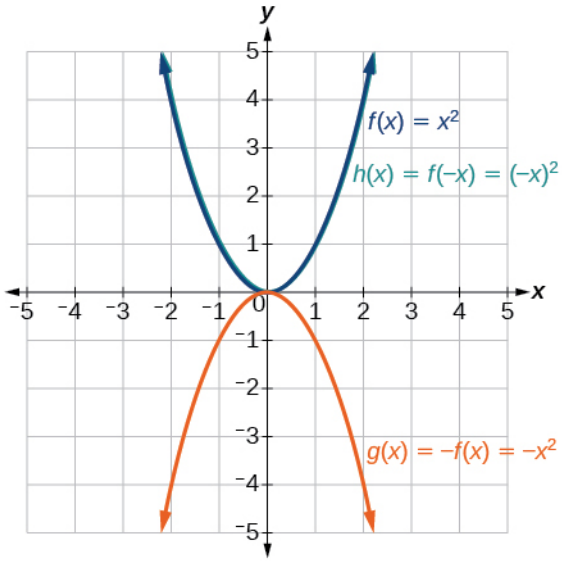
| $x$ | -2 | 0 | 2 | 4 |
|---|---|---|---|---|
| $g(x)$ | $-5$ | $-10$ | $-15$ | $-20$ |

b.

$h(x) = f(-x)$

| $x$ | -2 | 0 | 2 | 4 |
|---|---|---|---|---|
| $h(x)$ | 15 | 10 | 5 | unknown |

**3.38**.




Notice:
$g(x) = f(-x)$ looks the same as
$f(x)$.

**3.39**. even

**3.40**.

| | | | | |
|---|---|---|---|---|
| $x$ | 2 | 4 | 6 | 8 |
| $g(x)$ | 9 | 12 | 15 | 0 |

**3.41**.
$g(x) = 3x - 2$

**3.42**.
$g(x) = f\left(\frac{1}{3}x\right)$ so using the square root function we get
$g(x) = \sqrt{\frac{1}{3}x}$

**3.43**. using the variable
$p$ for passing,
$|p - 80| \leq 20$

**3.44**.
$f(x) = -|x + 2| + 3$

**3.45**.
$x = -1$ or
$x = 2$

**3.46**.
$h(2) = 6$

**3.47**. Yes

**3.48**. Yes

**3.49**. The domain of function
$f^{-1}$ is
$(-\infty, -2)$ and the range of function
$f^{-1}$ is
$(1, \infty)$.

**3.50**.

a.

$f(60) = 50.$ In 60 minutes, 50 miles are traveled.

b.

$f^{-1}(60) = 70.$ To travel 60 miles, it will take 70 minutes.

**3.51**. a. 3; b. 5.6

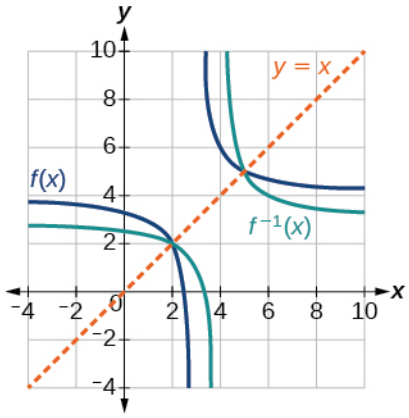
**3.52**.

$x = 3y + 5$

**3.53**.

$f^{-1}(x) = (2 - x)^2;$ domain of $f$ : $[0, \infty);$ domain of $f^{-1}$ : $(-\infty, 2]$

**3.54**.




## Section Exercises

**1**. A relation is a set of ordered pairs. A function is a special kind of relation in which no two ordered pairs have the same first coordinate.

**3**. When a vertical line intersects the graph of a relation more than once, that indicates that for that input there is more than one output. At any particular input value, there can be only one output if the relation is to be a function.

**5**. When a horizontal line intersects the graph of a function more than once, that indicates that for that output there is more than one input. A function is one-to-one if each output corresponds to only one input.

**7**. function

**9**. function

**11**. function

**13**. function

**15**. function

**17**. function

**19**. function

**21**. function

**23**. function

**25**. not a function

**27**.

$f(-3) = -11;\ f(2) = -1;\ f(-a) = -2a - 5;\ -f(a) = -2a + 5;\quad f(a + h) = 2a + 2h - 5$

**29**.

$f(-3) = \sqrt{5} + 5;\ f(2) = 5;\ f(-a) = \sqrt{2 + a} + 5;\ -f(a) = -\sqrt{2 - a} - 5;\quad f(a + h) =$

$\sqrt{2 - a - h} + 5$

**31**.

$f(-3) = 2;\ f(2) = 1 - 3 = -2;\ f(-a) = |-a - 1| - |-a + 1|;\ -f(a) = -|a - 1| + |a + 1|;\quad f(a + h) = |a + h - 1| - |a + h + 1|$

**33**.

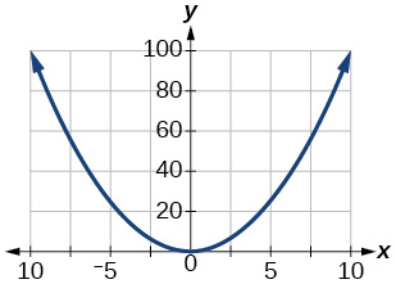
$\frac{g(x) - g(a)}{x - a} = x + a + 2,\ x \neq a$

**35**. a.

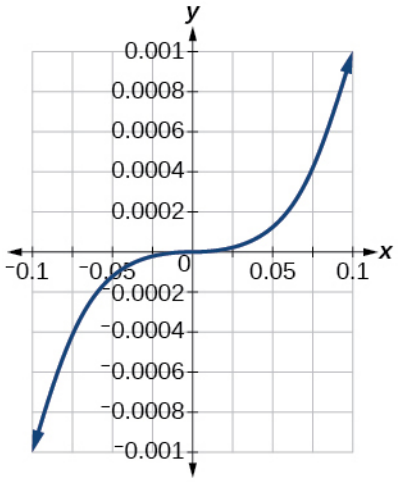
$f(-2) = 14;$ b.

$x = 3$

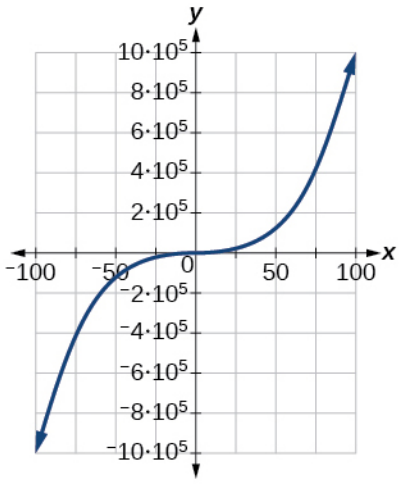
**37**. a.
$f(5) = 10$; b.
$x = -1$ or
$x = 4$
**39**. a.
$f(t) = 6 - \frac{2}{3}t$; b.
$f(-3) = 8$; c.
$t = 6$
**41**. not a function
**43**. function
**45**. function
**47**. function
**49**. function
**51**. function
**53**. a.
$f(0) = 1$; b.
$f(x) = -3,\ x = -2$ or
$x = 2$
**55**. not a function so it is also not a one-to-one function
**57**. one-to- one function
**59**. function, but not one-to-one
**61**. function
**63**. function
**65**. not a function
**67**.
$f(x) = 1,\ x = 2$
**69**.
$f(-2) = 14;\ f(-1) = 11;\ f(0) = 8;\ f(1) = 5;\ f(2) = 2$
**71**.
$f(-2) = 4;\quad f(-1) = 4.414;\ f(0) = 4.732;\ f(1) = 4.5;\ f(2) = 5.236$
**73**.
$f(-2) = \frac{1}{9};\ f(-1) = \frac{1}{3};\ f(0) = 1;\ f(1) = 3;\ f(2) = 9$
**75**. 20
**77**.
$[0,\ 100]$



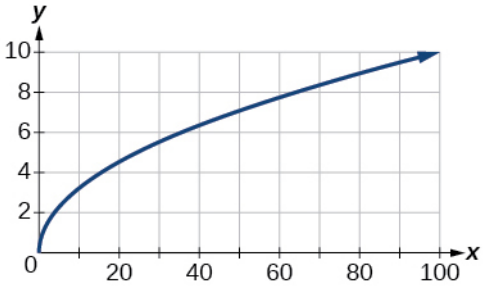

**79**.
$[-0.001,\ 0.001]$



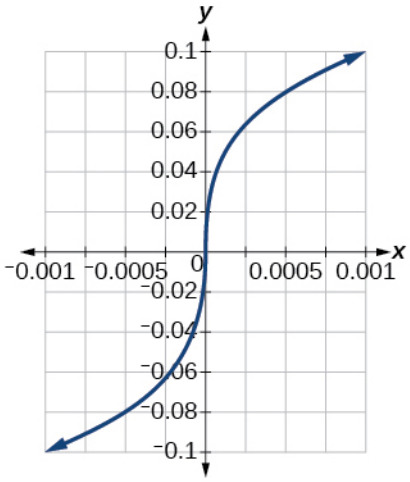

**81**.
$[-1,\ 000,\ 000,\ \ 1{,}000{,}000]$



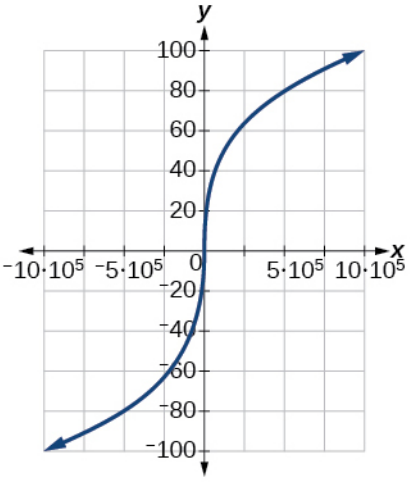

**83**.
$[0,\ \ 10]$




**85**.
$[-0.1,\ 0.1]$




**87**.
$[-100,\ 100]$




**89**. a.
$g(5000) = 50;$ b. The number of cubic yards of dirt required for a garden of 100 square feet is 1.
**91**. a. The height of a rocket above ground after 1 second is 200 ft. b. the height of a rocket above ground after 2 seconds is 350 ft.
**93**. The domain of a function depends upon what values of the independent variable make the function undefined or imaginary.
**95**. There is no restriction on
$x$ for
$f(x) = \sqrt[3]{x}$ because you can take the cube root of any real number. So the domain is all real numbers,
$(-\infty, \infty).$ When dealing with the set of real numbers, you cannot take the square root of negative numbers. So
$x$-values are restricted for
$f(x) = \sqrt{x}$ to nonnegative numbers and the domain is
$[0, \infty)$.
**97**. Graph each formula of the piecewise function over its corresponding domain. Use the same scale for the
$x$-axis and
$y$-axis for each graph. Indicate inclusive endpoints with a solid circle and exclusive endpoints with an open circle. Use an arrow to indicate
$-\infty$ or
$\infty.$ Combine the graphs to find the graph of the piecewise function.
**99**.
$(-\infty, \infty)$
**101**.
$(-\infty, 3]$

**103**.
$(-\infty, \infty)$

**105**.
$(-\infty, \infty)$

**107**.
$(-\infty, -\frac{1}{2}) \cup (-\frac{1}{2}, \infty)$

**109**.
$(-\infty, -11) \cup (-11, 2) \cup (2, \infty)$

**111**.
$(-\infty, -3) \cup (-3, 5) \cup (5, \infty)$

**113**.
$(-\infty, 5)$

**115**.
$[6, \infty)$

**117**.
$(-\infty, -9) \cup (-9, 9) \cup (9, \infty)$

**119**. domain:
$(2, 8]$, range
$[6, 8)$

**121**. domain:
$[-4, 4]$, range:
$[0, 2]$

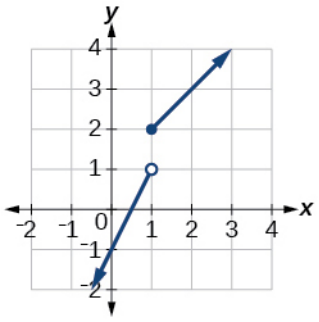
**123**. domain:
$[-5, 3)$, range:
$[0, 2]$

**125**. domain:
$(-\infty, 1]$, range:
$[0, \infty)$

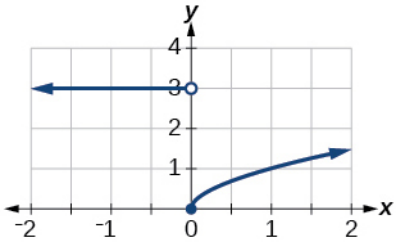
**127**. domain:
$\left[-6, -\frac{1}{6}\right] \cup \left[\frac{1}{6}, 6\right]$; range:
$\left[-6, -\frac{1}{6}\right] \cup \left[\frac{1}{6}, 6\right]$

**129**. domain:
$[-3, \infty)$; range:
$[0, \infty)$

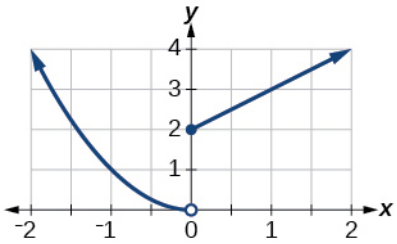
**131**. domain:
$(-\infty, \infty)$



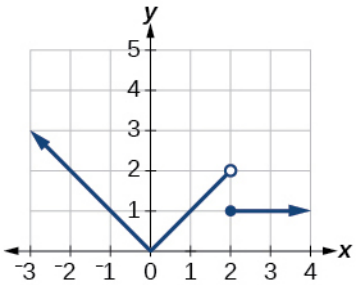

**133**. domain:
$(-\infty, \infty)$




**135**. domain:
$(-\infty, \infty)$




**137**. domain:
$(-\infty, \infty)$



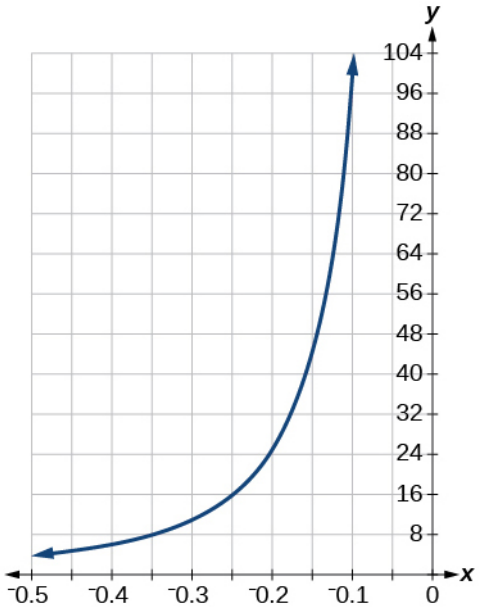

**139**.
$f(-3) = 1;\ f(-2) = 0;\ f(-1) = 0;\ f(0) = 0$

**141**.
$f(-1) = -4;\ f(0) = 6;\ f(2) = 20;\ f(4) = 34$

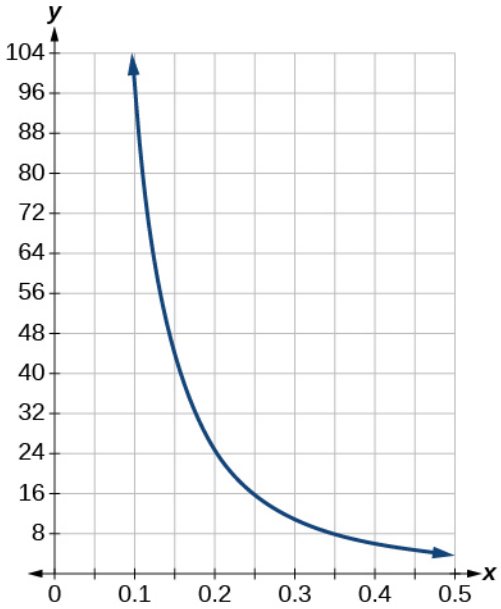
**143**.
$f(-1) = -5;\ f(0) = 3;\ f(2) = 3;\ f(4) = 16$

**145**. domain:
$(-\infty, 1) \cup (1, \infty)$

**147**.




window:
$[-0.5,\ -0.1]$; range:
$[4,\ 100]$




window:
$[0.1,\ 0.5]$; range:
$[4,\ 100]$

**149**.
$[0,\ 8]$

**151**. Many answers. One function is
$f(x) = \frac{1}{\sqrt{x-2}}$.

**152**. The domain is
$[0,\ 6]$; it takes 6 seconds for the projectile to leave the ground and return to the ground

**154**. Yes, the average rate of change of all linear functions is constant.

**156**. The absolute maximum and minimum relate to the entire graph, whereas the local extrema relate only to a specific region around an open interval.

**158**.
$4(b+1)$

**160**. 3

**162**.
$4x+2h$

**164**.
$\frac{-1}{13(13+h)}$

**166**.
$3h^2+9h+9$

**168**.
$4x+2h-3$

**170**.
$\frac{4}{3}$

**172**. increasing on
$(-\infty,\ -2.5)\cup(1,\ \infty)$, decreasing on
$(-2.5,\ 1)$

**174**. increasing on
$(-\infty,\ 1)\cup(3,\ 4)$, decreasing on
$(1,\ 3)\cup(4,\ \infty)$

**176**. local maximum:
$(-3,\ 60)$, local minimum:
$(3,\ -60)$

**178**. absolute maximum at approximately
$(7,\ 150)$, absolute minimum at approximately
$(-7.5,\ -220)$

**180**. a. –3000; b. –1250

**182**. -4

**184**. 27

**186**. –0.167

**188**. Local minimum at
$(3,\ -22)$, decreasing on
$(-\infty,\ 3)$, increasing on
$(3,\ \infty)$

**190**. Local minimum at
$(-2,\ -2)$, decreasing on
$(-3,\ -2)$, increasing on
$(-2,\ \infty)$

**192**. Local maximum at
$(-0.5,\ 6)$, local minima at
$(-3.25,\ -47)$ and
$(2.1,\ -32)$, decreasing on
$(-\infty,\ -3.25)$ and
$(-0.5,\ 2.1)$, increasing on
$(-3.25,\ -0.5)$ and
$(2.1,\ \infty)$

**194**. A

**196**.
$b=5$

**198**. 2.7 gallons per minute

**200**. approximately –0.6 milligrams per day

**201**. Find the numbers that make the function in the denominator $g$ equal to zero, and check for any other domain restrictions on $f$ and $g$, such as an even-indexed root or zeros in the denominator.

**203**. Yes. Sample answer: Let $f(x) = x + 1$ and $g(x) = x - 1$. Then $f(g(x)) = f(x - 1) = (x - 1) + 1 = x$ and $g(f(x)) = g(x + 1) = (x + 1) - 1 = x$. So $f \circ g = g \circ f$.

**205**.

$(f + g)(x) = 2x + 6$, domain: $(-\infty, \infty)$

$(f - g)(x) = 2x^2 + 2x - 6$, domain: $(-\infty, \infty)$

$(fg)(x) = -x^4 - 2x^3 + 6x^2 + 12x$, domain: $(-\infty, \infty)$

$\left(\frac{f}{g}\right)(x) = \frac{x^2 + 2x}{6 - x^2}$, domain: $(-\infty, -\sqrt{6}) \cup (-\sqrt{6}, \sqrt{6}) \cup (\sqrt{6}, \infty)$

**207**.

$(f + g)(x) = \frac{4x^3 + 8x^2 + 1}{2x}$, domain: $(-\infty, 0) \cup (0, \infty)$

$(f - g)(x) = \frac{4x^3 + 8x^2 - 1}{2x}$, domain: $(-\infty, 0) \cup (0, \infty)$

$(fg)(x) = x + 2$, domain: $(-\infty, 0) \cup (0, \infty)$

$\left(\frac{f}{g}\right)(x) = 4x^3 + 8x^2$, domain: $(-\infty, 0) \cup (0, \infty)$

**209**.

$(f + g)(x) = 3x^2 + \sqrt{x - 5}$, domain: $[5, \infty)$

$(f - g)(x) = 3x^2 - \sqrt{x - 5}$, domain: $[5, \infty)$

$(fg)(x) = 3x^2\sqrt{x - 5}$, domain: $[5, \infty)$

$\left(\frac{f}{g}\right)(x) = \frac{3x^2}{\sqrt{x - 5}}$, domain: $(5, \infty)$

**211**. a. 3; b. $f(g(x)) = 2(3x - 5)^2 + 1$; c. $f(g(x)) = 6x^2 - 2$; d. $(g \circ g)(x) = 3(3x - 5) - 5 = 9x - 20$; e. $(f \circ f)(-2) = 163$

**213**.

$f(g(x)) = \sqrt{x^2+3}+2,\ g(f(x)) = x + 4\sqrt{x} + 7$

**215**.

$f(g(x)) = \sqrt[3]{\frac{x+1}{x^3}} = \frac{\sqrt[3]{x+1}}{x},\ g(f(x)) = \frac{\sqrt[3]{x}+1}{x}$

**217**.

$(f \circ g)(x) = \frac{1}{\frac{2}{x}+4-4} = \frac{x}{2},\ (g \circ f)(x) = 2x - 4$

**219**.

$f(g(h(x))) = \left(\frac{1}{x+3}\right)^2 + 1$

**221**. a.

$(g \circ f)(x) = -\frac{3}{\sqrt{2-4x}}$; b.

$\left(-\infty, \frac{1}{2}\right)$

**223**. a.

$(0,\ 2) \cup (2,\ \infty)$; b.

$(-\infty,\ -2) \cup (2,\ \infty)$; c.

$(0,\ \infty)$

**225**.

$(1,\ \infty)$

**227**. sample:

$f(x) = x^3$

$g(x) = x - 5$

**229**. sample:

$f(x) = \frac{4}{x}$

$g(x) = (x+2)^2$

**231**. sample:

$f(x) = \sqrt[3]{x}$

$g(x) = \frac{1}{2x-3}$

**233**. sample:

$f(x) = \sqrt[4]{x}$

$g(x) = \frac{3x-2}{x+5}$

**235**. sample:

$f(x) = \sqrt{x}$

$g(x) = 2x + 6$

**237**. sample:

$f(x) = \sqrt{x}$

$g(x) = (x-1)$

**239**. sample:

$f(x) = x$

$g(x) = \frac{1}{x-2}$

**241**. sample:

$f(x) = \sqrt{x}$

$g(x) = \frac{2x-1}{3x+4}$

**243**. 2

**245**. 5
**247**. 4
**249**. 0
**251**. 2
**253**. 1
**255**. 4
**257**. 4
**259**. 9
**261**. 4
**263**. 2
**265**. 3
**267**. 11
**269**. 0
**271**. 7
**273**.
$f(g(0)) = 27,\ g(f(0)) = -94$
**275**.
$f(g(0)) = \frac{1}{5},\ g(f(0)) = 5$
**277**.
$18x^2 + 60x + 51$
**279**.
$g \circ g(x) = 9x + 20$
**281**. 2
**283**.
$(-\infty, \infty)$
**285**. False
**287**.
$(f \circ g)(6) = 6$;
$(g \circ f)(6) = 6$
**289**.
$(f \circ g)(11) = 11,\ (g \circ f)(11) = 11$
**291**. c
**293**.
$A(t) = \pi\left(25\sqrt{t+2}\right)^2$ and
$A(2) = \pi\left(25\sqrt{4}\right)^2 = 2500\pi$ square inches
**295**.
$A(5) = \pi(2(5)+1)^2 = 121\pi$ square units
**297**. a.
$N(T(t)) = 23(5t + 1.5)^2 - 56(5t + 1.5) + 1;$ b. 3.38 hours
**298**. A horizontal shift results when a constant is added to or subtracted from the input. A vertical shifts results when a constant is added to or subtracted from the output.
**300**. A horizontal compression results when a constant greater than 1 is multiplied by the input. A vertical compression results when a constant between 0 and 1 is multiplied by the output.
**302**. For a function
$f,$ substitute
$(-x)$ for
$(x)$ in
$f(x).$ Simplify. If the resulting function is the same as the original function,
$f(-x) = f(x),$ then the function is even. If the resulting function is the opposite of the original function,
$f(-x) = -f(x),$ then the original function is odd. If the function is not the same or the opposite, then the function is neither odd nor even.

**304**.
$g(x) = |x - 1| - 3$

**306**.
$g(x) = \frac{1}{(x+4)^2} + 2$

**308**. The graph of
$f(x + 43)$ is a horizontal shift to the left 43 units of the graph of
$f$.

**310**. The graph of
$f(x - 4)$ is a horizontal shift to the right 4 units of the graph of
$f$.

**312**. The graph of
$f(x) + 8$ is a vertical shift up 8 units of the graph of
$f$.

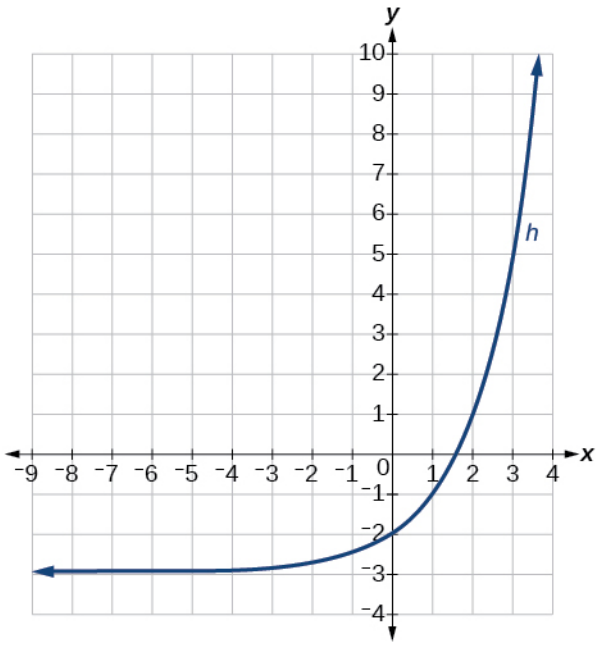
**314**. The graph of
$f(x) - 7$ is a vertical shift down 7 units of the graph of
$f$.

**316**. The graph of
$f(x + 4) - 1$ is a horizontal shift to the left 4 units and a vertical shift down 1 unit of the graph of
$f$.

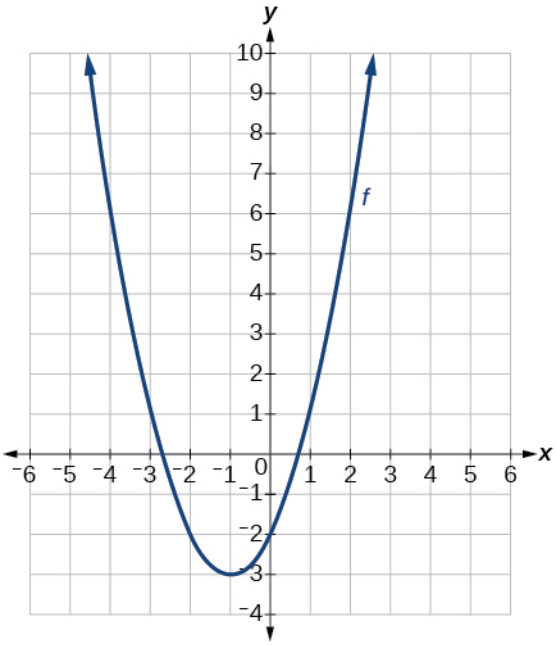
**318**. decreasing on
$(-\infty, -3)$ and increasing on
$(-3, \infty)$

**320**. decreasing on
$(0, \infty)$

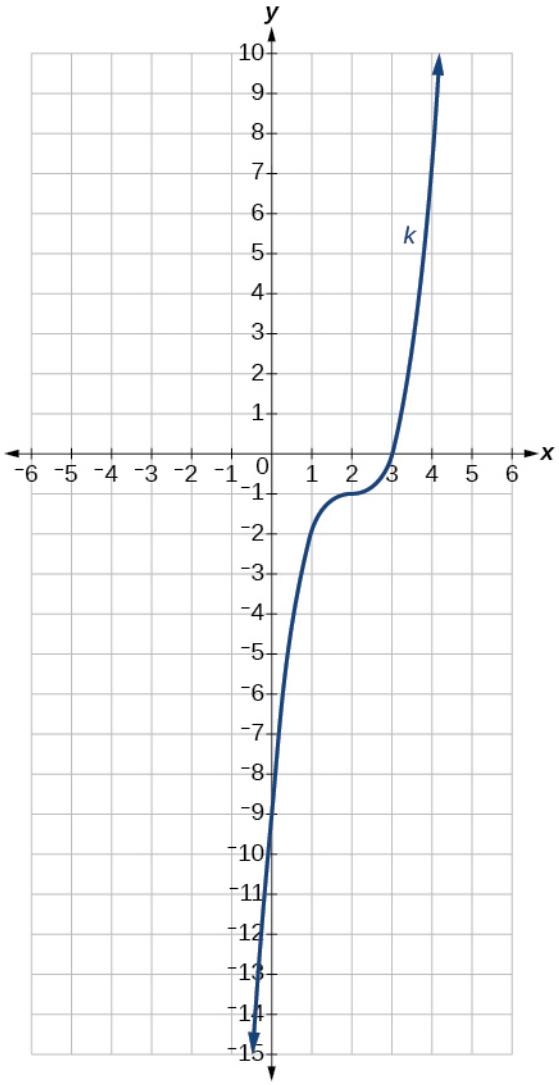
**322**.




**324**.




**326**.



**328**.
$g(x) = f(x - 1),\ h(x) = f(x) + 1$

**330**.
$f(x) = |x - 3| - 2$

**332**.
$f(x) = \sqrt{x + 3} - 1$

**334**.
$f(x) = (x - 2)^2$

**336**.
$f(x) = |x + 3| - 2$

**338**.
$f(x) = -\sqrt{x}$

**340**.
$f(x) = -(x + 1)^2 + 2$

**342**.
$f(x) = \sqrt{-x} + 1$

**344**. even
**346**. odd
**348**. even
**350**. The graph of $g$ is a vertical reflection (across the $x$-axis) of the graph of $f$.

**352**. The graph of $g$ is a vertical stretch by a factor of 4 of the graph of $f$.

**354**. The graph of $g$ is a horizontal compression by a factor of $\frac{1}{5}$ of the graph of $f$.

**356**. The graph of $g$ is a horizontal stretch by a factor of 3 of the graph of $f$.

**358**. The graph of $g$ is a horizontal reflection across the $y$-axis and a vertical stretch by a factor of 3 of the graph of $f$.

**360**.
$g(x) = |-4x|$

**362**.
$g(x) = \frac{1}{3(x + 2)^2} - 3$

**364**.
$g(x) = \frac{1}{2}(x - 5)^2 + 1$

**366**. The graph of the function $f(x) = x^2$ is shifted to the left 1 unit, stretched vertically by a factor of 4, and shifted down 5 units.

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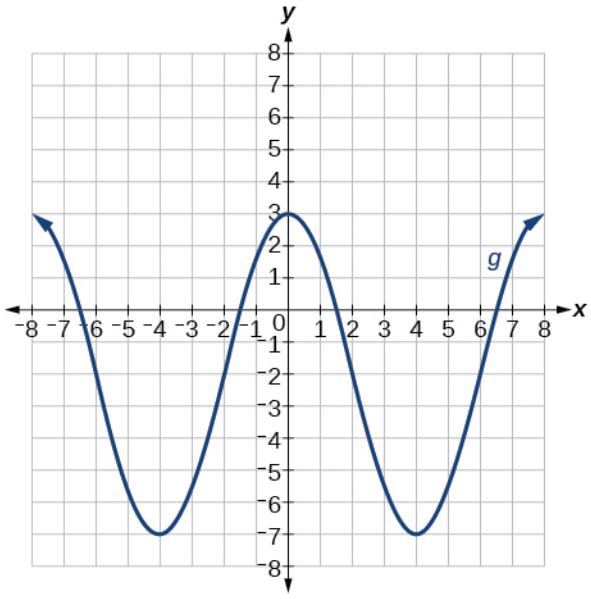
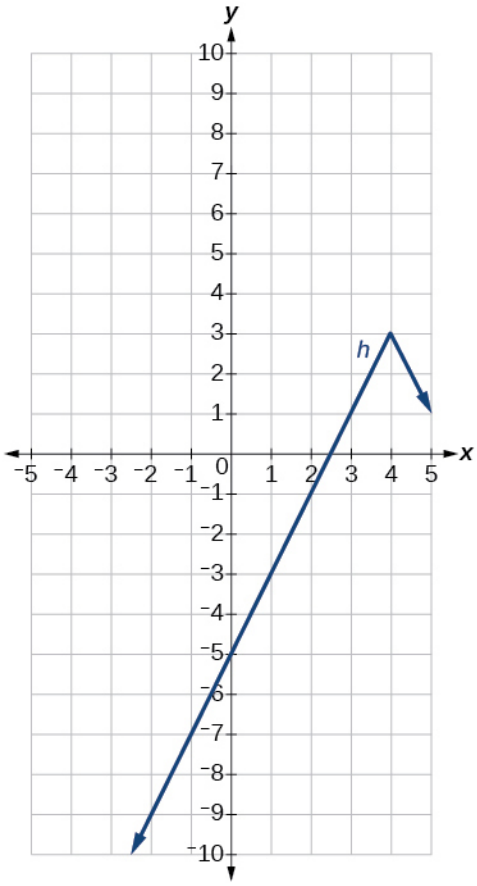
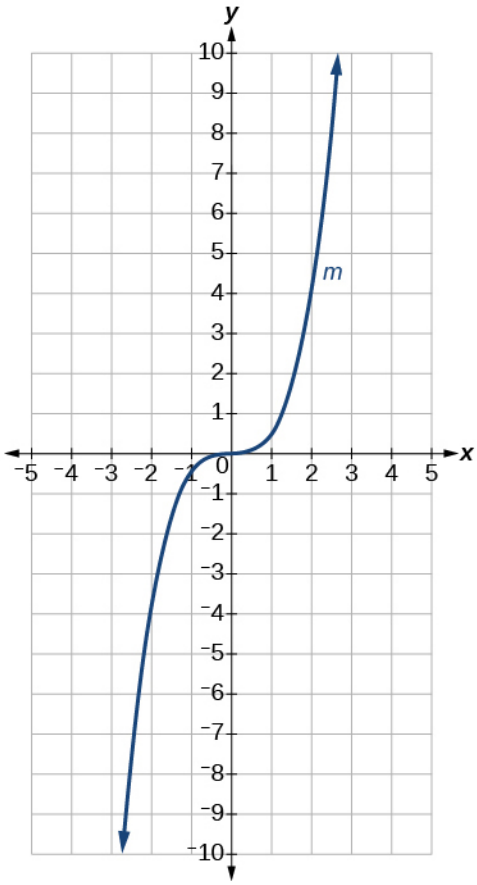


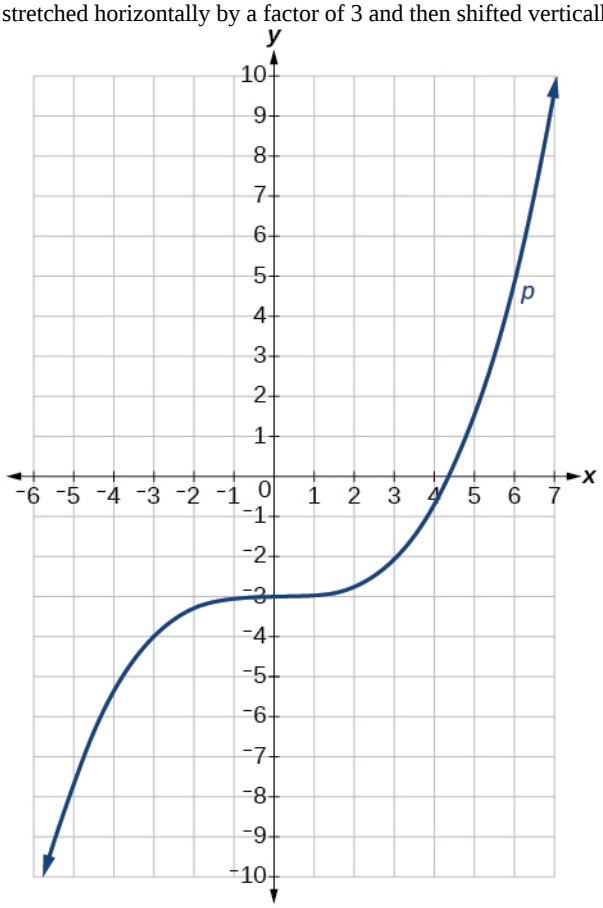

**368**. The graph of $f(x) = |x|$ is stretched vertically by a factor of 2, shifted horizontally 4 units to the right, reflected across the horizontal axis, and then shifted vertically 3 units up.



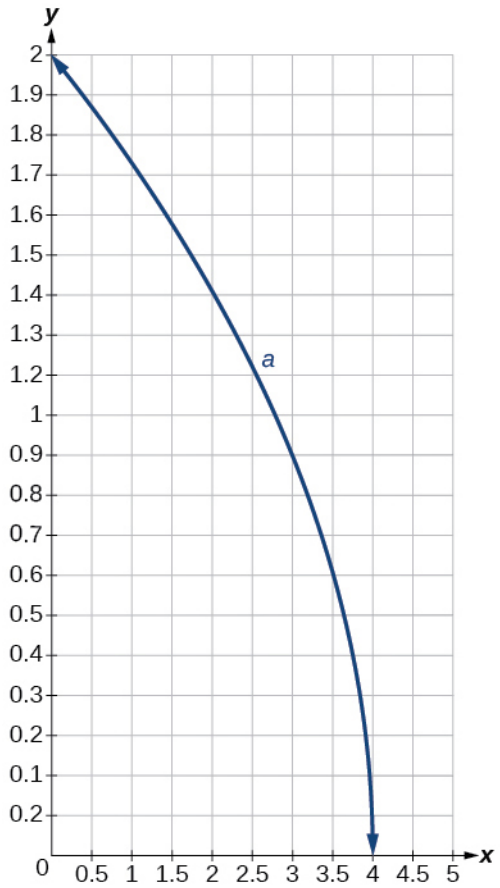

**370**. The graph of the function $f(x) = x^3$ is compressed vertically by a factor of $\frac{1}{2}$.



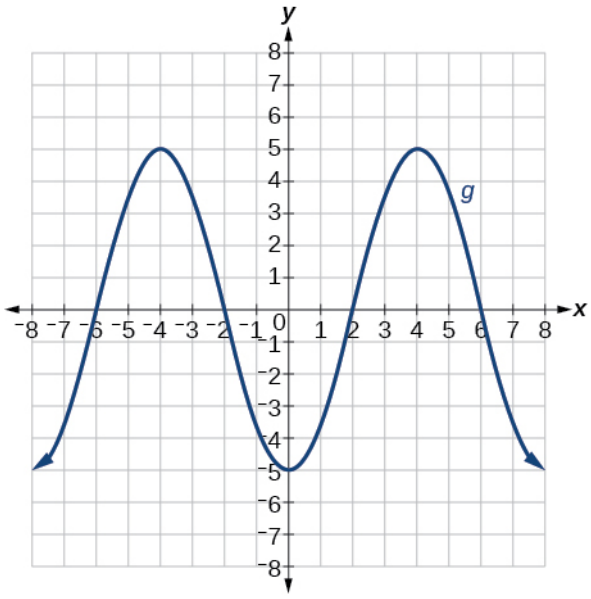

**372**. The graph of the function is stretched horizontally by a factor of 3 and then shifted vertically downward by 3 units.



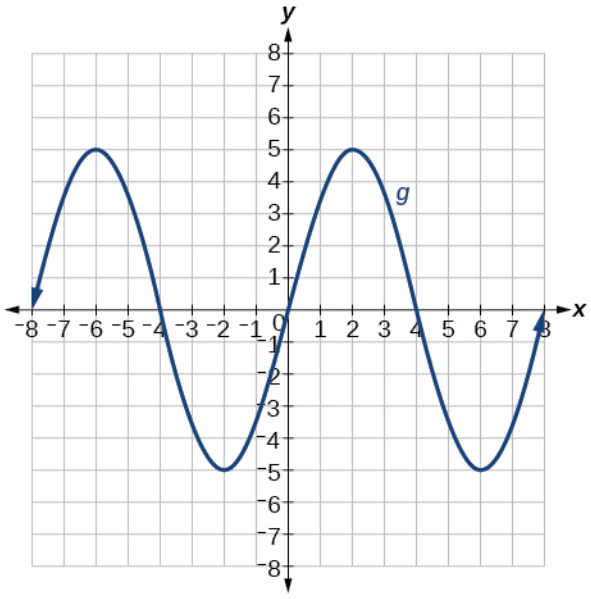
**374**. The graph of $f(x) = \sqrt{x}$ is shifted right 4 units and then reflected across the vertical line $x = 4$.




**376**.




**378**.




**379**. Isolate the absolute value term so that the equation is of the form $|A| = B$. Form one equation by setting the expression inside the absolute value symbol, $A$, equal to the expression on the other side of the equation, $B$. Form a second equation by setting $A$ equal to the opposite of the expression on the other side of the equation, $-B$. Solve each equation for the variable.

**381**. The graph of the absolute value function does not cross the $x$-axis, so the graph is either completely above or completely below the $x$-axis.

**384**.

$|x + 4| = \frac{1}{2}$

**386**.

$|f(x) - 8| < 0.03$

**388**.

$(0, -7)$; no $x$-intercepts

**390**.

$(0, 5), (1, 0), (-5, 0)$

**391**.

$(0, -4), (4, 0), (-2, 0)$

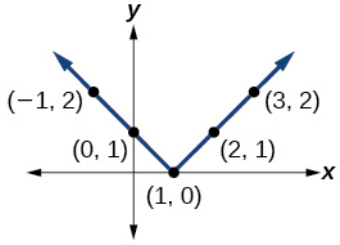
**392**.

$(0, -12), (-6, 0), (7, 0)$

**393**.

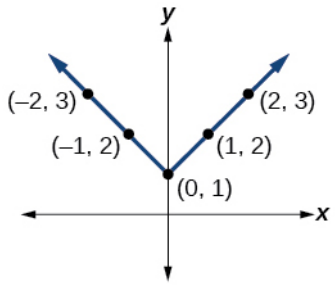
$(0, 7), (25, 0), (-7, 0)$

**394**.

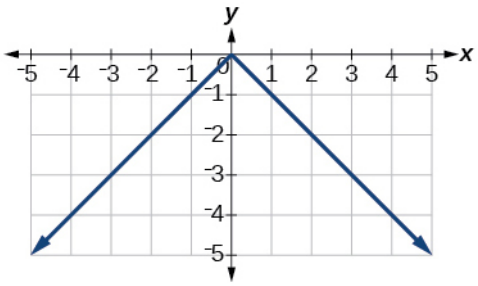



**396**.

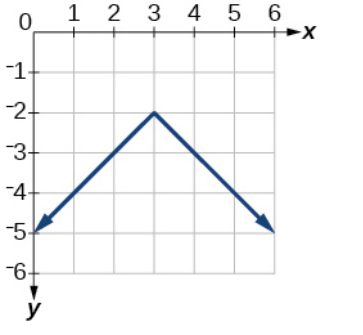
**398**.



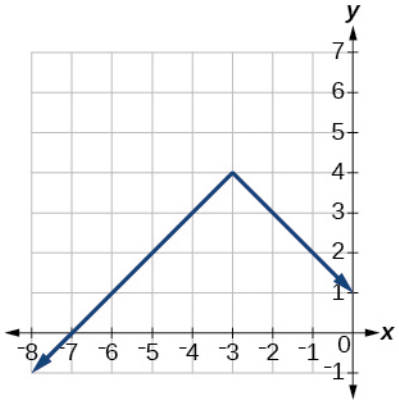

**400**.

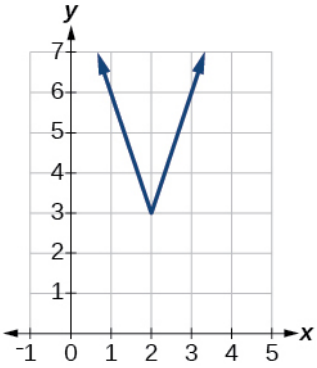



**402**.

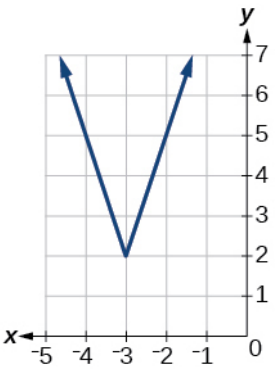



**404**.

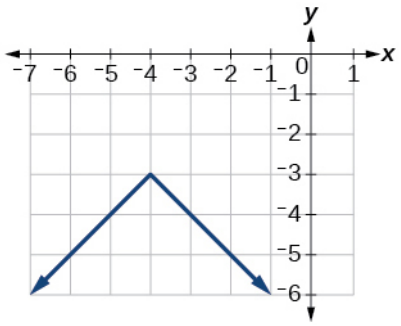


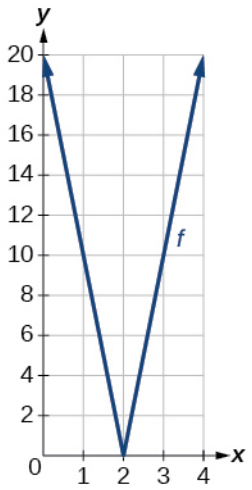
**406**.



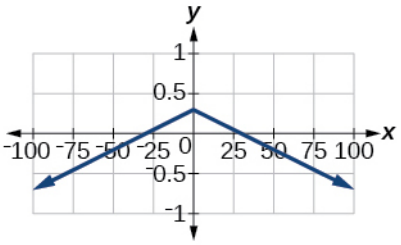

**408**.




**410**. range:
$[0, 20]$




**412**.
$x$- intercepts:




**415**. There is no solution for
$a$ that will keep the function from having a
$y$-intercept. The absolute value function always crosses the
$y$-intercept when
$x = 0$.

**417**.
$|p - 0.08| \leq 0.015$

**419**.
$|x - 5.0| \leq 0.01$

**421**. Each output of a function must have exactly one output for the function to be one-to-one. If any horizontal line crosses the graph of a function more than once, that means that $y$-values repeat and the function is not one-to-one. If no horizontal line crosses the graph of the function more than once, then no $y$-values repeat and the function is one-to-one.

**423**. Yes. For example, $f(x) = \frac{1}{x}$ is its own inverse.

**425**. Given a function $y = f(x)$, solve for $x$ in terms of $y$. Interchange the $x$ and $y$. Solve the new equation for $y$. The expression for $y$ is the inverse, $y = f^{-1}(x)$.

**427**.
$f^{-1}(x) = x - 3$

**429**.
$f^{-1}(x) = 2 - x$

**431**.
$f^{-1}(x) = \frac{-2x}{x - 1}$

**433**. domain of $f(x): [-7, \infty)$; $f^{-1}(x) = \sqrt{x} - 7$

**435**. domain of $f(x): [0, \infty)$; $f^{-1}(x) = \sqrt{x + 5}$

**436**. a. $f(g(x)) = x$ and $g(f(x)) = x$. b. This tells us that $f$ and $g$ are inverse functions

**437**.
$f(g(x)) = x, \ g(f(x)) = x$

**439**. one-to-one

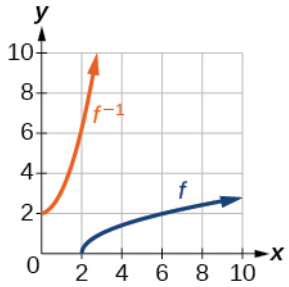
**441**. one-to-one

**443**. not one-to-one

**445**.
3

**447**.
2

**449**.




**451**.
$[2, 10]$

**453**.
6

**455**.
$-4$

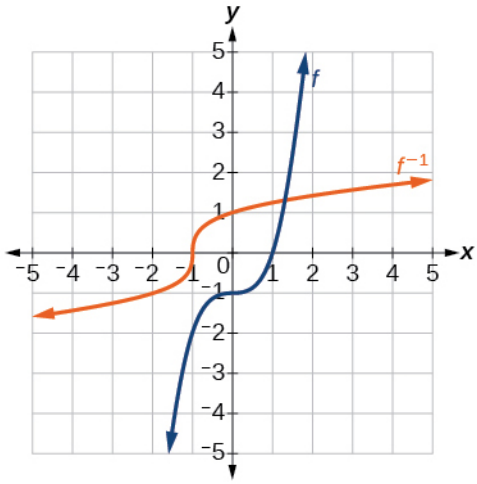
**457**.
0

**459**.
1

**461**.

| $x$ | 1 | 4 | 7 | 12 | 16 |
|---|---|---|---|---|---|
| $f^{-1}(x)$ | 3 | 6 | 9 | 13 | 14 |

**463**.
$f^{-1}(x) = (1 + x)^{1/3}$




**465**.
$f^{-1}(x) = \frac{5}{9}(x - 32)$. Given the Fahrenheit temperature, $x$, this formula allows you to calculate the Celsius temperature.

**467**.
$t(d) = \frac{d}{50}$,

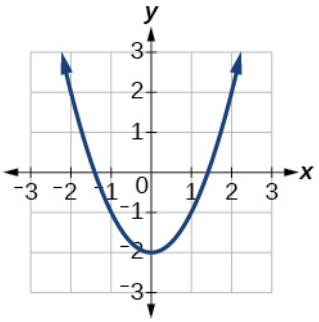
$t(180) = \frac{180}{50}$. The time for the car to travel 180 miles is 3.6 hours.

## Review Exercises

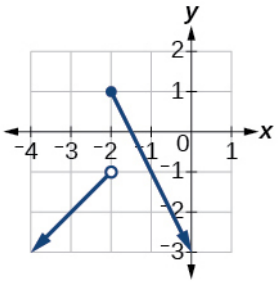
**468**. function

**470**. not a function
**472**.
$f(-3) = -27;$
$f(2) = -2;$
$f(-a) = -2a^2 - 3a;$

$-f(a) = 2a^2 - 3a;$
$f(a+h) = -2a^2 + 3a - 4ah + 3h - 2h^2$
**474**. one-to-one
**476**. function
**478**. function
**480**.




**482**.
2
**484**.
$x = -1.8$ or
or $x = 1.8$
**486**.
$\frac{-64 + 80a - 16a^2}{-1 + a} = -16a + 64$
**488**.
$(-\infty, -2) \cup (-2, 6) \cup (6, \infty)$
**490**.




**492**.
31
**494**. increasing
$(2, \infty)$; decreasing
$(-\infty, 2)$
**496**. increasing
$(-3, 1)$; constant
$(-\infty, -3) \cup (1, \infty)$
**498**. local minimum
$(-2, -3)$; local maximum
$(1, 3)$

**500**.
$(-1.8,\ 10)$

**502**.
$(f \circ g)(x) = 17 - 18x;\ (g \circ f)(x) = -7 - 18x$

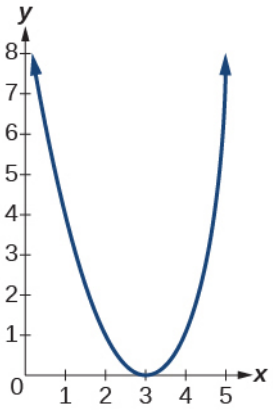
**504**.
$(f \circ g)(x) = \sqrt{\frac{1}{x} + 2};\ (g \circ f)(x) = \frac{1}{\sqrt{x+2}}$

**506**.
$(f \circ g)(x) = \frac{1+x}{1+4x}, \quad x \neq 0, \quad x \neq -\frac{1}{4}$

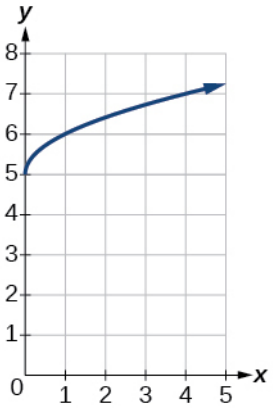
**508**.
$(f \circ g)(x) = \frac{1}{\sqrt{x}},\ x > 0$

**510**. sample:
$g(x) = \frac{2x-1}{3x+4};\ f(x) = \sqrt{x}$

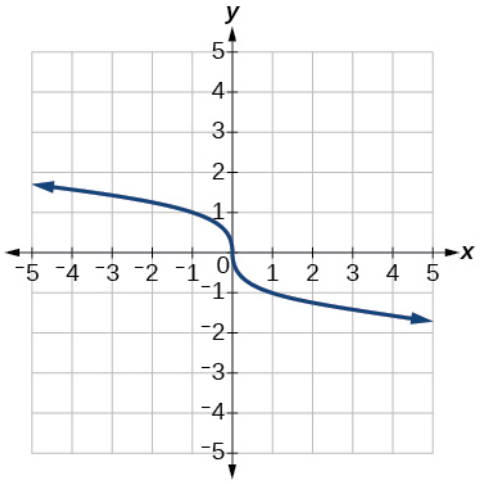
**512**.



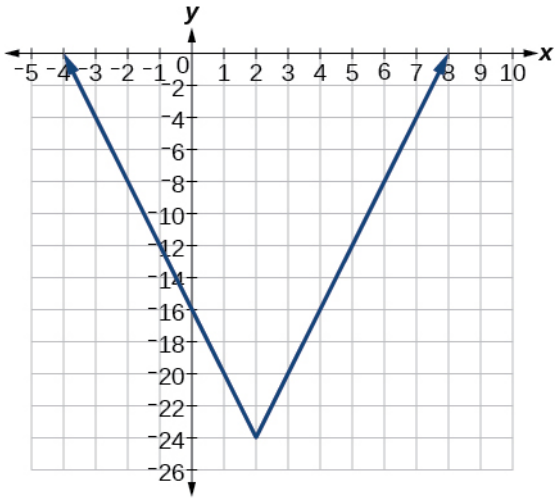

**514**.



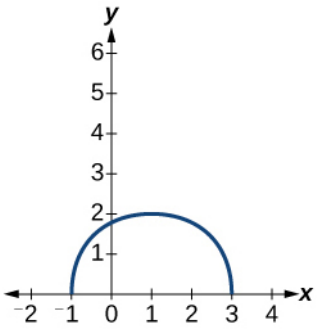

**516**.




**518**.




**520**.

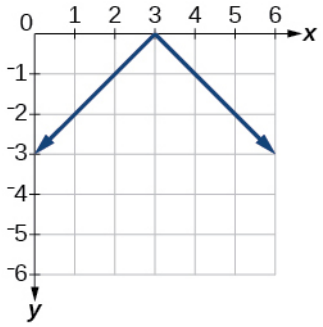



**522**.
$f(x) = |x - 3|$

**524**. even
**526**. odd
**528**. even
**530**.
$f(x) = \frac{1}{2}|x + 2| + 1$

**532**.
$f(x) = -3|x - 3| + 3$

**534**.




**537**.

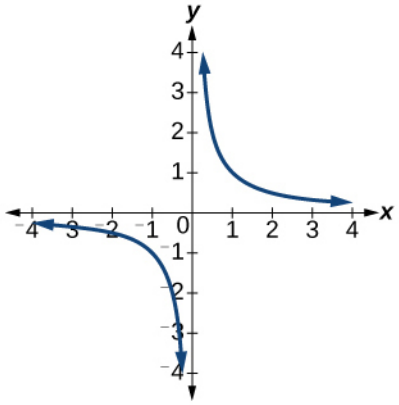
$f^{-1}(x) = \frac{-2x}{x-1}$

**539**.

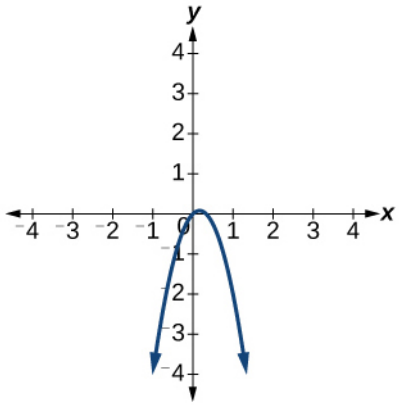
a. $f(g(x)) = x$ and $g(f(x)) = x$.

b. This tells us that $f$ and $g$ are inverse functions

**540**. The function is one-to-one.




**541**. The function is not one-to-one.




**542**.

5

## Practice Test

**544**. The relation is a function.

**546**. −16

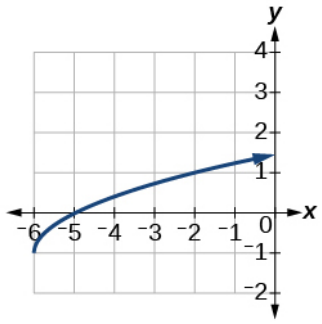
**548**. The graph is a parabola and the graph fails the horizontal line test.

**550**.
$2a^2 - a$

**552**.
$-2(a+b)+1$

**554**.
$\sqrt{2}$

**556**.




**558**.
even

**560**.
odd

**562**.
$f^{-1}(x) = \frac{x+5}{3}$

**564**.
$(-\infty, -1.1)$ and $(1.1, \infty)$

**566**.
$(1.1, -0.9)$

**568**.
$f(2) = 2$

**570**.
$f(x) = \begin{cases} |x| & \text{if } x \le 2 \\ 3 & \text{if } x > 2 \end{cases}$

**572**.
$x = 2$

**574**. yes

**576**.
$f^{-1}(x) = -\frac{x-11}{2}$

## Chapter 4

## Try It

**4.1**.
$m = \frac{4-3}{0-2} = \frac{1}{-2} = -\frac{1}{2}$; decreasing because $m < 0$.

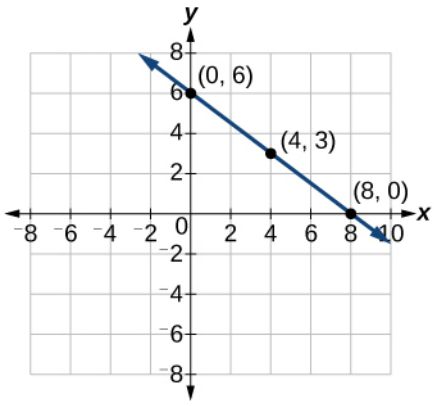
**4.2**. (4.10)

$$m = \frac{1,\,868 - 1,\,442}{2,\,012 - 2,\,009} = \frac{426}{3} = 142 \text{ people per year}$$

**4.3**.
$y = -7x + 3$

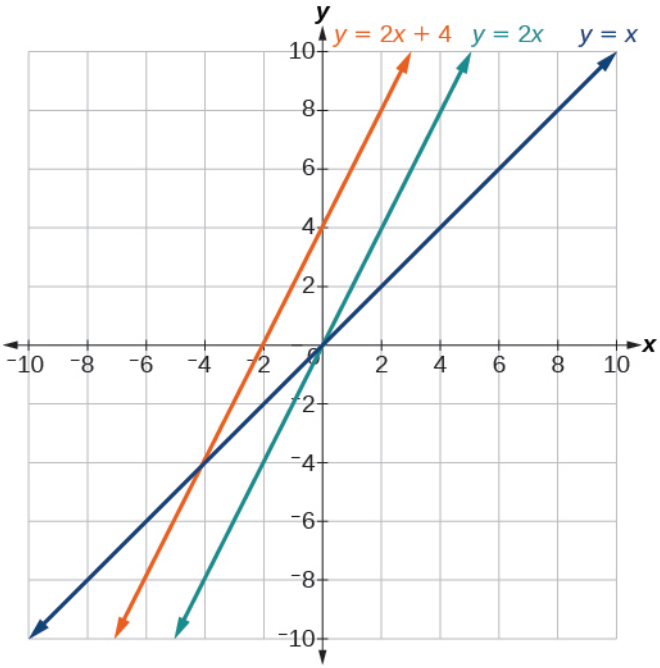
**4.4**.
$H(x) = 0.5x + 12.5$

**4.5**.




**4.6**. Possible answers include $(-3, 7)$, $(-6, 9)$, or $(-9, 11)$.

**4.7**.




**4.8**. $(16, 0)$

**4.9**. a. $f(x) = 2x$; b. $g(x) = -\frac{1}{2}x$

**4.10**. $y = -\frac{1}{3}x + 6$

**4.11**. a. $C(x) = 0.25x + 25,000$ b. The $y$-intercept is $(0, 25,000)$. If the company does not produce a single doughnut, they still incur a cost of $25,000.

**4.12**.

a. 41,100
b. 2020

**4.13**. 21.15 miles

**4.14**.
$54\ ^\circ\text{F}$

**4.15**. 150.871 billion gallons; extrapolation

## Section Exercises

**1**. Terry starts at an elevation of 3000 feet and descends 70 feet per second.
**3**.
$d(t) = 100 - 10t$

**5**. The point of intersection is
$(a,\ a)$. This is because for the horizontal line, all of the
$y$ coordinates are
$a$ and for the vertical line, all of the
$x$ coordinates are
$a$. The point of intersection is on both lines and therefore will have these two characteristics.
**7**. Yes
**9**. Yes
**11**. No
**13**. Yes
**15**. Increasing
**17**. Decreasing
**19**. Decreasing
**21**. Increasing
**23**. Decreasing
**25**. 2
**27**. –2
**29**.
$y = \frac{3}{5}x - 1$

**31**.
$y = 3x - 2$

**33**.
$y = -\frac{1}{3}x + \frac{11}{3}$

**35**.
$y = -1.5x - 3$

**37**. perpendicular
**39**. parallel
**41**.
$f(0) = -(0) + 2$
$f(0) = 2$
$y - \text{int} : (0,\ 2)$

$0 = -x + 2$
$x - \text{int} : (2,\ 0)$

**43**.
$h(0) = 3(0) - 5$
$h(0) = -5$
$y - \text{int} : (0,\ -5)$

$0 = 3x - 5$
$x - \text{int} : \left(\frac{5}{3},\ 0\right)$

**45**.
$-2x + 5y = 20$
$-2(0) + 5y = 20$
$5y = 20$
$y = 4$
$y - \text{int} : (0, 4)$
$-2x + 5(0) = 20$
$x = -10$
$x - \text{int} : (-10, 0)$

**47**. Line 1: $m = -10$ Line 2: $m = -10$ Parallel
**49**. Line 1: $m = -2$ Line 2: $m = 1$ Neither
**51**.
Line 1 : $m = -2$ Line 2 : $m = -2$ Parallel
**53**.
$y = 3x - 3$
**55**.
$y = -\frac{1}{3}t + 2$
**57**. 0
**59**.
$y = -\frac{5}{4}x + 5$
**61**.
$y = 3x - 1$
**63**.
$y = -2.5$
**65**. F
**67**. C
**69**. A
**71**.

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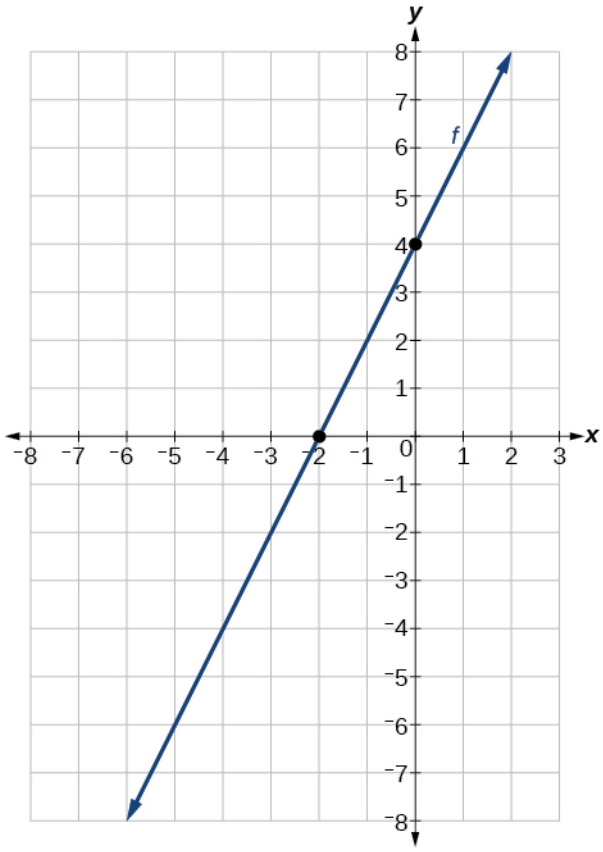
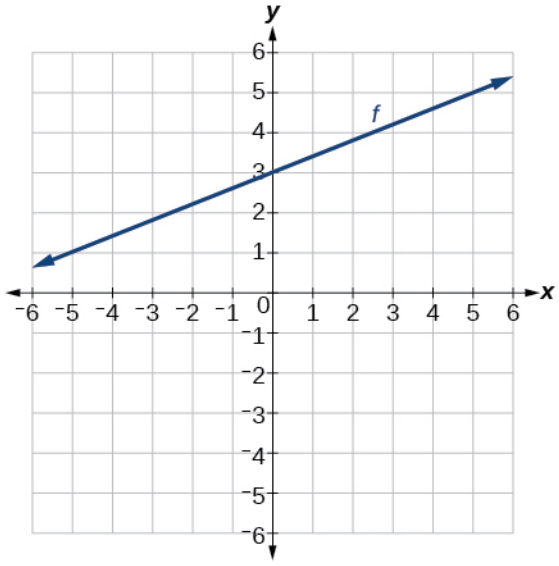
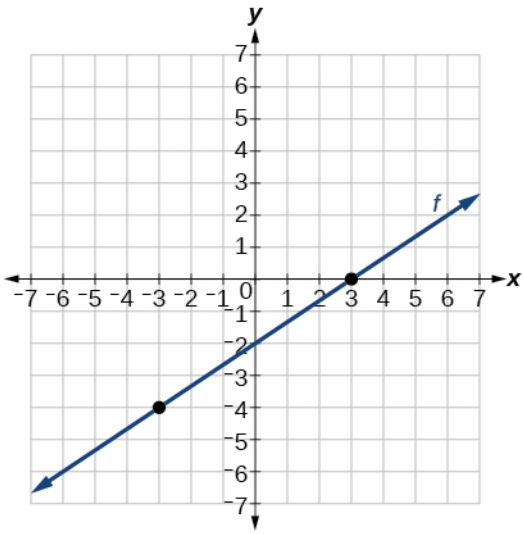

y
x
f
8
7
6
5
4
3
2
1
0
-1
-2
-3
-4
-5
-6
-7
-8
-8 -7 -6 -5 -4 -3 -2 -1 1 2 3

**73**.

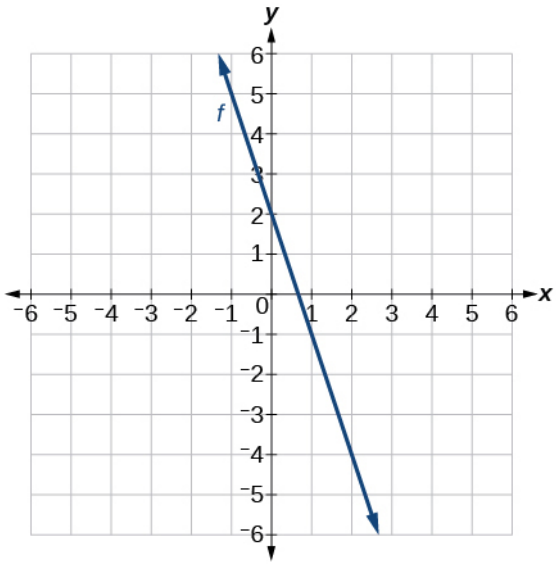

y
x
f
6
5
4
3
2
1
0
-1
-2
-3
-4
-5
-6
-6 -5 -4 -3 -2 -1 1 2 3 4 5 6

**75**.

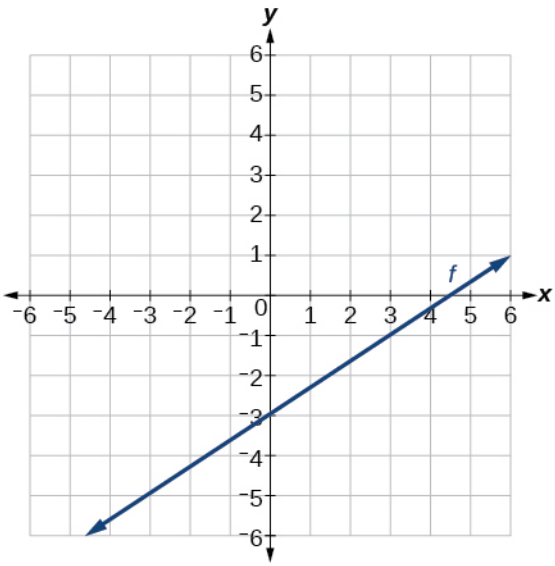
**77**.

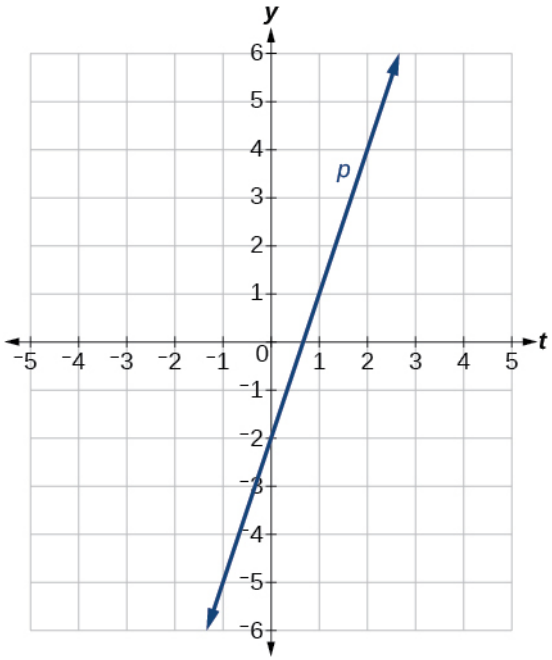



**79**.

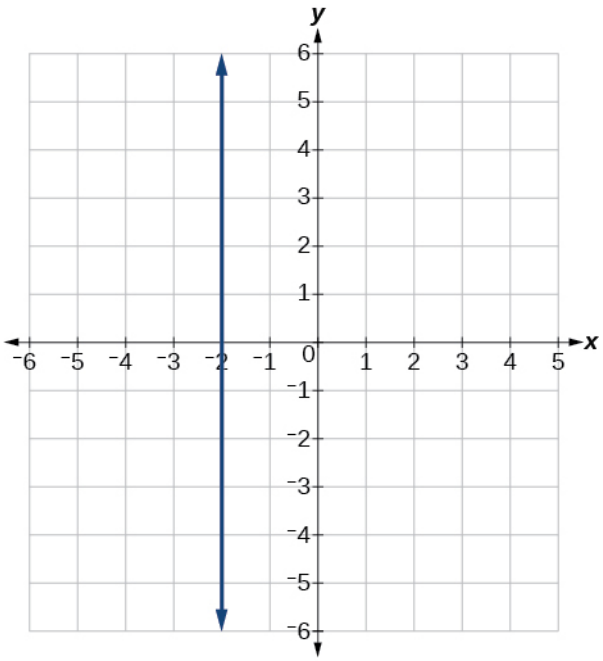



**81**.






**83**.




**85**.
$y = 3$

**87**.
$x = -3$

**89**. Linear,
$g(x) = -3x + 5$

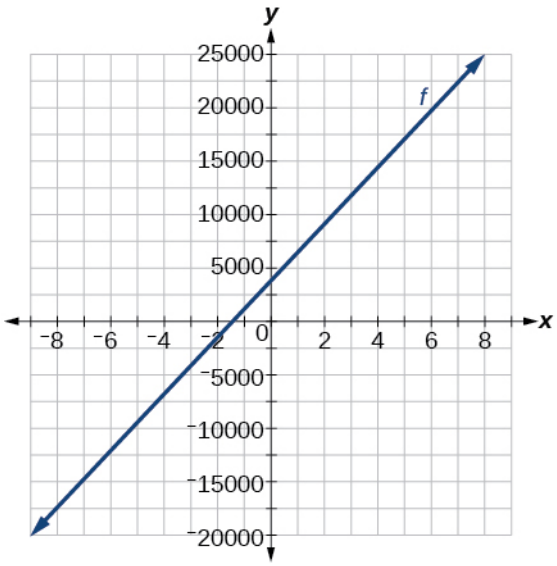
**91**. Linear,
$f(x) = 5x - 5$

**93**. Linear,
$g(x) = -\frac{25}{2}x + 6$

**95**. Linear,
$f(x) = 10x - 24$

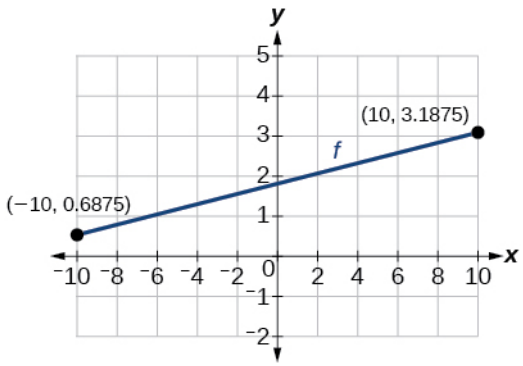
**97**.
$f(x) = -58x + 17.3$

**99**.

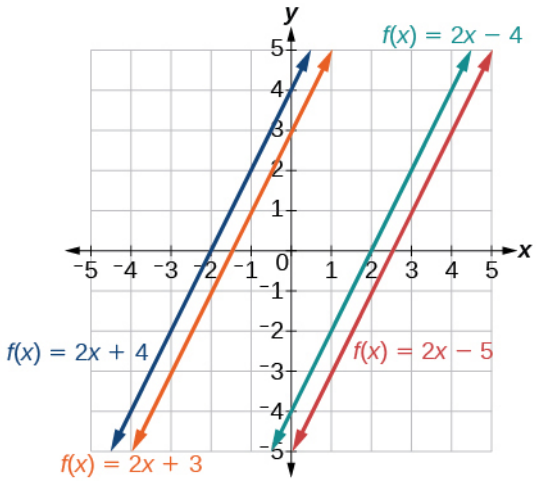



**100**.
$y = 3.613x - 6.129$

**102**.




**104**.




**106**. $y = 175$
**108**.
$y = -\frac{1}{2}x + b + 2$

**110**. $y = –0.01x + 2.01$

**112**.
$\left(\frac{1999}{201}, \frac{400,001}{2010}\right)$

**114**.
$20 + 0.5n$

**116**.
$p(n) = -0.004n + 34$

**118**.
$y = -0.5n + 45$

**120**.
$P(t) = 1700t + 45,000$

**122**.

a.
$$\text{Rate of change} = \frac{\Delta F}{\Delta C} = \frac{212 - 32}{100 - 0} = 1.8 \text{ degrees F for one degree change in C}$$

b.
$$F(28) = 1.8(28) + 32 = 82.4 \text{ degrees F is 28 degrees C}$$

c.
$$F(-40) = 1.8(-40) + 32 = -40 \text{ degrees F is -40 degrees C}$$

**123**. Determine the independent variable. This is the variable upon which the output depends.
**125**. To determine the initial value, find the output when the input is equal to zero.
**127**. 6 square units
**129**. 20.01 square units
**131**. 2,300
**133**. 64,170
**135**.
$P(t) = 75,000 + 2500t$

**137**. (–30, 0) Thirty years before the start of this model, the town had no citizens. (0, 75,000) Initially, the town had a population of 75,000.
**139**. Ten years after the model began
**141**.
$W(t) = 0.5t + 7.5$

**143**.
$(-15, 0)$ : The *x*-intercept is not a plausible set of data for this model because it means the baby weighed 0 pounds 15 months prior to birth.
$(0, 7.5)$ : The baby weighed 7.5 pounds at birth.
**145**. At age 5.8 months
**147**.
$C(t) = 12,025 - 205t$

**149**.
$(58.7, 0)$ : In roughly 59 years, the number of people inflicted with the common cold would be 0.
$(0, 12,025)$ Initially there were 12,025 people afflicted by the common cold.
**151**. 2063
**153**.
$y = -2t + 180$

**155**. In 2070, the company's profit will be zero.
**157**.
$y = 30t - 300$

**158**.
$(0, -300)$; In 1980, the company lost $300,000.
**160**.
$y = 30t - 300$ of form
$y = mx + b$, m = 30. For each year after 1980, the company's profits increased $30,000 per year
**162**. $80,640
**164**. Alabama

**166**. 2328

**168**.

a. $2134 - 1431 = 703$ people

b. $2007 - 2003 = 4$ years

c. Average rate of growth $= \frac{703}{4} = 175.75$ people per year So, using $y = mx + b,$ we have $y = 175.75x + 1431.$

d. The year 2000 corresponds to $t = -3.$ So, $y = 175.75(-3) + 1431 = 903.75$ or roughly 904 people in year 2000

e. If the year 2000 corresponds to $t$=0, then we have ordered pair $(0,\ 903.75)$ $y = 175.75x + 903.75$ corresponds to $P(t) = 175.75t + 903.75$

f. The year 2014 corresponds to $t = 14.$ Therefore, $P(14) = 175.75(14) + 903.75 = 3364.25$. So, a population of 3364.

**170**.

a. Ordered pairs are $(20,\ 11.20)$ and $(130,\ 17.80)$

$$\begin{aligned} m &= \frac{17.80 - 11.20}{130 - 20} = 0.06 \text{ and } (0,\ 10) \\ y &= mx + b \\ y &= 0.06x + 10 \text{ or } C(x) = 0.06x + 10 \end{aligned}$$

b. 0.06 For every MB, the client is charged 6 cents. $(0,\ 10)$ If no usage occurs, the client is charged \$10

c.

$$\begin{aligned} C(250) &= 0.06(250) + 10 \\ &= \$25 \end{aligned}$$

**172**.

a. Ordered pairs are $(0,\ 340)$ and $(4,\ 285)$

$$\begin{aligned} m &= \frac{285 - 340}{4 - 0} = -13.75 \text{ and } (0,\ 340) \\ y &= mx + b \\ y &= -13.75x + 340 \text{ or } P(t) = -13.75t + 340 \end{aligned}$$

b. The year 2012 corresponds to t = 9

$$\begin{aligned} P(9) &= -13.75(9) + 340 \\ &= 216.25 \text{ or } 216 \text{ owls} \end{aligned}$$

**174**.

a. The year 2014 corresponds to t = 0.

We have $m = -25$ and $(0,\ 1820)$

$$\begin{aligned} y &= mx + b \\ y &= -25x + 1820 \text{ or } R(t) = -25t + 1820 \end{aligned}$$

b.

$$\begin{aligned} R(7) &= -25(7) + 1820 \\ &= 645 \text{ billion cubic feet} \end{aligned}$$

c.

$$\begin{aligned} 0 &= -25t + 1820 \\ -1820 &= -25t \\ 72.8 &= t \Rightarrow 2014 + 72.8 = 2086.8. \text{ So, in the year 2086} \end{aligned}$$

**176**. (4.81)

Plan 1: $y = 5x$ where x is number of windows

Plan 2: $y = 3x + 40$ where x is number of windows

$$\begin{aligned} 3x + 40 &\leq 5x \\ 40 &\leq 2x \\ 20 &\leq x \end{aligned}$$

So, more than 20 windows

**178**. (4.82)

Option A: $y = 0.10x + 14{,}000$ where $x$ is dollars of sales.

Option B: $y = 0.04x + 19,\ 000$ where $x$ is dollars of sales

$$\begin{aligned} 0\,.10x + 14,\ 000 &\geq 0.04x + 19,\ 000 \\ 0.06x + 14,\ 000 &\geq 19,\ 000 \\ 0.06x &\geq 5,\ 000 \\ x &\geq 83,\ 333.33 \end{aligned}$$

So, more than \$83,333.33 in sales

**180**. (4.83)

Option A: $y = 0.09x + 10{,}000$ where x is dollars of sales

Option B: $y = 0.04x + 20,\ 000$ where x is dollars of sales

$$\begin{aligned} 0.09x + 10,\ 000 &\geq 0.04x + 20,\ 000 \\ 0.05x + 10,\ 000 &\geq 20,\ 000 \\ 0.05x &\geq 10,\ 000 \\ x &\geq 200,\ 000 \end{aligned}$$

So, more than \$200,000 in sales.

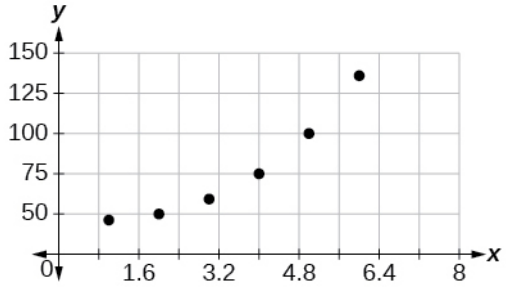
**181**. When our model no longer applies, after some value in the domain, the model itself doesn't hold.

**183**. We predict a value outside the domain and range of the data.

**185**. The closer the number is to 1, the less scattered the data, the closer the number is to 0, the more scattered the data.

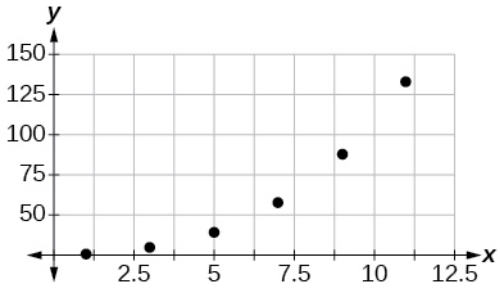
**187**. 61.966 years

**189**.



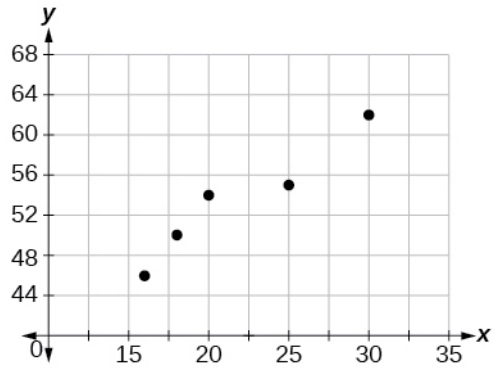

No.

**191**.




No.

**193**.




Interpolation. About
$60\ ^{\circ}F$.

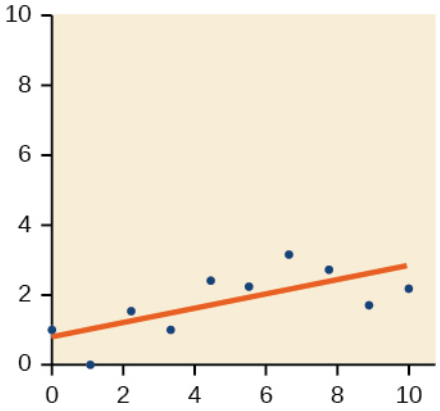
**195**.

This value of r indicates a strong negative correlation or slope, so C

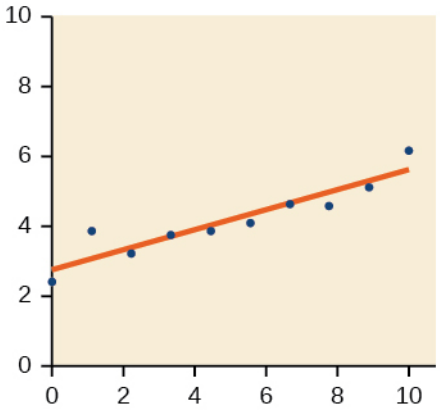
**197**.

This value of r indicates a weak negative correlation, so B

**199**.




**201**.

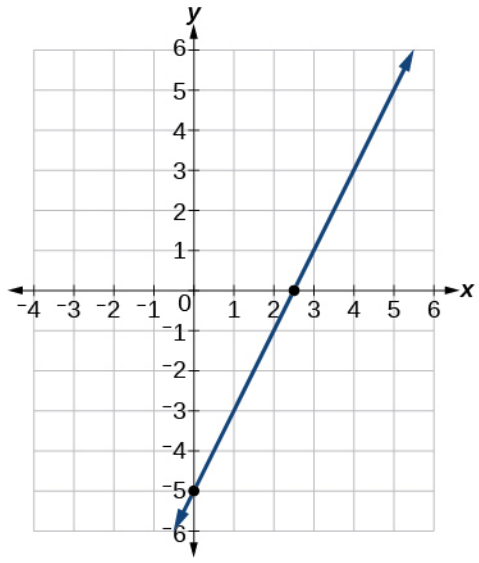


**203**. Yes, trend appears linear because
$r = 0.985$ and will exceed 12,000 near midyear, 2016, 24.6 years since 1992.
**205**.
$y = 1.640x + 13.800,$
$r = 0.987$
**207**.
$y = -0.962x + 26.86, \qquad r = -0.965$
**209**.
$y = -1.981x + 60.197;$
$r = -0.998$
**211**.
$y = 0.121x - 38.841,\ r = 0.998$
**213**.
$(-2, -6), (1, -12), (5, -20), (6, -22), (9, -28);$ Yes, the function is a good fit.
**215**.
$(189.8, 0)$ If 18,980 units are sold, the company will have a profit of zero dollars.
**217**.
$y = 0.00587x + 1985.41$
**219**.
$y = 20.25x - 671.5$
**221**.
$y = -10.75x + 742.50$

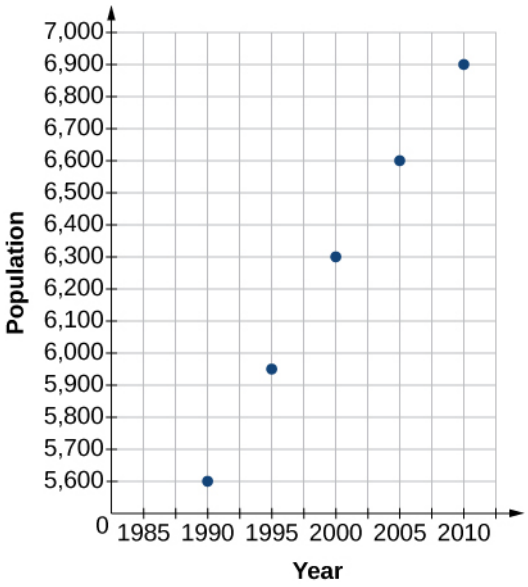
## Review Exercises

**223**. Yes
**225**. Increasing
**227**.
$y = -3x + 26$
**229**. 3
**231**.
$y = 2x - 2$
**233**. Not linear.
**235**. parallel
**237**.
(–9, 0); (0, –7)
**239**. Line 1:
$m = -2;$ Line 2:
$m = -2;$ Parallel
**241**.
$y = -0.2x + 21$

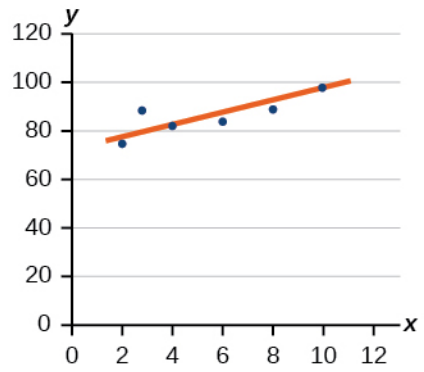
**243**.




**245**. More than 250
**247**. 118,000
**249**.
$y = -300x + 11,500$
**251**. a) 800 b) 100 students per year c)
$P(t) = 100t + 1700$
**253**. 18,500
**255**. $91,625
**257**. Extrapolation



**259**.




**261**. 2023
**263**.
$y = -1.294x + 49.412; \quad r = -0.974$

**265**. 2027
**267**. 7,660

## Practice Test

**269**. Decreasing
**271**.
$y = 5x - 24$

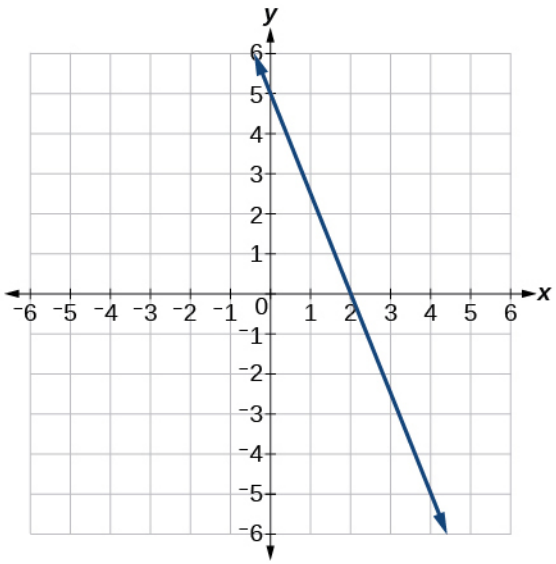
**273**.
$m = -2$

**275**. Yes,
$y = 3x + 32$

**277**.
$y = 30x + 120$

**279**. Neither
**281**. Line 1:
$m = 4;$ Line 2:
$m = 4;$ parallel

**283**.



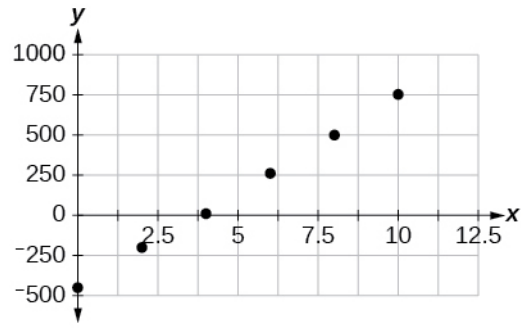
**285**.
$(7,\ 5)$

**287**. 16.94 square units

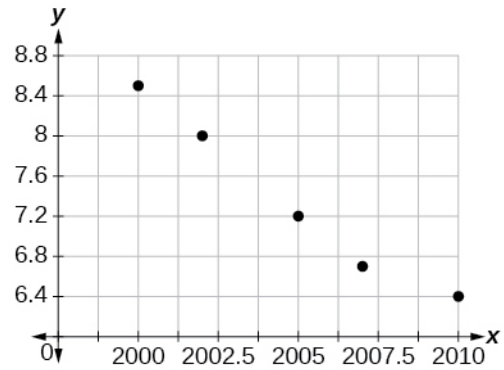
**289**. 2083

**291**.
$(0,\ 10,\ 625);$ In 1980, the profit was $10,625,000.

**293**.




**295**.




$y = -0.219x + 445.990$

**297**.
$y = 1.820x + 77.349,\ r = 0.986$

**299**. 2070

## Chapter 5

## Try It

**5.1**. The path passes through the origin and has vertex at $(-4,\ 7),$ so
$(h)x = -\frac{7}{16}(x+4)^2 + 7.$ To make the shot, $h(-7.5)$ would need to be about 4 but $h(-7.5) \approx 1.64;$ he doesn't make it.

**5.2**.
$g(x) = x^2 - 6x + 13$ in general form;
$g(x) = (x-3)^2 + 4$ in standard form

**5.3**. The domain is all real numbers. The range is $f(x) \geq \frac{8}{11},$ or $\left[\frac{8}{11}, \infty\right).$

**5.4**. $y$-intercept at (0, 13), No $x$-intercepts

**5.5**. 3 seconds; 256 feet; 7 seconds

**5.6**. $f(x)$ is a power function because it can be written as $f(x) = 8x^5.$ The other functions are not power functions.

**5.7**. As $x$ approaches positive or negative infinity, $f(x)$ decreases without bound: as $x \to \pm\infty, \quad f(x) \to -\infty$ because of the negative coefficient.

**5.8**. The degree is 6. The leading term is $-x^6.$ The leading coefficient is $-1.$

**5.9**. As $x \to \infty, \quad f(x) \to -\infty; \quad as \quad x \to -\infty, \quad f(x) \to -\infty.$ It has the shape of an even degree power function with a negative coefficient.

**5.10**. The leading term is $0.2x^3,$ so it is a degree 3 polynomial. As $x$ approaches positive infinity, $f(x)$ increases without bound; as $x$ approaches negative infinity, $f(x)$ decreases without bound.

**5.11**. $y$-intercept $(0, 0);$ $x$-intercepts $(0, 0), (-2, 0),$ and $(5, 0)$

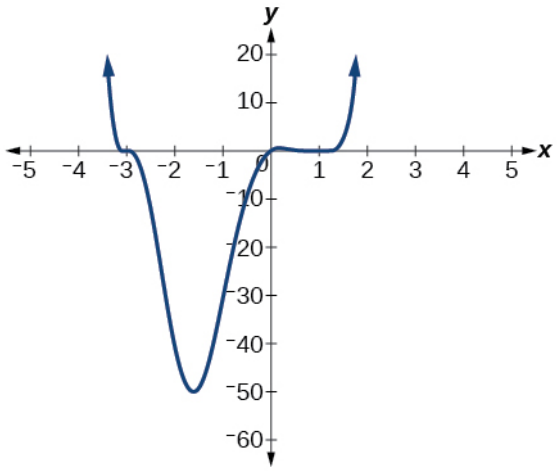
**5.12**. There are at most 12 $x$-intercepts and at most 11 turning points.

**5.13**. The end behavior indicates an odd-degree polynomial function; there are 3 $x$-intercepts and 2 turning points, so the degree is odd and at least 3. Because of the end behavior, we know that the lead coefficient must be negative.

**5.14**. The $x$-intercepts are $(2, 0), (-1, 0),$ and $(5, 0),$ the $y$-intercept is $(0, 2),$ and the graph has at most 2 turning points.

**5.15**. $y$-intercept $(0, 0);$ $x$-intercepts $(0, 0), (-5, 0), (2, 0),$ and $(3, 0)$

**5.16**. The graph has a zero of –5 with multiplicity 1, a zero of –1 with multiplicity 2, and a zero of 3 with even multiplicity.




**5.17**.

**5.18**. Because
$f$ is a polynomial function and since
$f(1)$ is negative and
$f(2)$ is positive, there is at least one real zero between
$x = 1$ and
$x = 2$.

**5.19**.

$$f(x) = -\frac{1}{8}(x-2)^3(x+1)^2(x-4)$$

**5.20**. The minimum occurs at approximately the point
$(0, -6.5)$, and the maximum occurs at approximately the point
$(3.5, 7)$.

**5.21**.

$$4x^2 - 8x + 15 - \frac{78}{4x+5}$$

**5.22**.

$$3x^3 - 3x^2 + 21x - 150 + \frac{1,090}{x+7}$$

**5.23**.

$$3x^2 - 4x + 1$$

**5.24**.

$$f(-3) = -412$$

**5.25**. The zeros are 2, –2, and –4.

**5.26**. There are no rational zeros.

**5.27**. The zeros are
$-4, \frac{1}{2}$, and 1.

**5.28**.

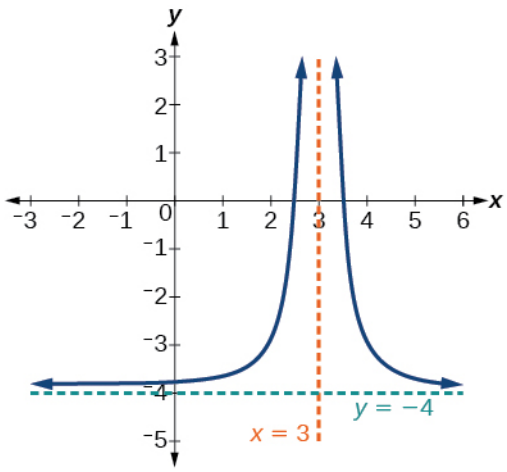
$$f(x) = -\frac{1}{2}x^3 + \frac{5}{2}x^2 - 2x + 10$$

**5.29**. There must be 4, 2, or 0 positive real roots and 0 negative real roots. The graph shows that there are 2 positive real zeros and 0 negative real zeros.

**5.30**. 3 meters by 4 meters by 7 meters

**5.31**. End behavior: as
$x \to \pm\infty, \quad f(x) \to 0;$ Local behavior: as
$x \to 0, \quad f(x) \to \infty$ (there are no $x$- or $y$-intercepts)

**5.32**.




The function and the asymptotes are shifted 3 units right and 4 units down. As $x \to 3,\ f(x) \to \infty,$ and as $x \to \pm\infty,\ f(x) \to -4.$ The function is $f(x) = \frac{1}{(x-3)^2} - 4.$

**5.33**.
$\frac{12}{11}$

**5.34**. The domain is all real numbers except $x = 1$ and $x = 5.$

**5.35**. Removable discontinuity at $x = 5.$ Vertical asymptotes: $x = 0,\ x = 1.$

**5.36**. Vertical asymptotes at $x = 2$ and $x = -3;$ horizontal asymptote at $y = 4.$

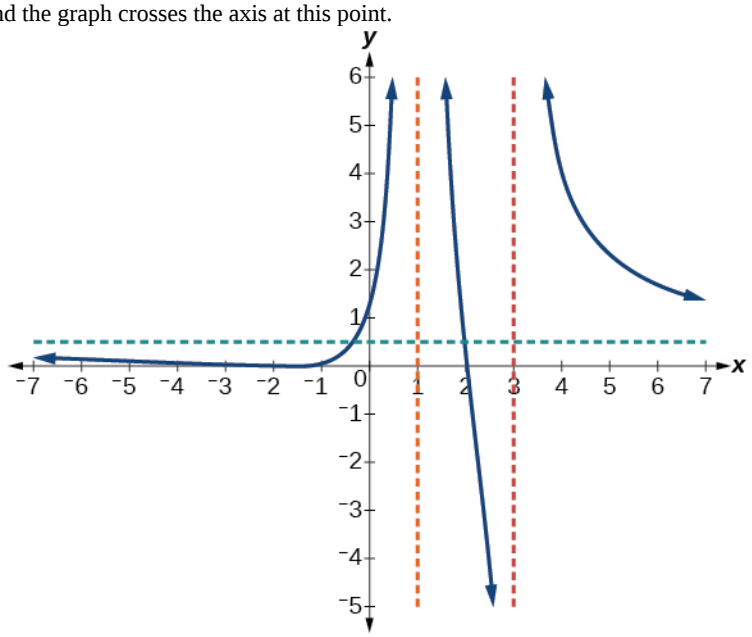
**5.37**. For the transformed reciprocal squared function, we find the rational form.

$$f(x) = \frac{1}{(x-3)^2} - 4 = \frac{1 - 4(x-3)^2}{(x-3)^2} = \frac{1 - 4(x^2 - 6x + 9)}{(x-3)(x-3)} = \frac{-4x^2 + 24x - 35}{x^2 - 6x + 9}$$

Because the numerator is the same degree as the denominator we know that as $x \to \pm\infty,\quad f(x) \to -4;\quad$ so $\quad y = -4$ is the horizontal asymptote. Next, we set the denominator equal to zero, and find that the vertical asymptote is $x = 3,$ because as $x \to 3,\ f(x) \to \infty.$ We then set the numerator equal to 0 and find the $x$-intercepts are at $(2.5,\ 0)$ and $(3.5,\ 0).$ Finally, we evaluate the function at 0 and find the $y$-intercept to be at $\left(0,\ \frac{-35}{9}\right).$

**5.38**. Horizontal asymptote at $y = \frac{1}{2}.$ Vertical asymptotes at $x = 1\quad$ and $\quad x = 3.$ $y$-intercept at $\left(0,\ \frac{4}{3}.\right)$ $x$-intercepts at $(2,\ 0)\quad$ and $(-2,\ 0).$

$(-2, 0)$ is a zero with multiplicity 2, and the graph bounces off the $x$-axis at this point.
$(2, 0)$ is a single zero and the graph crosses the axis at this point.




**5.39**.
$f^{-1}(f(x)) = f^{-1}\left(\frac{x+5}{3}\right) = 3\left(\frac{x+5}{3}\right) - 5 = (x-5) + 5 = x$ and
$f\left(f^{-1}(x)\right) = f(3x-5) = \frac{(3x-5)+5}{3} = \frac{3x}{3} = x$

**5.40**.
$f^{-1}(x) = x^3 - 4$

**5.41**.
$f^{-1}(x) = \sqrt{x-1}$

**5.42**.
$f^{-1}(x) = \frac{x^2-3}{2},\ x \geq 0$

**5.43**.
$f^{-1}(x) = \frac{2x+3}{x-1}$

**5.44**.
$\frac{128}{3}$

**5.45**.
$\frac{9}{2}$

**5.46**.
$x = 20$

## Section Exercises

**1**. When written in that form, the vertex can be easily identified.
**3**. If
$a = 0$ then the function becomes a linear function.
**5**. If possible, we can use factoring. Otherwise, we can use the quadratic formula.
**7**.
$f(x) = (x+1)^2 - 2,$ Vertex
$(-1, -4)$

**9**.

$f(x) = \left(x + \frac{5}{2}\right)^2 - \frac{33}{4},$ Vertex $\left(-\frac{5}{2},\ -\frac{33}{4}\right)$

**11**.

$f(x) = 3(x-1)^2 - 12,$ Vertex $(1,\ -12)$

**13**.

$f(x) = 3\left(x - \frac{5}{6}\right)^2 - \frac{37}{12},$ Vertex $\left(\frac{5}{6},\ -\frac{37}{12}\right)$

**15**. Minimum is $-\frac{17}{2}$ and occurs at $\frac{5}{2}$. Axis of symmetry is $x = \frac{5}{2}$.

**17**. Minimum is $-\frac{17}{16}$ and occurs at $-\frac{1}{8}$. Axis of symmetry is $x = -\frac{1}{8}$.

**19**. Minimum is $-\frac{7}{2}$ and occurs at $-3$. Axis of symmetry is $x = -3$.

**21**. Domain is $(-\infty, \infty)$. Range is $[2, \infty)$.

**23**. Domain is $(-\infty, \infty)$. Range is $[-5, \infty)$.

**25**. Domain is $(-\infty, \infty)$. Range is $[-12, \infty)$.

**26**.

$f(x) = x^2 - 4x + 4$

**28**.

$f(x) = x^2 + 1$

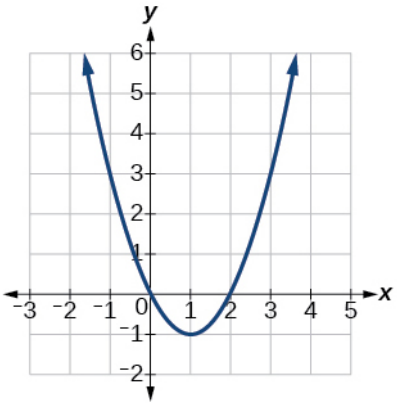
**30**.

$f(x) = \frac{6}{49}x^2 + \frac{60}{49}x + \frac{297}{49}$

**32**.

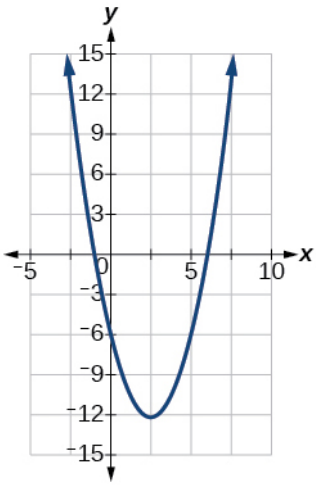
$f(x) = -x^2 + 1$

**34**.



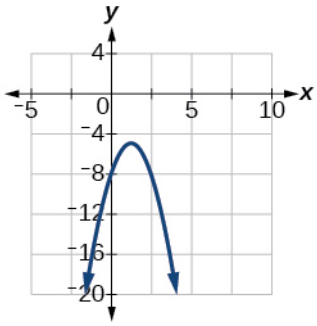

Vertex
$(1, \ -1)$, Axis of symmetry is
$x = 1.$ Intercepts are
$(0, \ 0), \quad (2, \ 0).$

**36**.



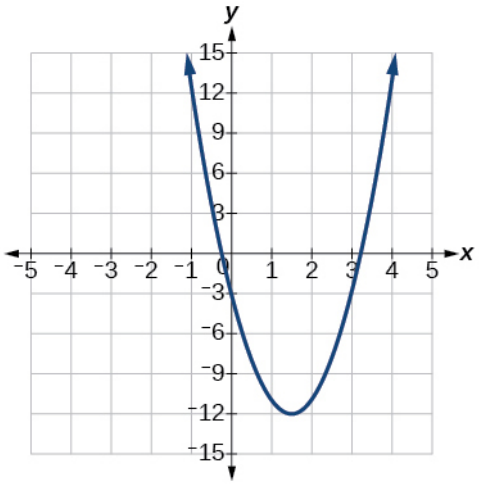

Vertex
$\left(\frac{5}{2}, \frac{-49}{4}\right)$, Axis of symmetry is
$(0, \ -6), \ (-1, \ 0), \ (6, \ 0).$

**38**.




Vertex
$\left(\frac{5}{4}, \quad -\frac{39}{8}\right)$, Axis of symmetry is
$x = \frac{5}{4}.$ Intercepts are
$(0, \quad -8).$

**39**.




**40**.
$f(x) = x^2 - 4x + 1$

**42**.
$f(x) = -2x^2 + 8x - 1$

**44**.
$f(x) = \frac{1}{2}x^2 - 3x + \frac{7}{2}$

**46**.
$f(x) = x^2 + 1$

**48**.
$f(x) = 2 - x^2$

**50**.
$f(x) = 2x^2$

**52**. The graph is shifted up or down (a vertical shift).

**54**. 50 feet

**56**. Domain is $(-\infty, \infty)$. Range is $[-2, \infty)$.

**58**. Domain is $(-\infty, \infty)$ Range is $(-\infty, 11]$.

**60**.
$f(x) = 2x^2 - 1$

**62**.
$f(x) = 3x^2 - 9$

**64**.
$f(x) = 5x^2 - 77$

**66**. 50 feet by 50 feet. Maximize
$f(x) = -x^2 + 100x$.

**68**. 125 feet by 62.5 feet. Maximize
$f(x) = -2x^2 + 250x$.

**70**.
6 and $-6$; product is –36; maximize
$f(x) = x^2 + 12x$.

**72**. 2909.56 meters

**74**. $10.70
**76**. The coefficient of the power function is the real number that is multiplied by the variable raised to a power. The degree is the highest power appearing in the function.
**78**. As $x$ decreases without bound, so does $f(x)$. As $x$ increases without bound, so does $f(x)$.
**80**. The polynomial function is of even degree and leading coefficient is negative.
**82**. Power function
**84**. Neither
**86**. Neither
**88**. Degree = 2, Coefficient = –2
**90**. Degree =4, Coefficient = –2
**92**.
$\text{As } x \to \infty,\ f(x) \to \infty,\ \text{as } x \to -\infty,\ f(x) \to \infty$
**94**.
$\text{As } x \to -\infty,\ f(x) \to -\infty,\ \text{as } x \to \infty,\ f(x) \to -\infty$
**96**.
$\text{As } x \to -\infty,\ f(x) \to -\infty,\ \text{as } x \to \infty,\ f(x) \to -\infty$
**98**.
$\text{As } x \to \infty,\ f(x) \to \infty,\ \text{as } x \to -\infty,\ f(x) \to -\infty$
**100**. $y$-intercept is $(0,\ 12)$, $t$-intercepts are $(1,\ 0);\ (-2,\ 0);$ and $(3,\ 0)$.
**102**. $y$-intercept is $(0,\ -16)$. $x$-intercepts are $(2,\ 0)$ and $(-2,\ 0)$.
**104**. $y$-intercept is $(0,\ 0)$. $x$-intercepts are $(0,\ 0),\ (4,\ 0),$ and $(-2,\ 0)$.
**106**. 3
**108**. 5
**110**. 3
**112**. 5
**114**. Yes. Number of turning points is 2. Least possible degree is 3.
**116**. Yes. Number of turning points is 1. Least possible degree is 2.
**118**. Yes. Number of turning points is 0. Least possible degree is 1.
**119**. No.
**120**. Yes. Number of turning points is 0. Least possible degree is 1.
**122**.

| $x$ | $f(x)$ |
|---|---|
| 10 | 9,500 |
| 100 | 99,950,000 |
| –10 | 9,500 |
| –100 | 99,950,000 |

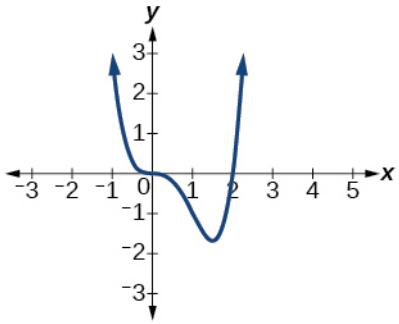
$\text{as } x \to -\infty,\ f(x) \to \infty,\ \text{as } x \to \infty,\ f(x) \to \infty$

**124**.

| $x$ | $f(x)$ |
|---|---|
| 10 | –504 |
| 100 | –941,094 |
| –10 | 1,716 |
| –100 | 1,061,106 |

$\text{as } x \to -\infty,\ f(x) \to \infty,\ \text{as } x \to \infty,\ f(x) \to -\infty$

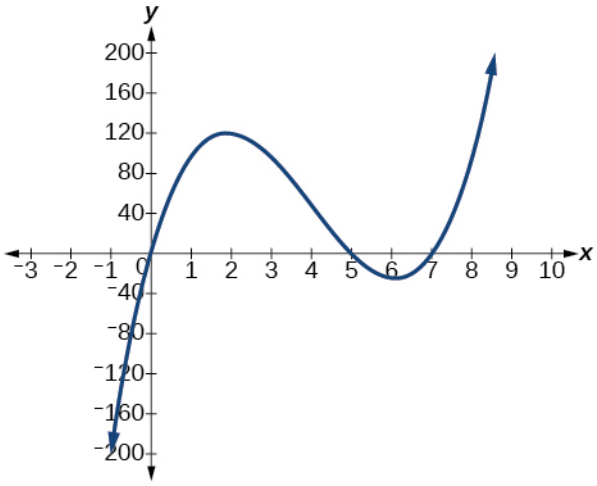
**126**.




The $y$-intercept is $(0,\ 0)$. The $x$-intercepts are $(0,\ 0),\ (2,\ 0)$.

$\text{As } x \to -\infty,\ f(x) \to \infty,\ \text{as } x \to \infty,\ f(x) \to \infty$

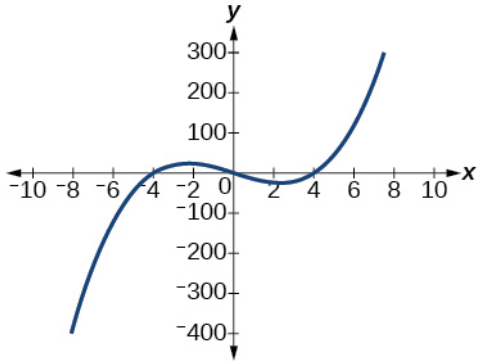
**128**.




The $y$-intercept is $(0,\ 0)$. The $x$-intercepts are $(0,\ \ 0),\ (5,\ \ 0),\ (7,\ \ 0)$.

As $x \to -\infty,\ f(x) \to -\infty,$ as $x \to \infty,\ f(x) \to \infty$

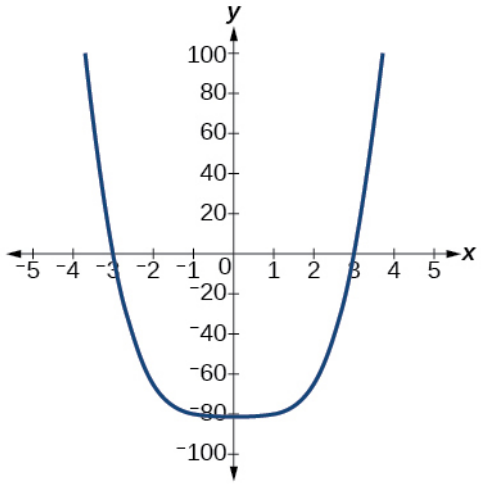
**130**.




The $y$-intercept is $(0,\ \ 0)$. The $x$-intercept is $(-4,\ \ 0),\ (0,\ \ 0),\ (4,\ \ 0)$.

As $x \to -\infty,\ f(x) \to -\infty,$ as $x \to \infty,\ f(x) \to \infty$

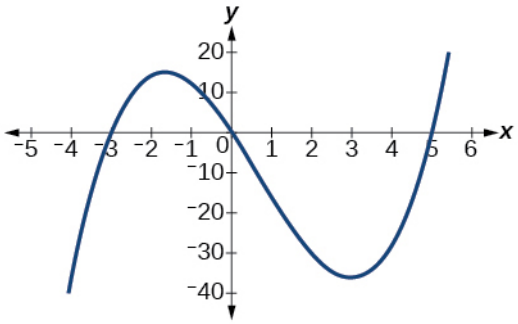
**132**.




The $y$-intercept is $(0, \ -81)$. The $x$-intercept are $(3, \ 0), \ (-3, \ 0)$.

As $x \to -\infty, \ f(x) \to \infty, \ \text{as} \ x \to \infty, \ f(x) \to \infty$

**134**.




The $y$-intercept is $(0, \ 0)$. The $x$-intercepts are $(-3, \ 0), \ (0, \ 0), \ (5, \ 0)$.

As $x \to -\infty, \ f(x) \to -\infty, \ \text{as} \ x \to \infty, \ f(x) \to \infty$

**136**.

$f(x) = x^2 - 4$

**138**.

$f(x) = x^3 - 4x^2 + 4x$

**140**.

$f(x) = x^4 + 1$

**142**.

$V(m) = 8m^3 + 36m^2 + 54m + 27$

**144**.

$V(x) = 4x^3 - 32x^2 + 64x$

**146**. The $x$-intercept is where the graph of the function crosses the $x$-axis, and the zero of the function is the input value for which $f(x) = 0$.

**148**. If we evaluate the function at $a$ and at $b$ and the sign of the function value changes, then we know a zero exists between $a$ and $b$.

**150**. There will be a factor raised to an even power.

**152**.
$(-2, 0), (3, 0), (-5, 0)$

**154**.
$(3, 0), (-1, 0), (0, 0)$

**156**.
$(0, 0), (-5, 0), (2, 0)$

**158**.
$(0, 0), (-5, 0), (4, 0)$

**160**.
$(2, 0), (-2, 0), (-1, 0)$

**162**.
$(-2, 0), (2, 0), \left(\frac{1}{2}, 0\right)$

**164**.
$(1, 0), (-1, 0)$

**166**.
$(0, 0), (\sqrt{3}, 0), (-\sqrt{3}, 0)$

**168**.
$(0, 0), (1, 0), (-1, 0), (2, 0), (-2, 0)$

**170**.
$f(2) = -10$ and $f(4) = 28.$ Sign change confirms.

**172**.
$f(1) = 3$ and $f(3) = -77.$ Sign change confirms.

**174**.
$f(0.01) = 1.000001$ and $f(0.1) = -7.999.$ Sign change confirms.

**176**. 0 with multiplicity 2, $-\frac{3}{2}$ with multiplicity 5, 4 with multiplicity 2

**178**. 0 with multiplicity 2, –2 with multiplicity 2

**180**.
$-\frac{2}{3}$ with multiplicity 5, 5 with multiplicity 2

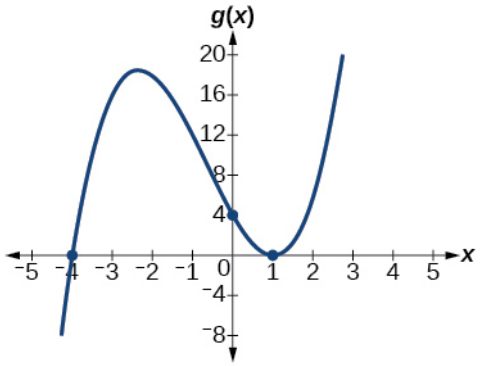
**182**.
0 with multiplicity 4, 2 with multiplicity 1, $-1$ with multiplicity 1

**184**.
$\frac{3}{2}$ with multiplicity 2, 0 with multiplicity 3

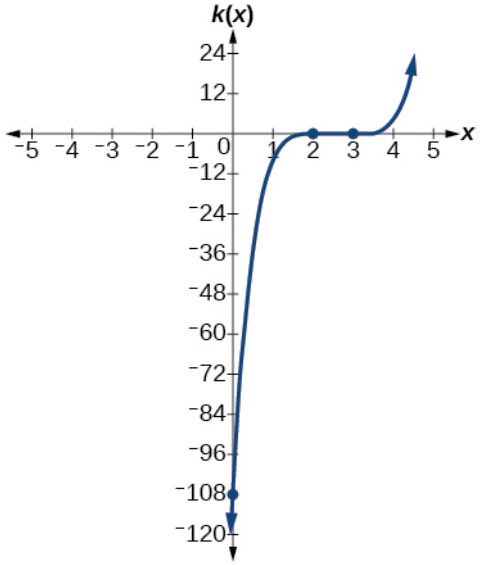
**186**.
0 with multiplicity 6, $\frac{2}{3}$ with multiplicity 2

**188**. $x$-intercepts,
$(1, 0)$ with multiplicity 2,
$(-4, 0)$ with multiplicity 1,
$y$-intercept

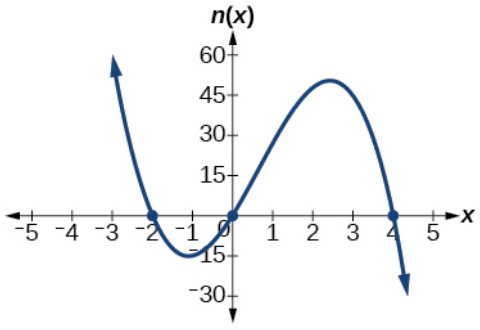
$(0, \ \ 4)$. As
$x \to -\infty, \ f(x) \to -\infty, \ \text{as } x \to \infty, \ f(x) \to \infty.$




**190**. $x$-intercepts
$(3, 0)$ with multiplicity 3,
$(2, 0)$ with multiplicity 2,
$y$-intercept
$(0, -108)$. As
$x \to -\infty, \ f(x) \to -\infty, \ \text{as } x \to \infty, \ f(x) \to \infty.$




**192**. $x$-intercepts
$(0, \ \ 0), \ (-2, \ \ 0), \ (4, 0)$ with multiplicity 1,
$y$-intercept
$(0, \ \ 0)$. As
$x \to -\infty, \ f(x) \to \infty, \ \text{as } x \to \infty, \ f(x) \to -\infty.$



**194**.
$f(x) = -\frac{2}{9}(x-3)(x+1)(x+3)$

**196**.
$f(x) = \frac{1}{4}(x+2)^2(x-3)$

**198**. –4, –2, 1, 3 with multiplicity 1
**200**. –2, 3 each with multiplicity 2
**202**.
$f(x) = -\frac{2}{3}(x+2)(x-1)(x-3)$

**204**.
$f(x) = \frac{1}{3}(x-3)^2(x-1)^2(x+3)$

**206**.
$f(x) = -15(x-1)^2(x-3)^3$

**208**.
$f(x) = -2(x+3)(x+2)(x-1)$

**210**.
$f(x) = -\frac{3}{2}(2x-1)^2(x-6)(x+2)$

**212**. local max
$(-.58, -.62)$, local min
$(.58, -1.38)$

**214**. global min
$(-.63, -.47)$

**216**. global min
$(.75, .89)$

**218**.
$f(x) = (x-500)^2(x+200)$

**220**.
$f(x) = 4x^3 - 36x^2 + 80x$

**222**.
$f(x) = 4x^3 - 36x^2 + 60x + 100$

**224**.
$f(x) = \pi(9x^3 + 45x^2 + 72x + 36)$

**225**. The binomial is a factor of the polynomial.
**227**.
$x + 6 + \frac{5}{x-1}$, quotient: $x+6$, remainder: 5

**229**.
$3x + 2$, quotient: $3x+2$, remainder: 0

**231**.
$x - 5$, quotient: $x-5$, remainder: 0

**233**.
$2x - 7 + \frac{16}{x+2}$, quotient: $2x-7$, remainder: 16

**235**.
$x - 2 + \frac{6}{3x+1}$, quotient: $x-2$, remainder: 6

**237**.
$2x^2 - 3x + 5$, quotient: $2x^2 - 3x + 5$, remainder: 0

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**239**.
$2x^2 + 2x + 1 + \frac{10}{x-4}$

**241**.
$2x^2 - 7x + 1 - \frac{2}{2x+1}$

**243**.
$3x^2 - 11x + 34 - \frac{106}{x+3}$

**245**.
$x^2 + 5x + 1$

**247**.
$4x^2 - 21x + 84 - \frac{323}{x+4}$

**249**.
$x^2 - 14x + 49$

**251**.
$3x^2 + x + \frac{2}{3x-1}$

**253**.
$x^3 - 3x + 1$

**255**.
$x^3 - x^2 + 2$

**257**.
$x^3 - 6x^2 + 12x - 8$

**259**.
$x^3 - 9x^2 + 27x - 27$

**261**.
$2x^3 - 2x + 2$

**263**. Yes
$(x-2)(3x^3 - 5)$

**265**. Yes
$(x-2)(4x^3 + 8x^2 + x + 2)$

**267**. No

**269**.
$(x-1)(x^2 + 2x + 4)$

**271**.
$(x-5)(x^2 + x + 1)$

**273**.
Quotient: $4x^2 + 8x + 16$, remainder: $-1$

**275**.
Quotient: $3x^2 + 3x + 5$, remainder: $0$

**277**.
Quotient: $x^3 - 2x^2 + 4x - 8$, remainder: $-6$

**279**.
$x^6 - x^5 + x^4 - x^3 + x^2 - x + 1$

**281**.
$x^3 - x^2 + x - 1 + \frac{1}{x+1}$

**283**.
$1 + \frac{1+i}{x-i}$

**285**.
$1+\frac{1-i}{x+i}$
**287**.
$x^2-ix-1+\frac{1-i}{x-i}$
**289**.
$2x^2+3$
**291**.
$2x+3$
**293**.
$x+2$
**295**.
$x-3$
**297**.
$3x^2-2$
**298**. The theorem can be used to evaluate a polynomial.
**300**. Rational zeros can be expressed as fractions whereas real zeros include irrational numbers.
**302**. Polynomial functions can have repeated zeros, so the fact that number is a zero doesn't preclude it being a zero again.
**304**.
$-106$
**306**.
$0$
**308**.
$255$
**310**.
$-1$
**312**.
$-2,\quad 1,\quad \frac{1}{2}$
**314**.
$-2$
**316**.
$-3$
**318**.
$-\frac{5}{2},\quad \sqrt{6},\quad -\sqrt{6}$
**320**.
$2,\quad -4,\quad -\frac{3}{2}$
**322**.
$4,\quad -4,\quad -5$
**324**.
$5,\quad -3,\quad -\frac{1}{2}$
**326**.
$\frac{1}{2},\quad \frac{1+\sqrt{5}}{2},\quad \frac{1-\sqrt{5}}{2}$
**328**.
$\frac{3}{2}$
**330**.
$2,\quad 3,\quad -1,\quad -2$
**332**.
$\frac{1}{2},\quad -\frac{1}{2},\quad 2,\quad -3$

**334**.
$-1, \quad -1, \quad \sqrt{5}, \quad -\sqrt{5}$

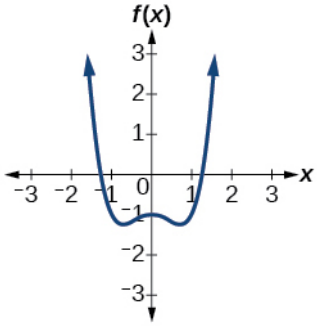
**336**.
$-\frac{3}{4}, \quad -\frac{1}{2}$

**338**.
$2, \quad 3+2i, \quad 3-2i$

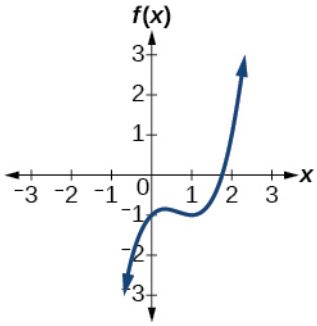
**340**.
$-\frac{2}{3}, \quad 1+2i, \quad 1-2i$

**342**.
$-\frac{1}{2}, \quad 1+4i, \quad 1-4i$

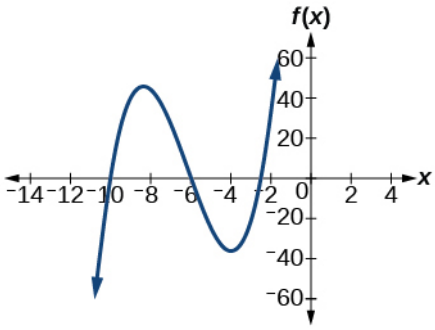
**344**. 1 positive, 1 negative



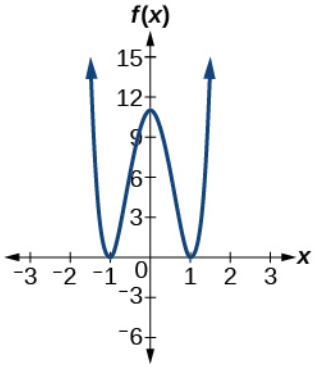

**346**. 3 or 1 positive, 0 negative



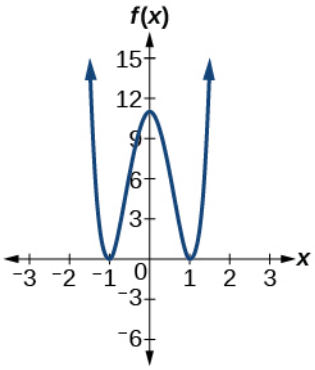

**348**. 0 positive, 3 or 1 negative




**350**. 2 or 0 positive, 2 or 0 negative




**352**. 2 or 0 positive, 2 or 0 negative




**354**.

$\pm 5, \quad \pm 1, \quad \pm \frac{5}{2}$

**356**.

$\pm 1, \quad \pm \frac{1}{2}, \quad \pm \frac{1}{3}, \quad \pm \frac{1}{6}$

**358**.

$1, \quad \frac{1}{2}, \quad -\frac{1}{3}$

**360**.

$2, \quad \frac{1}{4}, \quad -\frac{3}{2}$

**362**.

$\frac{5}{4}$

**364**.

$f(x) = \frac{4}{9}\left(x^3 + x^2 - x - 1\right)$

**366**.

$f(x) = -\frac{1}{5}\left(4x^3 - x\right)$

**368**. 8 by 4 by 6 inches
**370**. 5.5 by 4.5 by 3.5 inches
**372**. 8 by 5 by 3 inches
**374**. Radius = 6 meters, Height = 2 meters
**376**. Radius = 2.5 meters, Height = 4.5 meters
**378**. The rational function will be represented by a quotient of polynomial functions.
**380**. The numerator and denominator must have a common factor.
**382**. Yes. The numerator of the formula of the functions would have only complex roots and/or factors common to both the numerator and denominator.
**384**.

All reals $x \neq -1, \quad 1$

**386**.

All reals $x \neq -1, \quad -2, \quad 1, \quad 2$

**388**. V.A. at
$x = -\frac{2}{5}$; H.A. at
$y = 0$; Domain is all reals
$x \neq -\frac{2}{5}$

**390**. V.A. at
$x = 4, \quad -9$; H.A. at
$y = 0$; Domain is all reals
$x \neq 4, \quad -9$

**392**. V.A. at
$x = 0, \quad 4, \quad -4$; H.A. at
$y = 0$; Domain is all reals
$x \neq 0, 4, \quad -4$

**394**. V.A. at
$x = -5$; H.A. at
$y = 0$; Domain is all reals
$x \neq 5, -5$

**396**. V.A. at
$x = \frac{1}{3}$; H.A. at
$y = -\frac{2}{3}$; Domain is all reals
$x \neq \frac{1}{3}$.

**398**. none

**400**.
$x$-intercepts none, $y$-intercept $\left(0, \frac{1}{4}\right)$

**402**. Local behavior:
$x \to -\frac{1}{2}^{+}, f(x) \to -\infty, x \to -\frac{1}{2}^{-}, f(x) \to \infty$ End behavior:
$x \to \pm\infty, f(x) \to \frac{1}{2}$

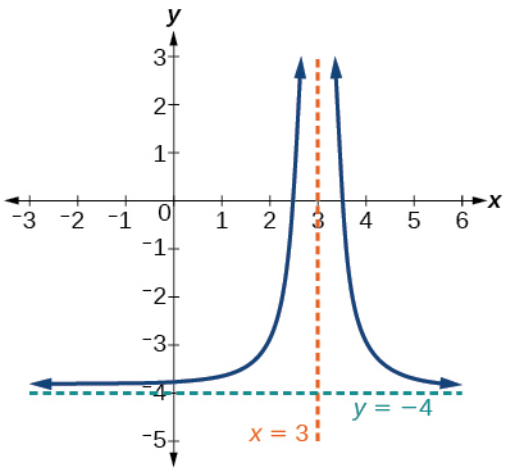
**404**. Local behavior:
$x \to 6^{+}, f(x) \to -\infty, x \to 6^{-}, f(x) \to \infty,$ End behavior:
$x \to \pm\infty, f(x) \to -2$

**406**. Local behavior:
$x \to -\frac{1}{3}^{+}, f(x) \to \infty, x \to -\frac{1}{3}^{-},$
$f(x) \to -\infty, x \to \frac{5}{2}^{-}, f(x) \to \infty, x \to \frac{5}{2}^{+}, f(x) \to -\infty$ End behavior:
$x \to \pm\infty, \ f(x) \to \frac{1}{3}$

**408**.
$y = 2x + 4$

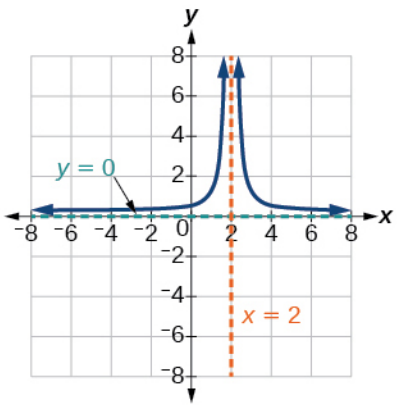
**410**.
$y = 2x$

**412**.
$V.A.\ x = 0, H.A.\ y = 2$



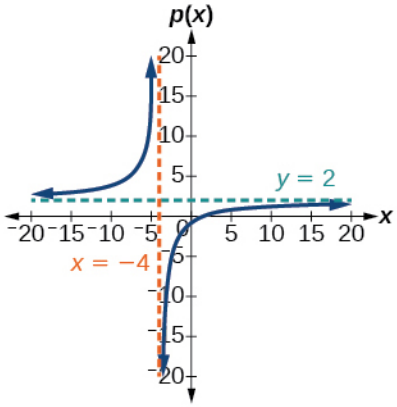

**414**.

$V.A.\ x = 2,\ \ H.A.\ y = 0$



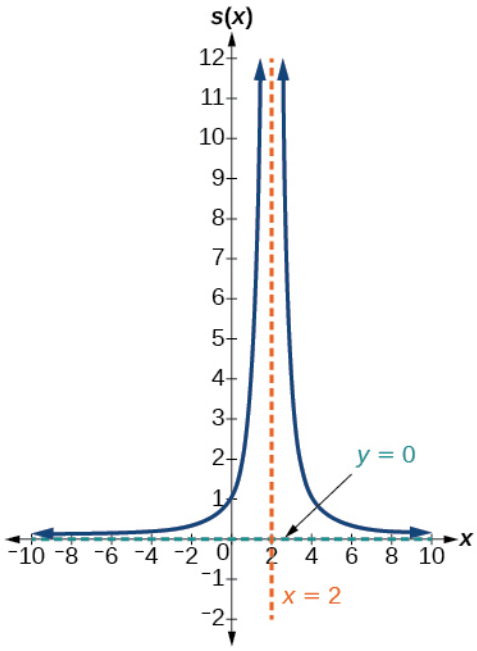

**416**.

$V.A.\ x = -4,\ \ H.A.\ y = 2; \left(\frac{3}{2}, 0\right); \left(0, -\frac{3}{4}\right)$



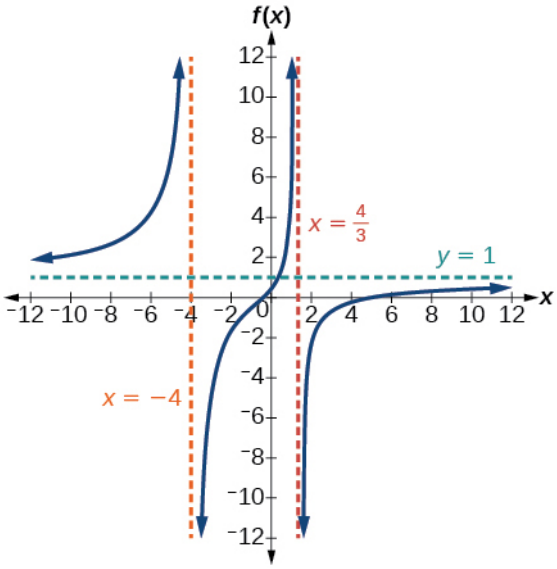

**418**.

$V.A.\ x = 2,\ \ H.A.\ y = 0,\ \ (0,\ 1)$



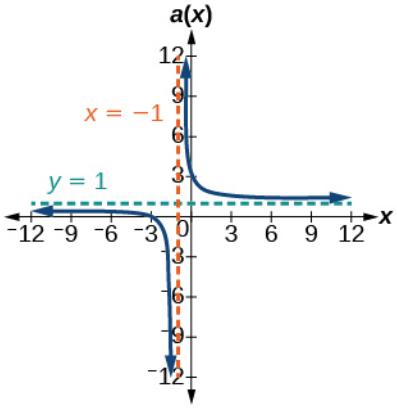

**420**.

$V.A.\ x = -4,\ x = \frac{4}{3},\ H.A.\ y = 1;\ (5,\ 0);\ \left(-\frac{1}{3},\ 0\right);\ \left(0,\ \frac{5}{16}\right)$



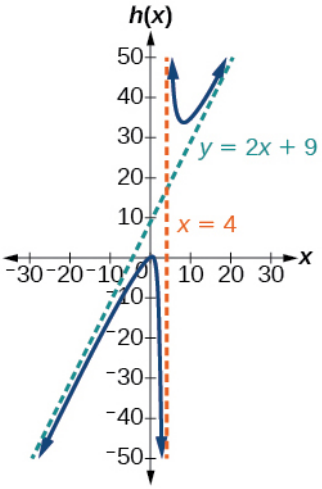

**422**.

$V.A.\ x = -1,\ H.A.\ y = 1;\ (-3,\ 0);\ (0,\ 3)$



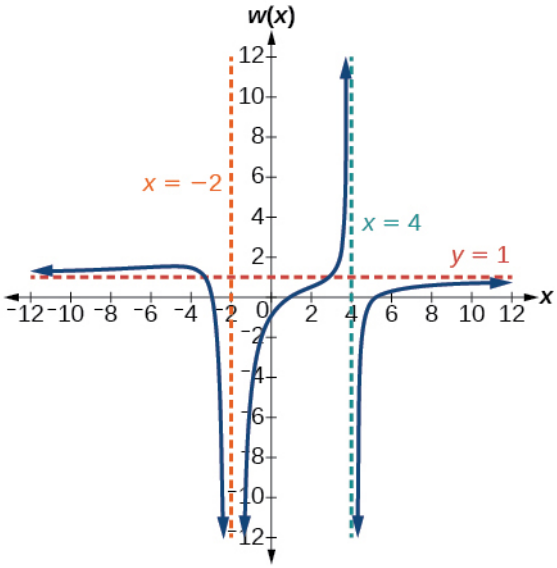
**424.**

$V.A.\ x = 4,\ \ S.A.\ y = 2x + 9;\ (-1,\ 0);\ \left(\frac{1}{2},\ 0\right);\ \left(0,\ \frac{1}{4}\right)$




**426.**

$V.A.\ x = -2,\ \ x = 4,\ \ H.A.\ y = 1,\ (1,\ 0);\ (5,\ 0);\ (-3,\ 0);\ \left(0,\ -\frac{15}{16}\right)$




**428.**

$y = 50\dfrac{x^2 - x - 2}{x^2 - 25}$

**430.**

$y = 7\dfrac{x^2 + 2x - 24}{x^2 + 9x + 20}$

**432.**

$y = \dfrac{1}{2}\dfrac{x^2 - 4x + 4}{x + 1}$

**434.**

$y = 4\dfrac{x - 3}{x^2 - x - 12}$

**436.**

$y = -9\dfrac{x - 2}{x^2 - 9}$

**438**.

$y = \frac{1}{3}\frac{x^2 + x - 6}{x - 1}$

**440**.

$y = -6\frac{(x-1)^2}{(x+3)(x-2)^2}$

**442**.

| $x$ | 2.01 | 2.001 | 2.0001 | 1.99 | 1.999 |
|---|---|---|---|---|---|
| $y$ | 100 | 1,000 | 10,000 | –100 | –1,000 |

| ***x*** | **10** | **100** | **1,000** | **10,000** | **100,000** |
|---|---|---|---|---|---|
| $y$ | .125 | .0102 | .001 | .0001 | .00001 |

Vertical asymptote $x = 2,$ Horizontal asymptote $y = 0$

**444**.

| $x$ | –4.1 | –4.01 | –4.001 | –3.99 | –3.999 |
|---|---|---|---|---|---|
| $y$ | 82 | 802 | 8,002 | –798 | –7998 |

| $x$ | 10 | 100 | 1,000 | 10,000 | 100,000 |
|---|---|---|---|---|---|
| $y$ | 1.4286 | 1.9331 | 1.992 | 1.9992 | 1.999992 |

Vertical asymptote $x = -4,$ Horizontal asymptote $y = 2$

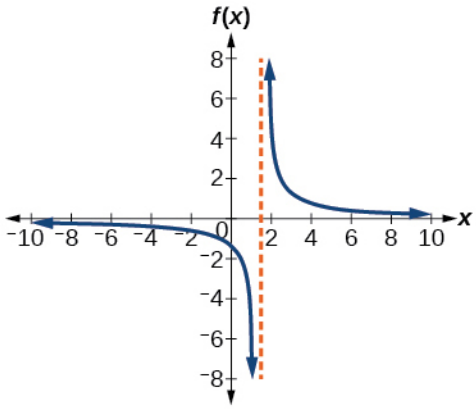
**446**.

| $x$ | –.9 | –.99 | –.999 | –1.1 | –1.01 |
|---|---|---|---|---|---|
| $y$ | 81 | 9,801 | 998,001 | 121 | 10,201 |

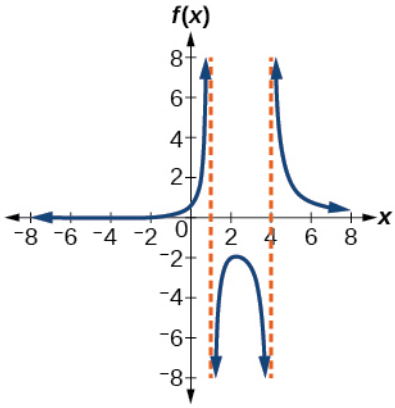
| $x$ | 10 | 100 | 1,000 | 10,000 | 100,000 |
|---|---|---|---|---|---|
| $y$ | .82645 | .9803 | .998 | .9998 | |

Vertical asymptote
$x = -1,$ Horizontal asymptote
$y = 1$

**448**.
$\left(\frac{3}{2}, \infty\right)$




**450**.
$(-2, 1) \cup (4, \infty)$




**452**.
$(2, 4)$

**454**.
$(2, 5)$

**456**.
$(-1, 1)$

**458**.
$C(t) = \frac{8 + 2t}{300 + 20t}$

**460**. After about 6.12 hours.

**462**.
$A(x) = 50x^2 + \frac{800}{x}$. 2 by 2 by 5 feet.

**464**.
$A(x) = \pi x^2 + \frac{100}{x}$. Radius = 2.52 meters.

**466**. It can be too difficult or impossible to solve for $x$ in terms of $y$.

**468**. We will need a restriction on the domain of the answer.

**470**.
$f^{-1}(x) = \sqrt{x} + 4$

**472**.
$f^{-1}(x) = \sqrt{x+3} - 1$

**473**.
$f^{-1}(x) = -\sqrt{\frac{x-5}{3}}$

**475**.
$f(x) = \sqrt{9-x}$

**477**.
$f^{-1}(x) = \sqrt[3]{x-5}$

**479**.
$f^{-1}(x) = \sqrt[3]{4-x}$

**481**.
$f^{-1}(x) = \frac{x^2-1}{2}, \ [0, \infty)$

**483**.
$f^{-1}(x) = \frac{(x-9)^2+4}{4}, \ [9, \infty)$

**485**.
$f^{-1}(x) = \left(\frac{x-9}{2}\right)^3$

**487**.
$f^{-1}(x) = \frac{2-8x}{x}$

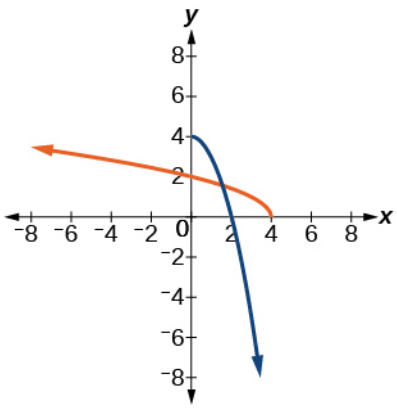
**489**.
$f^{-1}(x) = \frac{7x-3}{1-x}$

**491**.
$f^{-1}(x) = \frac{5x-4}{4x+3}$

**493**.
$f^{-1}(x) = \sqrt{x+1} - 1$

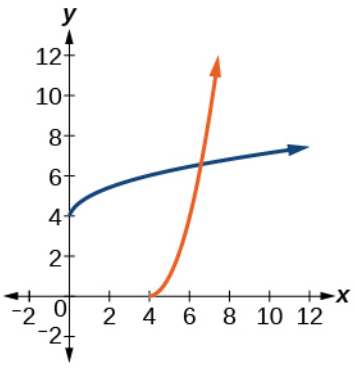
**495**.
$f^{-1}(x) = \sqrt{x+6} + 3$

**497**.
$f^{-1}(x) = \sqrt{4-x}$



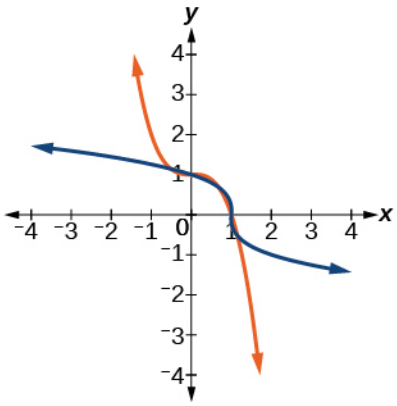

**499**.

$f^{-1}(x) = \sqrt{x} + 4$



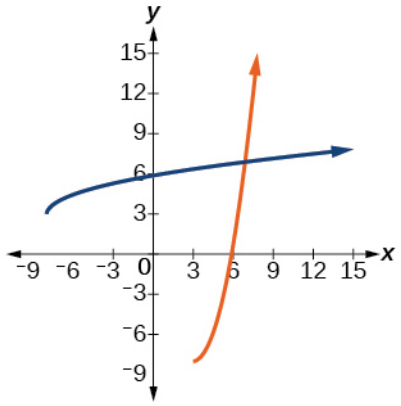

**501**.

$f^{-1}(x) = \sqrt[3]{1-x}$



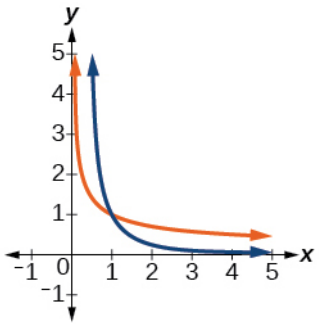

**503**.

$f^{-1}(x) = \sqrt{x+8} + 3$



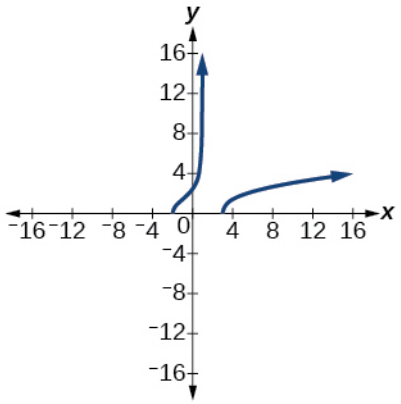

**505**.

$f^{-1}(x) = \sqrt[4]{\frac{1}{x}}$



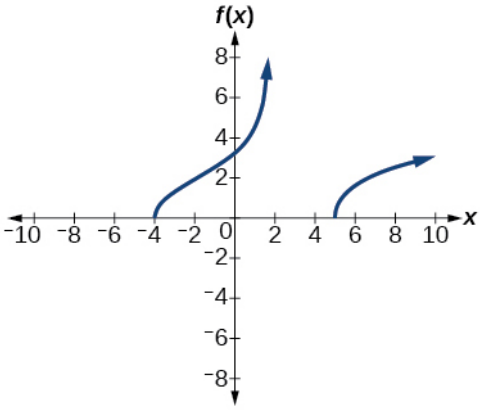

**507**.

$[-2, 1) \cup [3, \infty)$

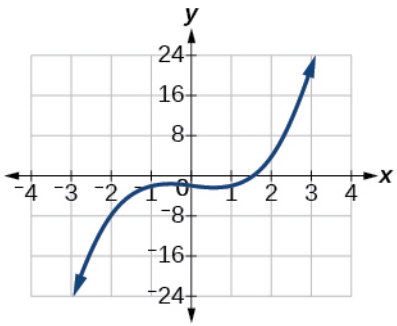



**509**.

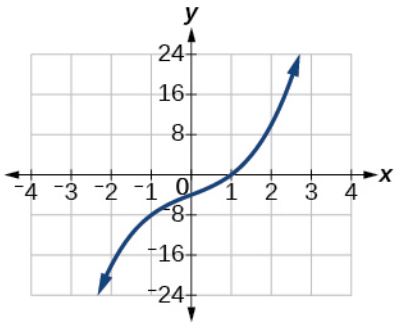
$[-4, 2) \cup [5, \infty)$



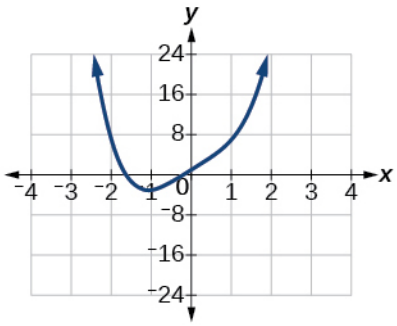
**511**.
$(-2, \quad 0); \quad (4, \quad 2); \quad (22, \quad 3)$




**513**.
$(-4, \quad 0); \quad (0, \quad 1); \quad (10, \quad 2)$




**515**.
$(-3, \quad -1); \quad (1, \quad 0); \quad (7, \quad 1)$




**517**.
$f^{-1}(x) = \sqrt{x + \frac{b^2}{4}} - \frac{b}{2}$

**519**.
$f^{-1}(x) = \frac{x^3 - b}{a}$

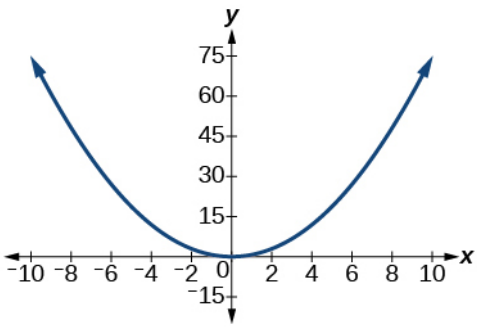
**521**.
$t(h) = \sqrt{\frac{200 - h}{4.9}}$, 5.53 seconds

**523**.
$r(V) = \sqrt[3]{\frac{3V}{4\pi}}$, 3.63 feet

**525**.
$n(C) = \frac{100C - 25}{.6 - C}$, 250 mL

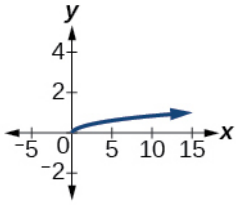
**527**.
$r(V) = \sqrt{\frac{V}{6\pi}}$, 3.99 meters

**529**.
$r(V) = \sqrt{\frac{V}{4\pi}}$, 1.99 inches
**531**. The graph will have the appearance of a power function.
**533**. No. Multiple variables may jointly vary.
**535**.
$y = 5x^2$
**537**.
$y = 10x^3$
**539**.
$y = 6x^4$
**541**.
$y = \frac{18}{x^2}$
**543**.
$y = \frac{81}{x^4}$
**545**.
$y = \frac{20}{\sqrt[3]{x}}$
**547**.
$y = 10xzw$
**549**.
$y = 10x\sqrt{z}$
**551**.
$y = 4\frac{xz}{w}$
**553**.
$y = 40\frac{xz}{\sqrt{w}t^2}$
**555**.
$y = 256$
**557**.
$y = 6$
**559**.
$y = 6$
**561**.
$y = 27$
**563**.
$y = 3$
**565**.
$y = 18$
**567**.
$y = 90$
**569**.
$y = \frac{81}{2}$
**571**.
$y = \frac{3}{4}x^2$



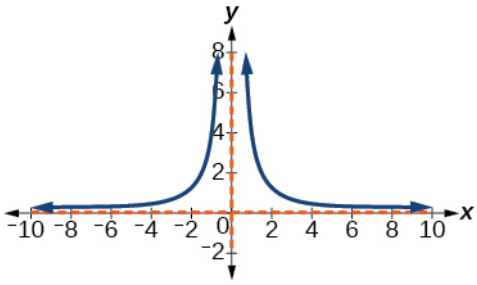

**573**.

$y = \frac{1}{3}\sqrt{x}$




**575**.

$y = \frac{4}{x^2}$

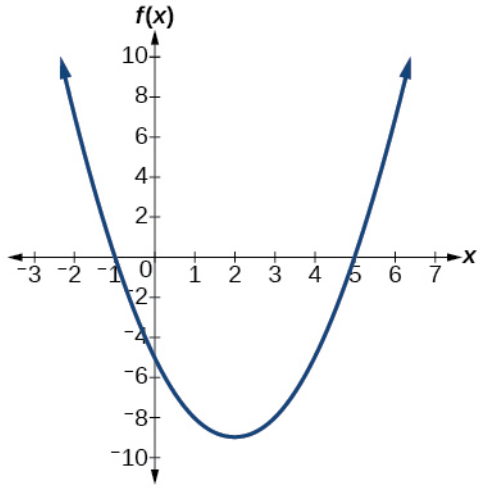



**577**. 1.89 years
**579**. 0.61 years
**581**. 3 seconds
**583**. 48 inches
**585**. 49.75 pounds
**587**. 33.33 amperes
**589**. 2.88 inches

## Review Exercises

**591**.

$f(x) = (x-2)^2 - 9$ vertex $(2, -9)$, intercepts $(5, 0)$; $(-1, 0)$; $(0, -5)$




**593**.

$f(x) = \frac{3}{25}(x+2)^2 + 3$

**595**. 300 meters by 150 meters, the longer side parallel to river.

**597**. Yes, degree = 5, leading coefficient = 4

**599**. Yes, degree = 4, leading coefficient = 1

**601**.

As $x \to -\infty$, $f(x) \to -\infty$, as $x \to \infty$, $f(x) \to \infty$

**603**. –3 with multiplicity 2,

$-\frac{1}{2}$ with multiplicity 1, –1 with multiplicity 3

**605**. 4 with multiplicity 1

**607**.

$\frac{1}{2}$ with multiplicity 1, 3 with multiplicity 3

**609**.

$x^2 + 4$ with remainder 12

**611**.

$x^2 - 5x + 20 - \frac{61}{x+3}$

**613**.

$2x^2 - 2x - 3$, so factored form is

$(x+4)(2x^2 - 2x - 3)$

**615**.

$\left\{-2, \quad 4, \quad -\frac{1}{2}\right\}$

**617**.

$\left\{1, \quad 3, \quad 4, \quad \frac{1}{2}\right\}$

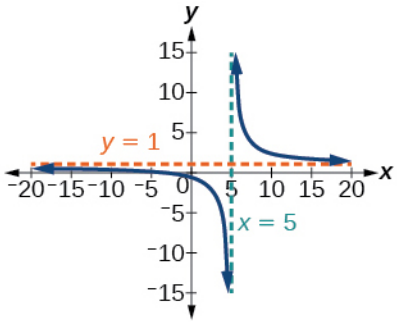
**619**. 0 or 2 positive, 1 negative

**621**. Intercepts

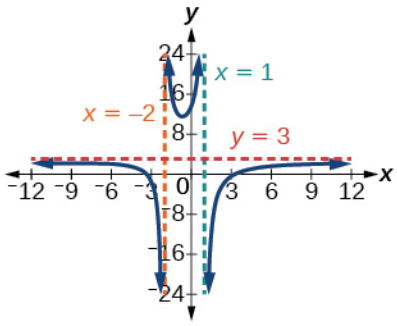
$(-2, 0)$ and $\left(0, -\frac{2}{5}\right)$, Asymptotes

$x = 5$ and

$y = 1.$




**623**. Intercepts (3, 0), (-3, 0), and $\left(0, \frac{27}{2}\right)$, Asymptotes $x = 1, \quad x = -2, \quad y = 3.$




**625**.
$y = x - 2$

**627**.
$f^{-1}(x) = \sqrt{x} + 2$

**629**.
$f^{-1}(x) = \sqrt{x + 11} - 3$

**631**.
$f^{-1}(x) = \frac{(x+3)^2 - 5}{4}, \ x \geq -3$

**633**.
$y = 64$

**635**.
$y = 72$

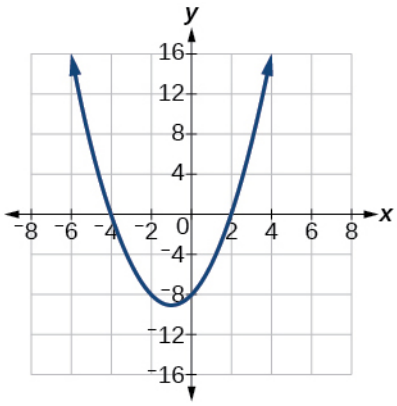
**637**. 148.5 pounds

## Practice Test

**640**.
$As\, x \to -\infty, \ f(x) \to -\infty, \ as\, x \to \infty, \ f(x) \to \infty$

**642**.
$f(x) = (x+1)^2 - 9$, vertex
$(-1, -9)$, intercepts

$(2,\ 0);\ (-4,\ 0);\ (0,\ -8)$




**644**. 60,000 square feet

**646**. 0 with multiplicity 4, 3 with multiplicity 2

**648**.

$2x^2 - 4x + 11 - \frac{26}{x+2}$

**650**.

$2x^2 - x - 4$. So factored form is

$(x+3)(2x^2 - x - 4)$

**652**.

$-\frac{1}{2}$ (has multiplicity 2),

$\frac{-1 \pm i\sqrt{15}}{2}$

**654**.

$-2$ (has multiplicity 3),

$\pm i$

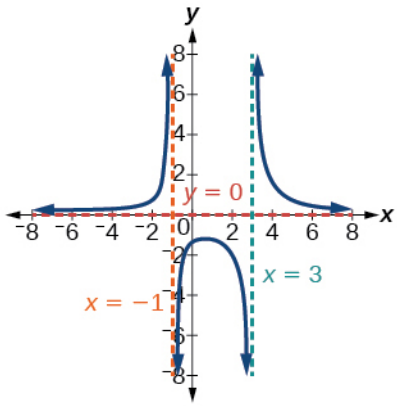
**656**.

$f(x) = 2(2x-1)^3(x+3)$

**658**. Intercepts

$(-4,\ 0),\ \left(0,\ -\frac{4}{3}\right)$, Asymptotes

$x = 3,\ x = -1,\ y = 0$.




**660**.

$y = x + 4$

**662**.
$f^{-1}(x) = \sqrt[3]{\frac{x+4}{3}}$

**664**.
$y = 18$

**666**. 4 seconds

# Chapter 6

## Try It

**6.1**.
$g(x) = 0.875^x$ and
$j(x) = 1095.6^{-2x}$ represent exponential functions.

**6.2**.
5.5556

**6.3**. About
1.548 billion people; by the year 2031, India's population will exceed China's by about 0.001 billion, or 1 million people.

**6.4**.
$(0, 129)$ and
$(2, 236)$; $N(t) = 129(1.3526)^t$

**6.5**.
$f(x) = 2(1.5)^x$

**6.6**.
$f(x) = \sqrt{2}(\sqrt{2})^x$. Answers may vary due to round-off error. The answer should be very close to
$1.4142(1.4142)^x$.

**6.7**.
$y \approx 12 \cdot 1.85^x$

**6.8**. about $3,644,675.88

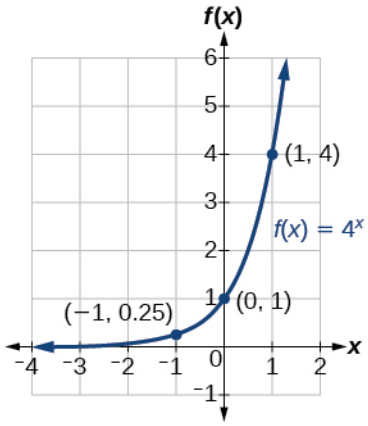
**6.9**. $13,693

**6.10**.
$e^{-0.5} \approx 0.60653$

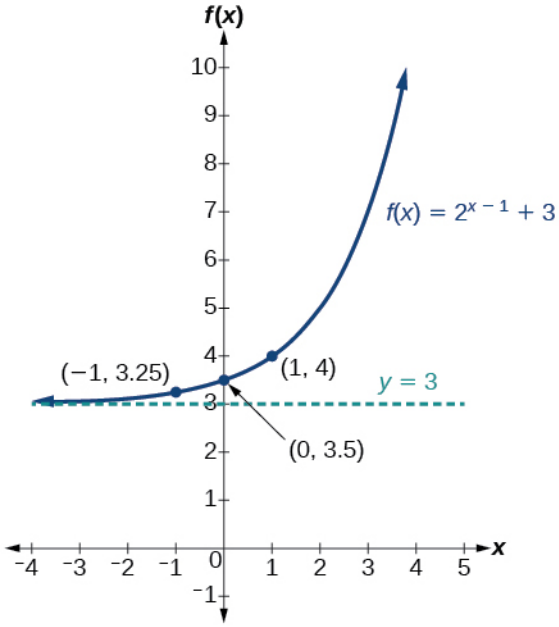
**6.11**. $3,659,823.44

**6.12**. 3.77E-26 (This is calculator notation for the number written as
$3.77 \times 10^{-26}$ in scientific notation. While the output of an exponential function is never zero, this number is so close to zero that for all practical purposes we can accept zero as the answer.)

**6.13**. The domain is
$(-\infty, \infty)$; the range is
$(0, \infty)$; the horizontal asymptote is
$y = 0$.

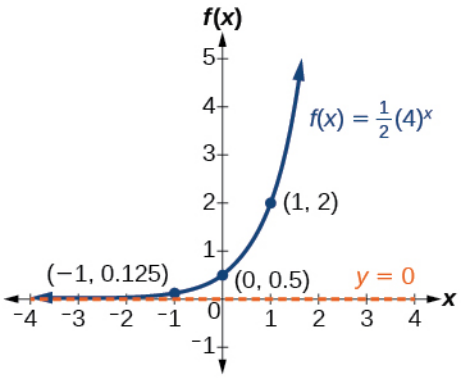


**6.14**. The domain is $(-\infty, \infty)$; the range is $(3, \infty)$; the horizontal asymptote is $y = 3$.

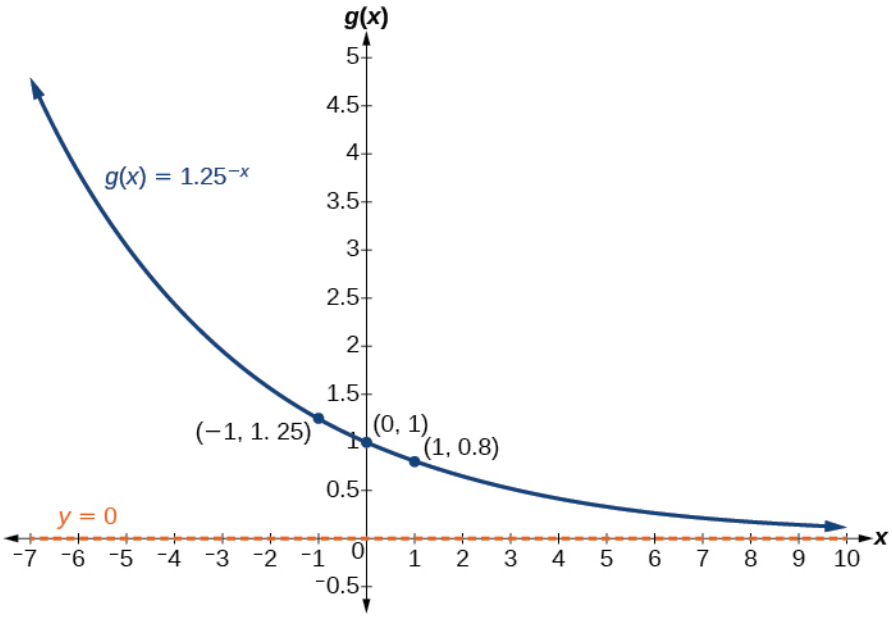



**6.15**.
$x \approx -1.608$

**6.16**. The domain is $(-\infty, \infty)$; the range is $(0, \infty)$; the horizontal asymptote is $y = 0$.




**6.17**. The domain is $(-\infty, \infty)$; the range is $(0, \infty)$; the horizontal asymptote is $y = 0$.




**6.18**.

$f(x) = -\frac{1}{3}e^x - 2;$ the domain is $(-\infty, \infty);$ the range is $(-\infty, 2);$ the horizontal asymptote is $y = 2.$

**6.19**.

a. $\log_{10}(1,\,000,\,000) = 6$ is equivalent to $10^6 = 1,\,000,\,000$

b. $\log_5(25) = 2$ is equivalent to $5^2 = 25$

**6.20**.

a. $3^2 = 9$ is equivalent to $\log_3(9) = 2$

b. $5^3 = 125$ is equivalent to $\log_5(125) = 3$

c. $2^{-1} = \frac{1}{2}$ is equivalent to $\log_2(\frac{1}{2}) = -1$

**6.21**.

$\log_{121}(11) = \frac{1}{2}$ (recalling that $\sqrt{121} = (121)^{\frac{1}{2}} = 11$ )

**6.22**.

$\log_2\left(\frac{1}{32}\right) = -5$

**6.23**.
$\log(1,\,000,\,000) = 6$

**6.24**.
$\log(123) \approx 2.0899$

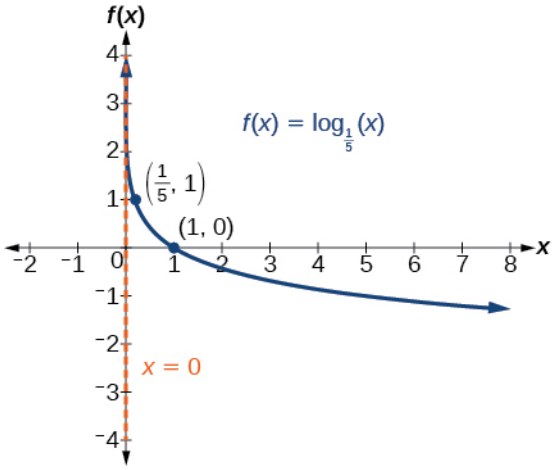
**6.25**. The difference in magnitudes was about
3.929.

**6.26**. It is not possible to take the logarithm of a negative number in the set of real numbers.

**6.27**.
$(2,\,\infty)$

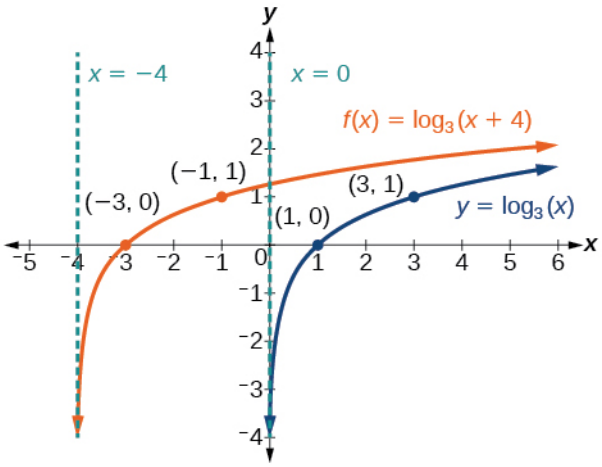
**6.28**.
$(5,\,\infty)$

**6.29**.



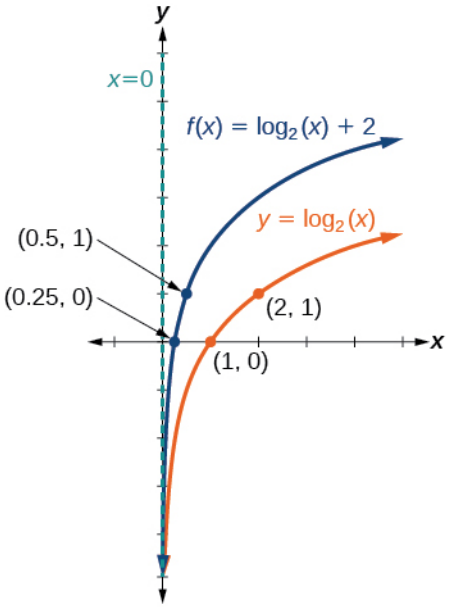

The domain is
$(0,\,\infty)$, the range is
$(-\infty,\,\infty)$, and the vertical asymptote is
$x = 0$.

**6.30**.



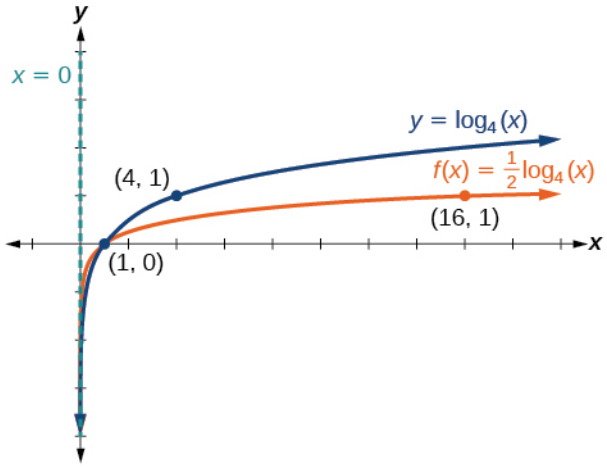

The domain is
$(-4,\,\infty)$, the range
$(-\infty,\,\infty)$, and the asymptote
$x = -4$.

**6.31**.



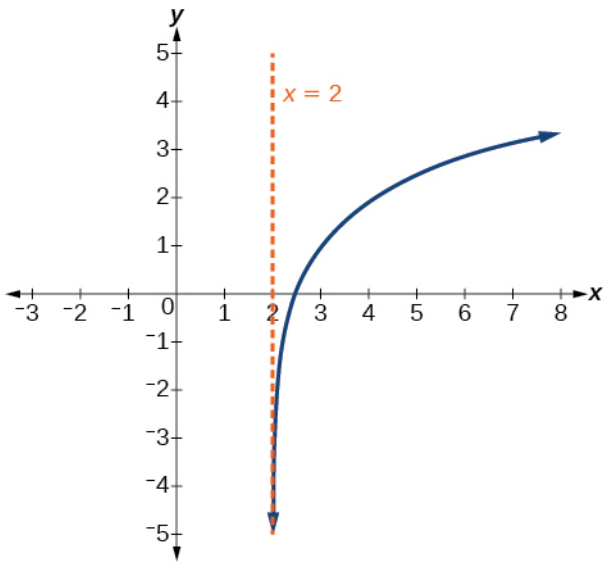

The domain is $(0, \infty)$, the range is $(-\infty, \infty)$, and the vertical asymptote is $x = 0$.

**6.32**.



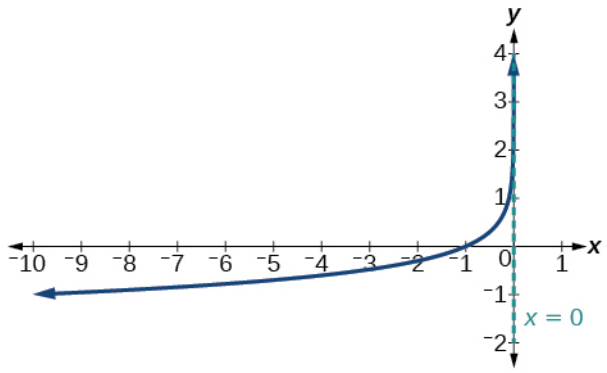

The domain is $(0, \infty)$, the range is $(-\infty, \infty)$, and the vertical asymptote is $x = 0$.

**6.33**.




The domain is $(2, \infty)$, the range is $(-\infty, \infty)$, and the vertical asymptote is $x = 2$.

**6.34**.




The domain is $(-\infty, 0)$, the range is $(-\infty, \infty)$, and the vertical asymptote is $x = 0$.

**6.35**.

$x \approx 3.049$

**6.36**.

$x = 1$

**6.37**.

$f(x) = 2\ln(x+3) - 1$

**6.38**.

$\log_b 2 + \log_b 2 + \log_b 2 + \log_b k = 3\log_b 2 + \log_b k$

**6.39**.

$\log_3(x+3) - \log_3(x-1) - \log_3(x-2)$

**6.40**.

$2\ln x$

**6.41**.

$-2\ln(x)$

**6.42**.

$\log_3 16$

**6.43**.
$2\log x + 3\log y - 4\log z$

**6.44**.
$\frac{2}{3}\ln x$

**6.45**.
$\frac{1}{2}\ln(x-1) + \ln(2x+1) - \ln(x+3) - \ln(x-3)$

**6.46**.
$\log\left(\frac{3 \cdot 5}{4 \cdot 6}\right)$; can also be written
$\log\left(\frac{5}{8}\right)$ by reducing the fraction to lowest terms.

**6.47**.
$\log\left(\frac{5(x-1)^3\sqrt{x}}{(7x-1)}\right)$

**6.48**.
$\log\frac{x^{12}(x+5)^4}{(2x+3)^4}$; this answer could also be written
$\log\left(\frac{x^3(x+5)}{(2x+3)}\right)^4$.

**6.49**. The pH increases by about 0.301.

**6.50**.
$\frac{\ln 8}{\ln 0.5}$

**6.51**.
$\frac{\ln 100}{\ln 5} \approx \frac{4.6051}{1.6094} = 2.861$

**6.52**.
$x = -2$

**6.53**.
$x = -1$

**6.54**.
$x = \frac{1}{2}$

**6.55**. The equation has no solution.

**6.56**.
$x = \frac{\ln 3}{\ln\left(\frac{2}{3}\right)}$

**6.57**.
$t = 2\ln\left(\frac{11}{3}\right)$ or
$\ln\left(\frac{11}{3}\right)^2$

**6.58**.
$t = \ln\left(\frac{1}{\sqrt{2}}\right) = -\frac{1}{2}\ln(2)$

**6.59**.
$x = \ln 2$

**6.60**.
$x = e^4$

**6.61**.
$x = e^5 - 1$

**6.62**.
$x \approx 9.97$
**6.63**.
$x = 1$ or
$x = -1$
**6.64**.
$t = 703,\ 800,\ 000 \times \frac{\ln(0.8)}{\ln(0.5)}$ years $\approx$ 226, 572, 993 years.

**6.65**.
$f(t) = A_0 e^{-0.0000000087t}$

**6.66**. less than 230 years, 229.3157 to be exact
**6.67**.
$f(t) = A_0 e^{\frac{\ln 2}{3}t}$

**6.68**. 6.026 hours
**6.69**. 895 cases on day 15
**6.70**. Exponential.
$y = 2e^{0.5x}$.

**6.71**.
$y = 3e^{(\ln 0.5)x}$

**6.72**.

a. The exponential regression model that fits these data is
$y = 522.88585984(1.19645256)^x$.

b. If spending continues at this rate, the graduate's credit card debt will be $4,499.38 after one year.

**6.73**.

a. The logarithmic regression model that fits these data is
$y = 141.91242949 + 10.45366573\ln(x)$

b. If sales continue at this rate, about 171,000 games will be sold in the year 2015.

**6.74**.

a. The logistic regression model that fits these data is
$y = \frac{25.65665979}{1 + 6.113686306e^{-0.3852149008x}}$.

b. If the population continues to grow at this rate, there will be about
25,634 seals in 2020.

c. To the nearest whole number, the carrying capacity is 25,657.

## Section Exercises

**1**. Linear functions have a constant rate of change. Exponential functions increase based on a percent of the original.
**3**. When interest is compounded, the percentage of interest earned to principal ends up being greater than the annual percentage rate for the investment account. Thus, the annual percentage rate does not necessarily correspond to the real interest earned, which is the very definition of *nominal*.
**5**. exponential; the population decreases by a proportional rate. .
**7**. not exponential; the charge decreases by a constant amount each visit, so the statement represents a linear function. .
**9**. The forest represented by the function
$B(t) = 82(1.029)^t$.

**11**. After
$t = 20$ years, forest A will have
43 more trees than forest B.
**13**. Answers will vary. Sample response: For a number of years, the population of forest A will increasingly exceed forest B, but because forest B actually grows at a faster rate, the population will eventually become larger than forest A and will remain that way as long as the population growth models hold. Some factors that might influence the long-term validity of the exponential growth model are drought, an epidemic that culls the population, and other environmental and biological factors.
**15**. exponential growth; The growth factor,
1.06, is greater than
1.

**17**. exponential decay; The decay factor, $0.97,$ is between $0$ and $1.$

**19**.
$f(x) = 2000(0.1)^x$

**21**.
$f(x) = \left(\frac{1}{6}\right)^{-\frac{3}{5}} \left(\frac{1}{6}\right)^{\frac{x}{5}} \approx 2.93(0.699)^x$

**23**. Linear

**25**. Neither

**27**. Linear

**29**.
$\$10,\ 250$

**31**.
$\$13,\ 268.58$

**33**.
$P = A(t) \cdot \left(1 + \frac{r}{n}\right)^{-nt}$

**35**.
$\$4{,}572.56$

**37**.
$4\%$

**39**. continuous growth; the growth rate is greater than $0.$

**41**. continuous decay; the growth rate is less than $0.$

**43**.
$\$669.42$

**45**.
$f(-1) = -4$

**47**.
$f(-1) \approx -0.2707$

**49**.
$f(3) \approx 483.8146$

**51**.
$y = 3 \cdot 5^x$

**53**.
$y \approx 18 \cdot 1.025^x$

**55**.
$y \approx 0.2 \cdot 1.95^x$

**57**.
$$\text{APY} = \frac{A(t) - a}{a} = \frac{a\left(1 + \frac{r}{365}\right)^{365(1)} - a}{a} = \frac{a\left[\left(1 + \frac{r}{365}\right)^{365} - 1\right]}{a} = \left(1 + \frac{r}{365}\right)^{365} - 1;$$
$$I(n) = \left(1 + \frac{r}{n}\right)^n - 1$$

**59**. Let $f$ be the exponential decay function $f(x) = a \cdot \left(\frac{1}{b}\right)^x$ such that $b > 1.$ Then for some number

$n > 0$,

$$f(x) = a \cdot \left(\frac{1}{b}\right)^x = a\left(b^{-1}\right)^x = a\left((e^n)^{-1}\right)^x = a(e^{-n})^x = a(e)^{-nx}.$$

**61**.
47, 622 fox

**63**.
1.39%;
$155, 368.09

**65**.
$35, 838.76

**67**.
$82, 247.78;
$449.75

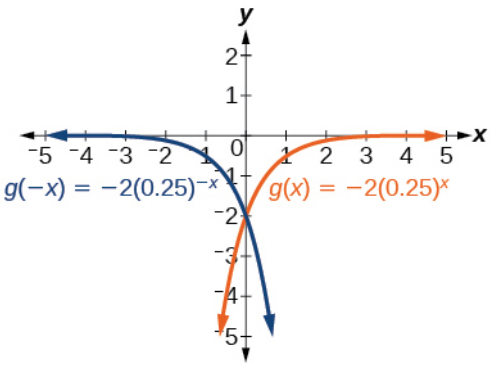
**69**. An asymptote is a line that the graph of a function approaches, as $x$ either increases or decreases without bound. The horizontal asymptote of an exponential function tells us the limit of the function's values as the independent variable gets either extremely large or extremely small.

**71**.
$g(x) = 4(3)^{-x}$; $y$-intercept: $(0, 4)$; Domain: all real numbers; Range: all real numbers greater than 0.

**73**.
$g(x) = -10^x + 7$; $y$-intercept: $(0, 6)$; Domain: all real numbers; Range: all real numbers less than 7.

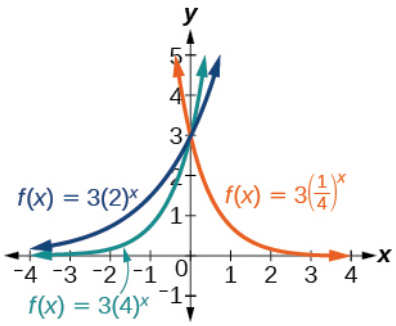
**75**.
$g(x) = 2\left(\frac{1}{4}\right)^x$; $y$-intercept: $(0, 2)$; Domain: all real numbers; Range: all real numbers greater than 0.

**77**.

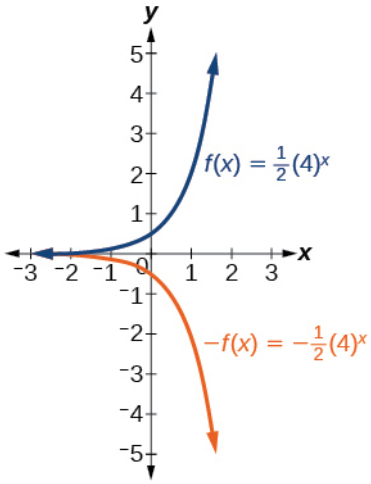



$y$-intercept: $(0, -2)$

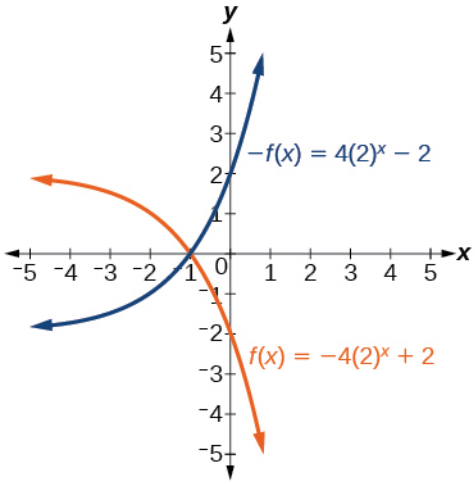
**79**.



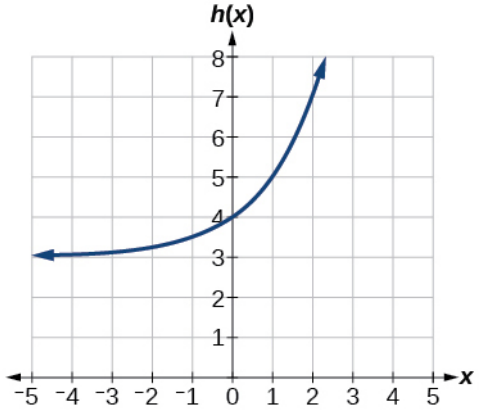

**81**. B
**83**. A
**85**. E
**87**. D
**89**. C
**91**.




**93**.




**95**.




Horizontal asymptote:
$h(x) = 3;$ Domain: all real numbers; Range: all real numbers strictly greater than 3.

**97**. As
$x \to \infty$,
$f(x) \to -\infty$;
As
$x \to -\infty$,
$f(x) \to -1$

**99**. As
$x \to \infty$,
$f(x) \to 2$;
As
$x \to -\infty$,
$f(x) \to \infty$

**101**.
$f(x) = 4^x - 3$

**103**.
$f(x) = 4^{x-5}$

**105**.
$f(x) = 4^{-x}$

**107**.
$y = -2^x + 3$

**109**.
$y = -2(3)^x + 7$

**111**.
$g(6) = 800 + \frac{1}{3} \approx 800.3333$

**113**.
$h(-7) = -58$

**115**.
$x \approx -2.953$

**117**.
$x \approx -0.222$

**119**. The graph of
$G(x) = \left(\frac{1}{b}\right)^x$ is the refelction about the *y*-axis of the graph of
$F(x) = b^x;$ For any real number
$b > 0$ and function
$f(x) = b^x,$ the graph of

$\left(\frac{1}{b}\right)^x$ is the the reflection about the *y*-axis,
$F(-x)$.

**121**. The graphs of
$g(x)$ and
$h(x)$ are the same and are a horizontal shift to the right of the graph of
$f(x)$; For any real number *n*, real number
$b > 0$, and function
$f(x) = b^x$, the graph of
$\left(\frac{1}{b^n}\right)b^x$ is the horizontal shift
$f(x-n)$.

**123**. A logarithm is an exponent. Specifically, it is the exponent to which a base
$b$ is raised to produce a given value. In the expressions given, the base
$b$ has the same value. The exponent,
$y$, in the expression
$b^y$ can also be written as the logarithm,
$\log_b x$, and the value of
$x$ is the result of raising
$b$ to the power of
$y$.

**125**. Since the equation of a logarithm is equivalent to an exponential equation, the logarithm can be converted to the exponential equation
$b^y = x$, and then properties of exponents can be applied to solve for
$x$.

**127**. The natural logarithm is a special case of the logarithm with base
$b$ in that the natural log always has base
$e$. Rather than notating the natural logarithm as
$\log_e(x)$, the notation used is
$\ln(x)$.

**129**.
$a^c = b$

**131**.
$x^y = 64$

**133**.
$15^b = a$

**135**.
$13^a = 142$

**137**.
$e^n = w$

**139**.
$\log_c(k) = d$

**141**.
$\log_{19} y = x$

**143**.
$\log_n(103) = 4$

**145**.
$\log_y\left(\frac{39}{100}\right) = x$

**147**.
$\ln(h) = k$

**149**.
$x = 2^{-3} = \frac{1}{8}$
**151**.
$x = 3^3 = 27$
**153**.
$x = 9^{\frac{1}{2}} = 3$
**155**.
$x = 6^{-3} = \frac{1}{216}$
**157**.
$x = e^2$
**159**.
32
**161**.
1.06
**163**.
14.125
**165**.
$\frac{1}{2}$
**167**.
4
**169**.
$-3$
**171**.
$-12$
**173**.
0
**175**.
10
**177**.
2.708
**179**.
0.151
**181**. No, the function has no defined value for $x = 0.$ To verify, suppose $x = 0$ is in the domain of the function $f(x) = \log(x).$ Then there is some number $n$ such that $n = \log(0).$ Rewriting as an exponential equation gives: $10^n = 0,$ which is impossible since no such real number $n$ exists. Therefore, $x = 0$ is *not* the domain of the function $f(x) = \log(x).$
**183**. Yes. Suppose there exists a real number $x$ such that $\ln x = 2.$ Rewriting as an exponential equation gives $x = e^2,$ which is a real number. To verify, let $x = e^2.$ Then, by definition, $\ln(x) = \ln\left(e^2\right) = 2.$

**185**. No;
$\ln(1) = 0,$ so
$\frac{\ln\left(e^{1.725}\right)}{\ln(1)}$ is undefined.
**187**.
2
**189**. Since the functions are inverses, their graphs are mirror images about the line
$y = x.$ So for every point
$(a, b)$ on the graph of a logarithmic function, there is a corresponding point
$(b, a)$ on the graph of its inverse exponential function.
**191**. Shifting the function right or left and reflecting the function about the y-axis will affect its domain.
**193**. No. A horizontal asymptote would suggest a limit on the range, and the range of any logarithmic function in general form is all real numbers.
**195**. Domain:
$\left(-\infty, \frac{1}{2}\right);$ Range:
$(-\infty, \infty)$
**197**. Domain:
$\left(-\frac{17}{4}, \infty\right);$ Range:
$(-\infty, \infty)$
**199**. Domain:
$(5, \infty);$ Vertical asymptote:
$x = 5$
**201**. Domain:
$\left(-\frac{1}{3}, \infty\right);$ Vertical asymptote:
$x = -\frac{1}{3}$
**203**. Domain:
$(-3, \infty);$ Vertical asymptote:
$x = -3$
**205**. Domain:
$\left(\frac{3}{7}, \infty\right)$;
Vertical asymptote:
$x = \frac{3}{7}$; End behavior: as
$x \to \left(\frac{3}{7}\right)^{+}, f(x) \to -\infty$ and as
$x \to \infty, f(x) \to \infty$
**207**. Domain:
$(-3, \infty)$; Vertical asymptote:
$x = -3$;
End behavior: as
$x \to -3^{+}$,
$f(x) \to -\infty$ and as
$x \to \infty$,
$f(x) \to \infty$
**209**. Domain:
$(1, \infty);$ Range:
$(-\infty, \infty);$ Vertical asymptote:

$x = 1$; $x$-intercept: $\left(\frac{5}{4}, 0\right)$; $y$-intercept: DNE

**211**. Domain: $(-\infty, 0)$; Range: $(-\infty, \infty)$; Vertical asymptote: $x = 0$; $x$-intercept: $\left(-e^2, 0\right)$; $y$-intercept: DNE

**213**. Domain: $(0, \infty)$; Range: $(-\infty, \infty)$; Vertical asymptote: $x = 0$; $x$-intercept: $\left(e^3, 0\right)$; $y$-intercept: DNE

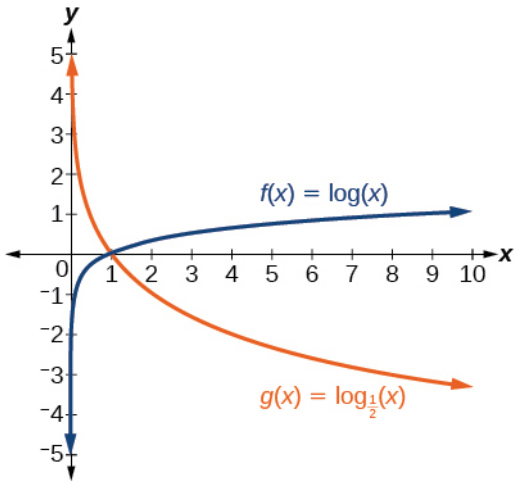
**215**. B

**217**. C

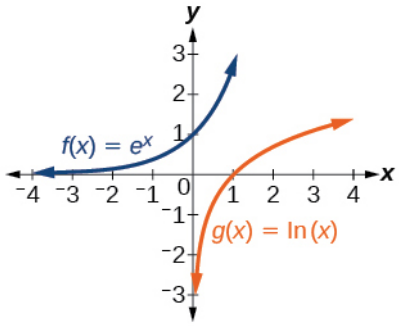
**219**. B

**221**. C

**223**.

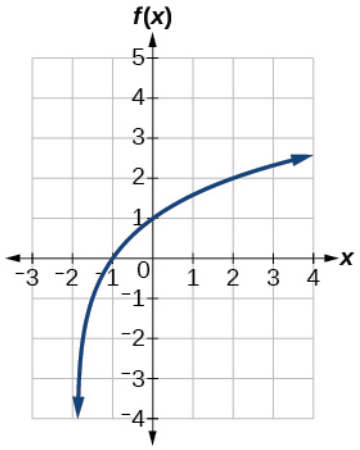



**225**.

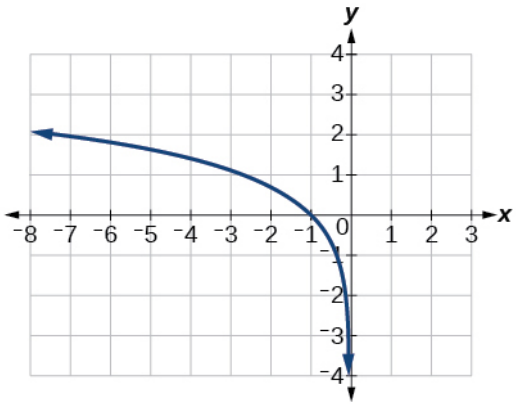



**227**. C

**229**.




**231**.

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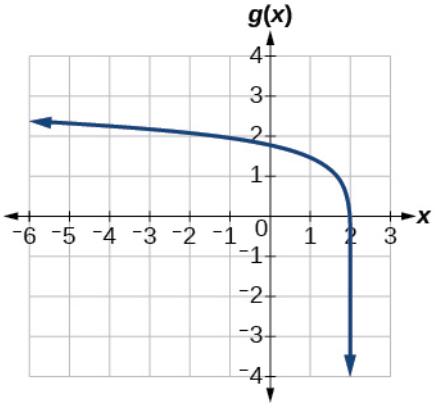



**235**.
$f(x) = \log_2(-(x-1))$

**237**.
$f(x) = 3\log_4(x+2)$

**239**.
$x = 2$

**241**.
$x \approx 2.303$

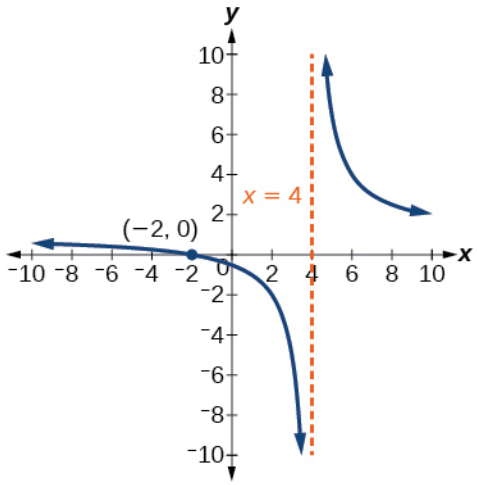
**243**.
$x \approx -0.472$

**245**. The graphs of
$f(x) = \log_{\frac{1}{2}}(x)$ and
$g(x) = -\log_2(x)$ appear to be the same; Conjecture: for any positive base

$b \neq 1$,

$\log_b(x) = -\log_{\frac{1}{b}}(x)$.

**247**. Recall that the argument of a logarithmic function must be positive, so we determine where $\frac{x+2}{x-4} > 0$. From the graph of the function $f(x) = \frac{x+2}{x-4}$, note that the graph lies above the $x$-axis on the interval $(-\infty, -2)$ and again to the right of the vertical asymptote, that is $(4, \infty)$. Therefore, the domain is $(-\infty, -2) \cup (4, \infty)$.




**249**. Any root expression can be rewritten as an expression with a rational exponent so that the power rule can be applied, making the logarithm easier to calculate. Thus,

$\log_b\left(x^{\frac{1}{n}}\right) = \frac{1}{n}\log_b(x)$.

**251**.

$\log_b(2) + \log_b(7) + \log_b(x) + \log_b(y)$

**253**.

$\log_b(13) - \log_b(17)$

**255**.

$-k\ln(4)$

**257**.

$\ln(7xy)$

**259**.

$\log_b(4)$

**261**.

$\log_b(7)$

**263**.

$15\log(x) + 13\log(y) - 19\log(z)$

**265**.

$\frac{3}{2}\log(x) - 2\log(y)$

**267**.

$\frac{8}{3}\log(x) + \frac{14}{3}\log(y)$

**269**.

$\ln(2x^7)$

**271**.

$\log\left(\frac{xz^3}{\sqrt{y}}\right)$

**273**.

$\log_7(15) = \frac{\ln(15)}{\ln(7)}$

**275**.

$\log_{11}(5) = \frac{\log_5(5)}{\log_5(11)} = \frac{1}{b}$

**277**.

$\log_{11}\left(\frac{6}{11}\right) = \frac{\log_5\left(\frac{6}{11}\right)}{\log_5(11)} = \frac{\log_5(6) - \log_5(11)}{\log_5(11)} = \frac{a-b}{b} = \frac{a}{b} - 1$

**279**.

3

**281**.

2.81359

**283**.

0.93913

**285**.

−2.23266

**287**.

$x = 4$; By the quotient rule:

$\log_6(x+2) - \log_6(x-3) = \log_6\left(\frac{x+2}{x-3}\right) = 1.$ Rewriting as an exponential equation and solving for $x$:

$$\begin{aligned} 6^1 &= \frac{x+2}{x-3} \\ 0 &= \frac{x+2}{x-3} - 6 \\ 0 &= \frac{x+2}{x-3} - \frac{6(x-3)}{(x-3)} \\ 0 &= \frac{x+2-6x+18}{x-3} \\ 0 &= \frac{x-4}{x-3} \\ x &= 4 \end{aligned}$$

Checking, we find that $\log_6(4+2) - \log_6(4-3) = \log_6(6) - \log_6(1)$ is defined, so $x = 4$.

**289**. Let $b$ and $n$ be positive integers greater than 1. Then, by the change-of-base formula,

$\log_b(n) = \frac{\log_n(n)}{\log_n(b)} = \frac{1}{\log_n(b)}.$

**291**. Determine first if the equation can be rewritten so that each side uses the same base. If so, the exponents can be set equal to each other. If the equation cannot be rewritten so that each side uses the same base, then apply the logarithm to each side and use properties of logarithms to solve.

**293**. The one-to-one property can be used if both sides of the equation can be rewritten as a single logarithm with the same base. If so, the arguments can be set equal to each other, and the resulting equation can be solved algebraically. The one-to-one property cannot be used when each side of the equation cannot be rewritten as a single logarithm with the same base.

**295**.

$x = -\frac{1}{3}$

**297**.

$n = -1$

**299**.
$b = \frac{6}{5}$

**301**.
$x = 10$

**303**. No solution

**305**.
$p = \log\left(\frac{17}{8}\right) - 7$

**307**.
$k = -\frac{\ln(38)}{3}$

**309**.
$x = \frac{\ln\left(\frac{38}{3}\right) - 8}{9}$

**311**.
$x = \ln 12$

**313**.
$x = \frac{\ln\left(\frac{3}{5}\right) - 3}{8}$

**315**. no solution

**317**.
$x = \ln(3)$

**319**.
$10^{-2} = \frac{1}{100}$

**321**.
$n = 49$

**323**.
$k = \frac{1}{36}$

**325**.
$x = \frac{9 - e}{8}$

**327**.
$n = 1$

**329**. No solution

**331**. No solution

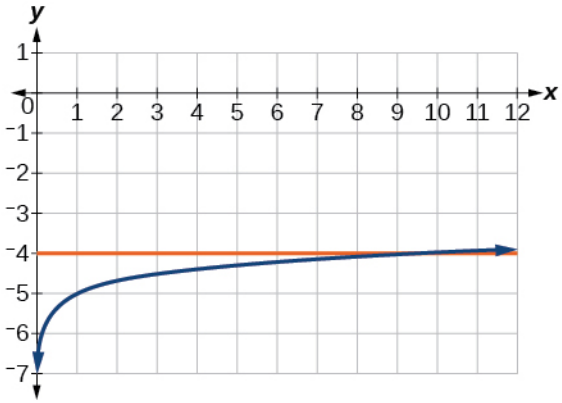
**333**.
$x = \pm\frac{10}{3}$

**335**.
$x = 10$

**337**.
$x = 0$

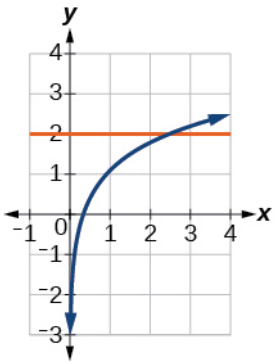
**339**.
$x = \frac{3}{4}$

**341**.
$x = 9$



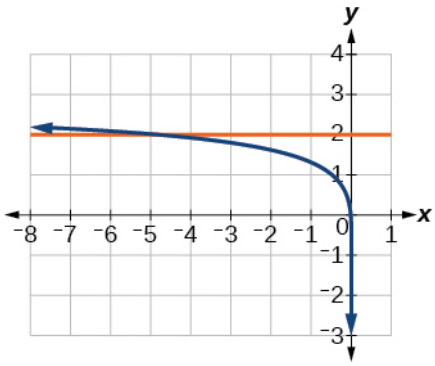

**343**.

$x = \frac{e^2}{3} \approx 2.5$



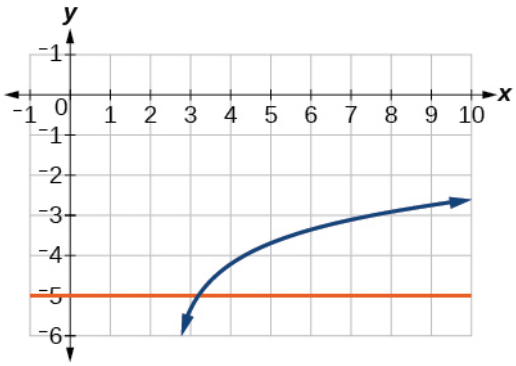

**345**.

$x = -5$

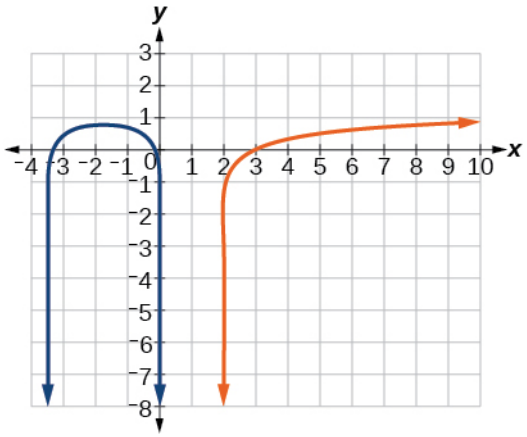



**347**.

$x = \frac{e + 10}{4} \approx 3.2$

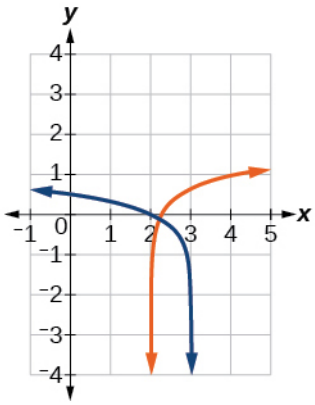



**349**. No solution



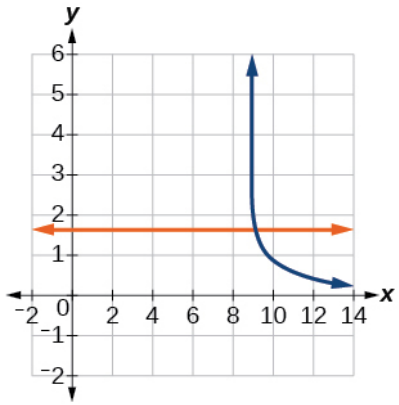

**351**.

$x = \frac{11}{5} \approx 2.2$



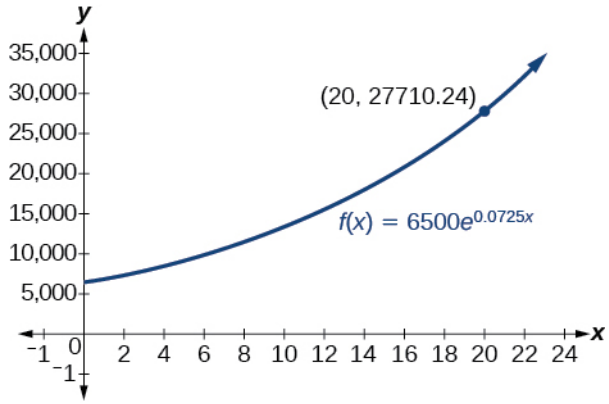

**353**.

$x = \frac{101}{11} \approx 9.2$

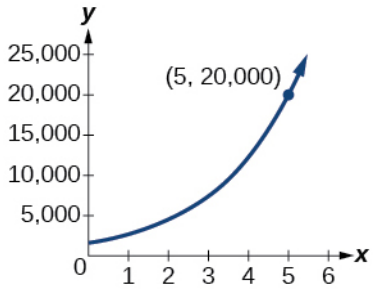



**355**. about

$27,710.24




**357**. about 5 years




**359**.

$\frac{\ln(17)}{5} \approx 0.567$

**361**.

$x = \frac{\log(38) + 5\log(3)}{4\log(3)} \approx 2.078$

**363**.

$x \approx 2.2401$

**365**.

$x \approx -44655.7143$

**367**. about

5.83

**369**.

$t = \ln\left(\left(\frac{y}{A}\right)^{\frac{1}{k}}\right)$

**371**.

$t = \ln\left(\left(\frac{T - T_s}{T_0 - T_s}\right)^{-\frac{1}{k}}\right)$

**372**. Half-life is a measure of decay and is thus associated with exponential decay models. The half-life of a substance or quantity is the amount of time it takes for half of the initial amount of that substance or quantity to decay.

**374**. Doubling time is a measure of growth and is thus associated with exponential growth models. The doubling time of a substance or quantity is the amount of time it takes for the initial amount of that substance or quantity to double in size.

**376**. An order of magnitude is the nearest power of ten by which a quantity exponentially grows. It is also an approximate position on a logarithmic scale; Sample response: Orders of magnitude are useful when making comparisons between numbers that differ by a great amount. For example, the mass of Saturn is 95 times greater than the mass of Earth. This is the same as saying that the mass of Saturn is about

$10^2$ times, or *2 orders of magnitude* greater, than the mass of Earth.

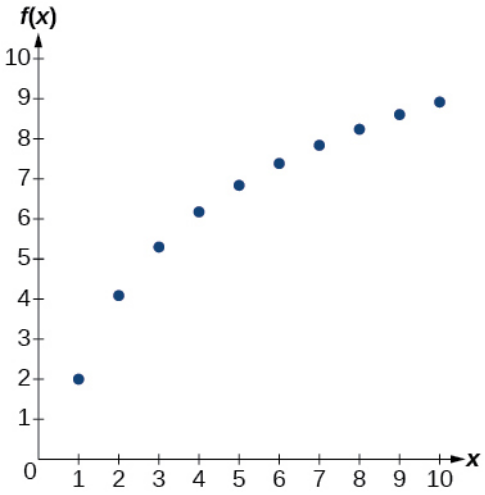
**378**.

$f(0) \approx 16.7;$ The amount initially present is about 16.7 units.

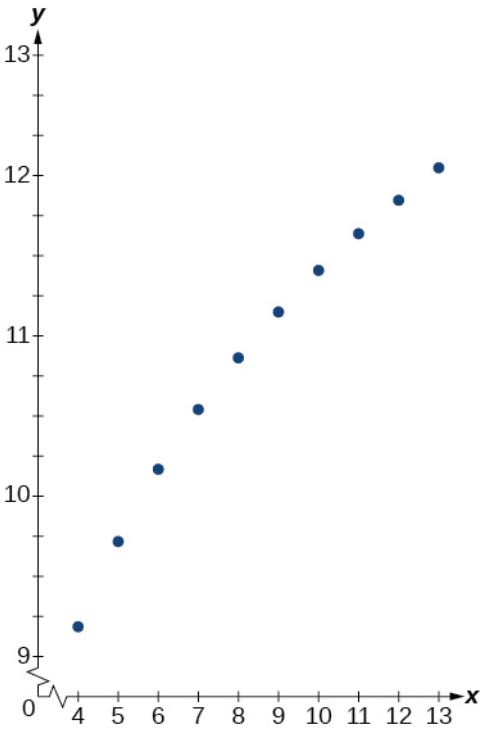
**380**. 150

**383**. exponential;
$f(x) = 1.2^x$

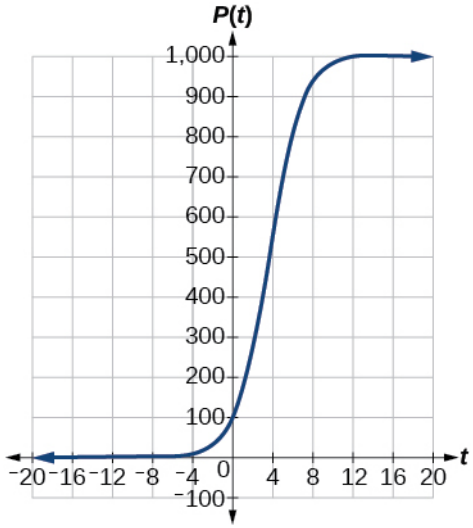
**385**. logarithmic




**387**. logarithmic




**389**.




**391**. about
$1.4$ years
**393**. about
$7.3$ years
**395**.
$4$ half-lives;
$8.18$ minutes
**397**.

$$M = \frac{2}{3}\log\left(\frac{S}{S_0}\right)$$

$$\log\left(\frac{S}{S_0}\right) = \frac{3}{2}M$$

$$\frac{S}{S_0} = 10^{\frac{3M}{2}}$$

$$S = S_0 10^{\frac{3M}{2}}$$

**399**. Let
$y = b^x$ for some non-negative real number
$b$ such that
$b \neq 1.$ Then,

$$\ln(y) = \ln(b^x)$$

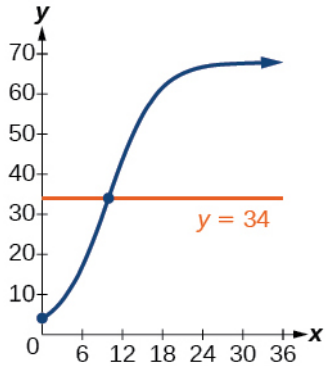
$$\ln(y) = x\ln(b)$$

$$e^{\ln(y)} = e^{x\ln(b)}$$

$$y = e^{x\ln(b)}$$

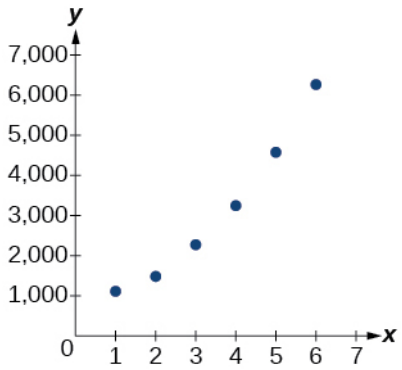
**401**.
$A = 125e^{(-0.3567t)}$; $A \approx 43$ mg
**403**. about
$60$ days
**405**.
$f(t) = 250e^{(-0.00914t)}$; half-life: about
$76$ minutes
**407**.
$r \approx -0.0667,$ So the hourly decay rate is about
$6.67\%$

**409**.
$f(t) = 1350e^{(0.03466t)}$; after 3 hours:
$P(180) \approx 691,\ 200$
**411**.
$f(t) = 256e^{(0.068110t)}$; doubling time: about
$10$ minutes
**413**. about
$88$ minutes
**415**.
$T(t) = 90e^{(-0.008377t)} + 75,$ where
$t$ is in minutes.
**417**. about
$113$ minutes
**419**.
$\log(x) = 1.5;\ x \approx 31.623$
**421**. MMS magnitude:
$5.82$
**423**.
$N(3) \approx 71$
**425**. C
**426**. Logistic models are best used for situations that have limited values. For example, populations cannot grow indefinitely since resources such as food, water, and space are limited, so a logistic model best describes populations.
**428**. Regression analysis is the process of finding an equation that best fits a given set of data points. To perform a regression analysis on a graphing utility, first list the given points using the STAT then EDIT menu. Next graph the scatter plot using the STAT PLOT feature. The shape of the data points on the scatter graph can help determine which regression feature to use. Once this is determined, select the appropriate regression analysis command from the STAT then CALC menu.
**430**. The $y$-intercept on the graph of a logistic equation corresponds to the initial population for the population model.
**432**. C
**434**. B
**436**.
$P(0) = 22$ ; 175
**438**.
$p \approx 2.67$
**440**. $y$-intercept:
$(0,\ 15)$
**442**.
$4$ koi
**444**. about
$6.8$ months.
**445**.




**447**.
$10$ wolves
**449**. about 5.4 years.

**451**.

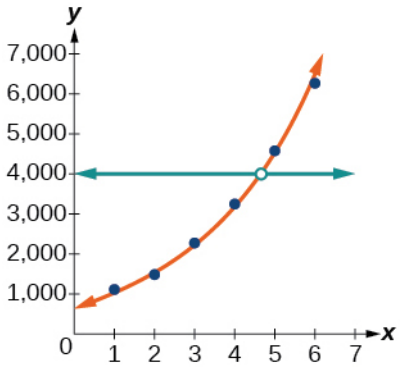



**453**.

$f(x) = 776.682e^{0.3549x}$

**455**. When $f(x) = 4000$,

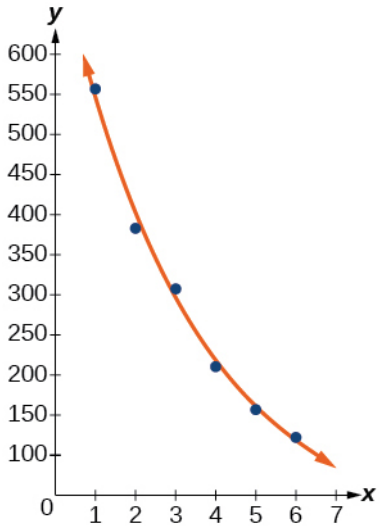
$x \approx 4.6$.



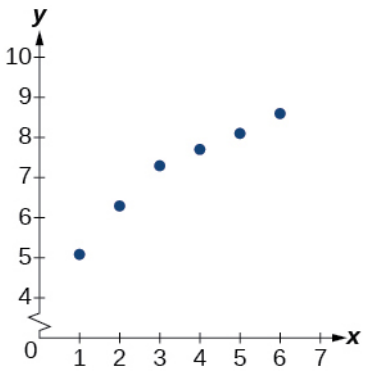

**457**.

$f(x) = 731.92(0.738)^x$

**459**.

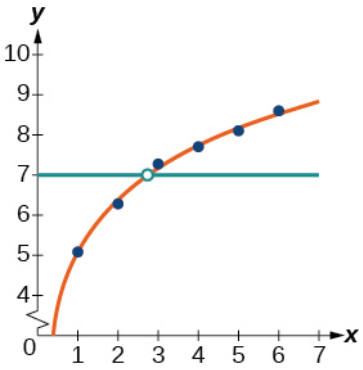



**461**.



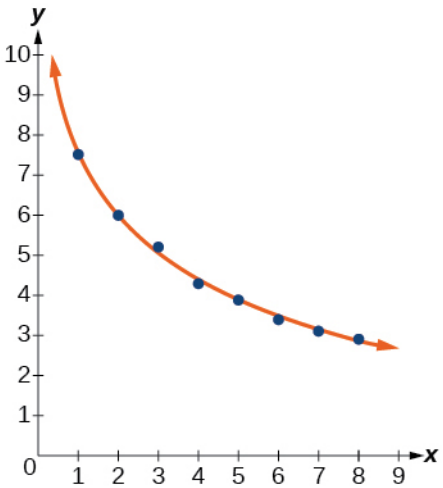

**463**.
$f(10) \approx 9.5$

**465**. When
$f(x) = 7,$
$x \approx 2.7.$

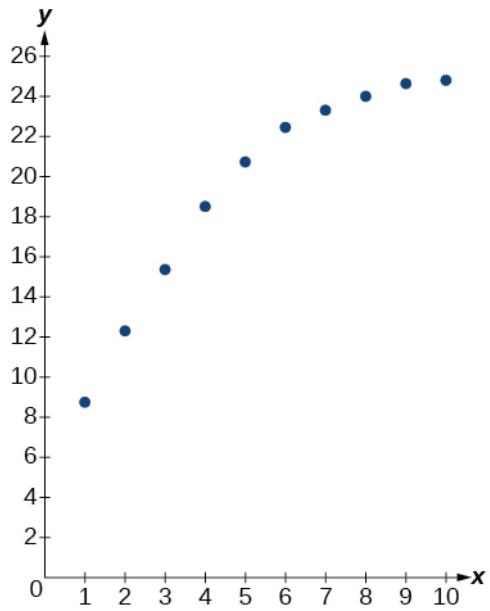



**467**.
$f(x) = 7.544 - 2.268\ln(x)$

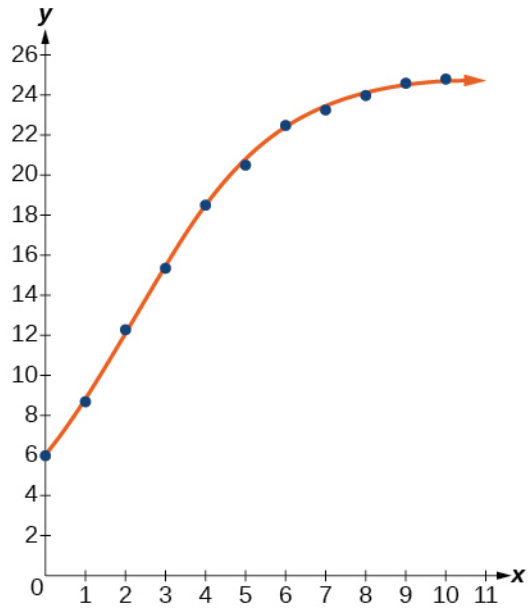
**469**.



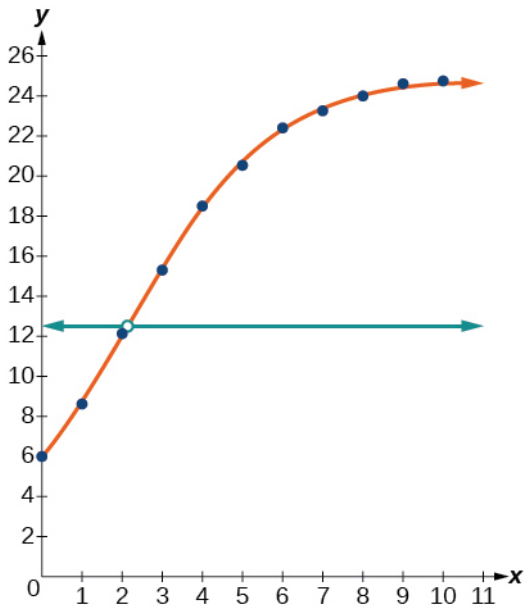

**471**.




**473**.




**475**. When $f(x) = 12.5$, $x \approx 2.1$.




**477**.

$$f(x) = \frac{136.068}{1 + 10.324e^{-0.480x}}$$

**479**. about 136

**481**. Working with the left side of the equation, we see that it can be rewritten as $ae^{-bt}$:

$$\frac{c - P(t)}{P(t)} = \frac{c - \frac{c}{1 + ae^{-bt}}}{\frac{c}{1 + ae^{-bt}}} = \frac{\frac{c\left(1 + ae^{-bt}\right) - c}{1 + ae^{-bt}}}{\frac{c}{1 + ae^{-bt}}} = \frac{\frac{c\left(1 + ae^{-bt} - 1\right)}{1 + ae^{-bt}}}{\frac{c}{1 + ae^{-bt}}} = 1 + ae^{-bt} - 1 = ae^{-bt}$$

Working with the right side of the equation we show that it can also be rewritten as $ae^{-bt}$. But first note that when $t = 0$,

$$P_0 = \frac{c}{1 + ae^{-b(0)}} = \frac{c}{1 + a}.$$

Therefore,

$$\frac{c - P_0}{P_0}e^{-bt} = \frac{c - \frac{c}{1 + a}}{\frac{c}{1 + a}}e^{-bt} = \frac{\frac{c(1 + a) - c}{1 + a}}{\frac{c}{1 + a}}e^{-bt} = \frac{\frac{c(1 + a - 1)}{1 + a}}{\frac{c}{1 + a}}e^{-bt} = (1 + a - 1)e^{-bt} = ae^{-bt}$$

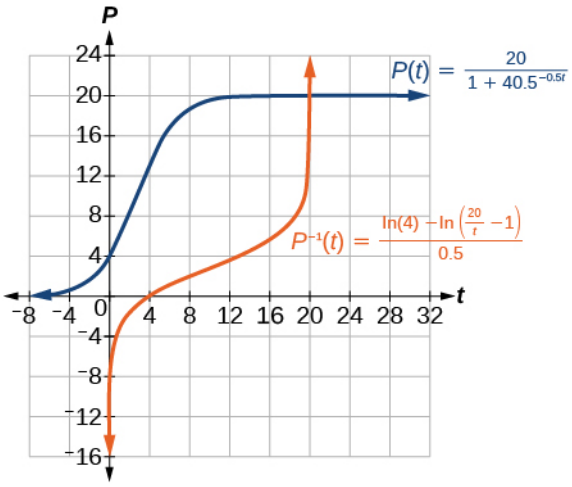
Thus,

$$\frac{c - P(t)}{P(t)} = \frac{c - P_0}{P_0}e^{-bt}.$$

**483**. First rewrite the exponential with base $e$: $f(x) = 1.034341e^{0.247800x}$. Then test to verify that $f(g(x)) = x$, taking rounding error into consideration:

$$\begin{aligned} g(f(x)) &= 4.035510\ln\left(1.034341e^{0.247800x}\right) - 0.136259 \\ &= 4.03551\left(\ln(1.034341) + \ln\left(e^{0.2478x}\right)\right) - 0.136259 \\ &= 4.03551(\ln(1.034341) + 0.2478x) - 0.136259 \\ &= 0.136257 + 0.999999x - 0.136259 \\ &= -0.000002 + 0.999999x \\ &\approx 0 + x \\ &= x \end{aligned}$$

**485**.




The graph of $P(t)$ has a $y$-intercept at (0, 4) and horizontal asymptotes at $y = 0$ and $y = 20$. The graph of $P^{-1}(t)$ has an $x$- intercept at (4, 0) and vertical asymptotes at $x = 0$ and $x = 20$.

## Review Exercises

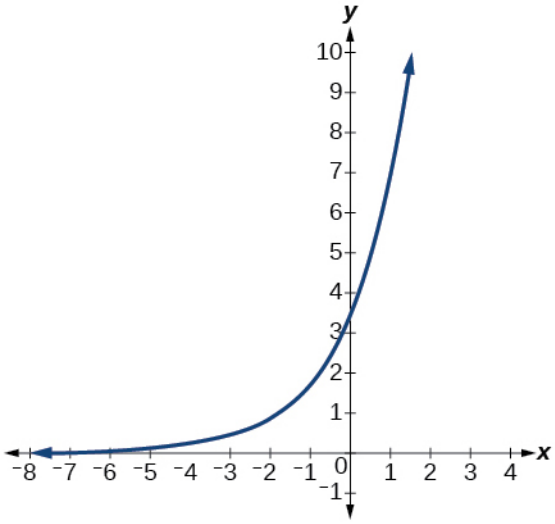
**486**. exponential decay; The growth factor, $0.825,$ is between $0$ and $1.$

**488**.
$y = 0.25(3)^x$

**490**.
$\$42,\ 888.18$

**492**. continuous decay; the growth rate is negative.

**494**. domain: all real numbers; range: all real numbers strictly greater than zero; $y$-intercept: (0, 3.5);




**496**.
$g(x) = 7(6.5)^{-x}$; $y$-intercept: $(0,\ 7)$; Domain: all real numbers; Range: all real numbers greater than $0.$

**498**.
$17^x = 4913$

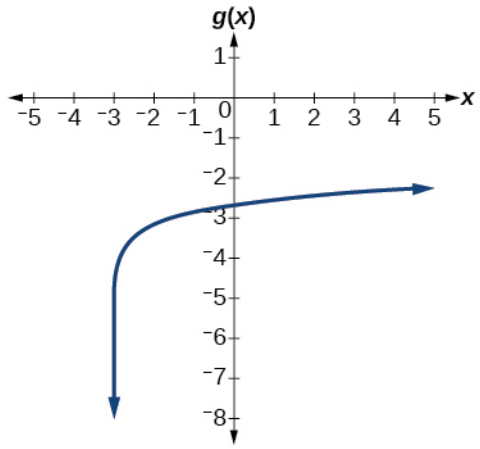
**500**.
$\log_a b = -\frac{2}{5}$

**502**.
$x = 64^{\frac{1}{3}} = 4$

**504**.
$\log(0.000001) = -6$

**506**.
$\ln\left(e^{-0.8648}\right) = -0.8648$

**508**.




**510**. Domain:
$x > -5$; Vertical asymptote:
$x = -5$; End behavior: as
$x \to -5^+$, $f(x) \to -\infty$ and as
$x \to \infty$, $f(x) \to \infty$.

**512**.
$\log_8(65xy)$

**514**.
$\ln\left(\frac{z}{xy}\right)$

**516**.
$\log_y(12)$

**518**.
$\ln(2) + \ln(b) + \frac{\ln(b+1) - \ln(b-1)}{2}$

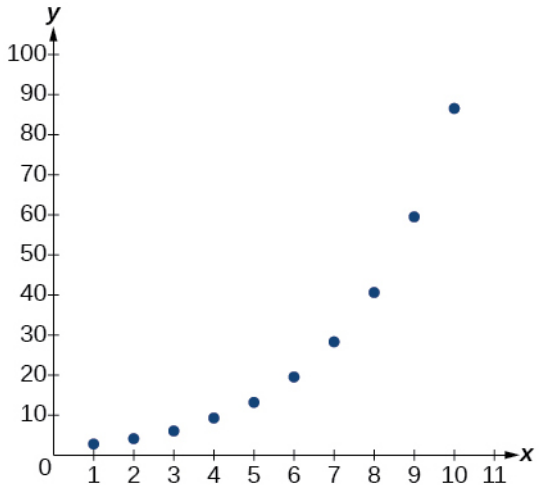
**520**.
$\log_7\left(\frac{y^3 w^6}{\sqrt[3]{u}}\right)$

**522**.
$x = \frac{\frac{\log(125)}{\log(5)} + 17}{12} = \frac{5}{3}$

**524**.
$x = -3$

**526**. no solution
**528**. no solution

**530**.
$x = \ln(11)$
**532**.
$a = e^4 - 3$
**534**.
$x = \pm\frac{9}{5}$
**536**. about
$5.45$ years
**538**.
$f^{-1}(x) = \sqrt[3]{2^{4x} - 1}$
**540**.
$f(t) = 300(0.83)^t$; $f(24) \approx 3.43\,g$
**542**. about
$45$ minutes
**544**. about
$8.5$ days
**546**. exponential




**548**.
$y = 4(0.2)^x$;
$y = 4e^{-1.609438x}$
**550**. about
$7.2$ days
**552**. logarithmic;
$y = 16.68718 - 9.71860\ln(x)$

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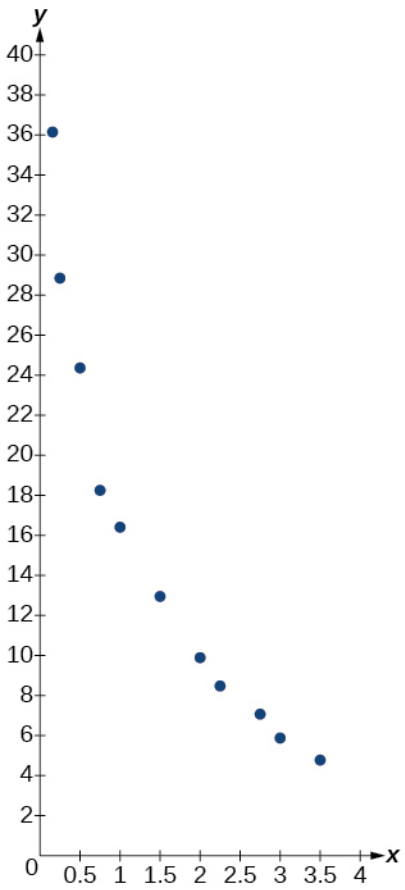
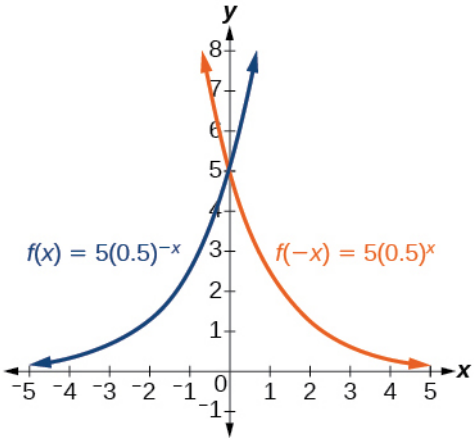



## Practice Test

**554**. About
13 dolphins.

**556**.
$1,947

**558**. *y*-intercept:
(0, 5)




**560**.
$8.5^a = 614.125$

**562**.
$x = \left(\frac{1}{7}\right)^2 = \frac{1}{49}$

**564**.
$\ln(0.716) \approx -0.334$

**566**. Domain:
$x < 3;$ Vertical asymptote:
$x = 3;$ End behavior:
$x \to 3^{-}, f(x) \to -\infty$ and
$x \to -\infty, f(x) \to \infty$

**568**.
$\log_t(12)$

**570**.
$3 \ln(y) + 2\ln(z) + \frac{\ln(x-4)}{3}$

**572**.
$x = \frac{\frac{\ln(1000)}{\ln(16)} + 5}{3} \approx 2.497$

**574**.
$a = \frac{\ln(4) + 8}{10}$

**576**. no solution

**578**.
$x = \ln(9)$

**580**.
$x = \pm \frac{3\sqrt{3}}{2}$

**582**.
$f(t) = 112e^{-.019792t};$ half-life: about
35 days

**584**.
$T(t) = 36e^{-0.025131t} + 35;\ T(60) \approx 43^{\circ}\text{F}$

**586**. logarithmic

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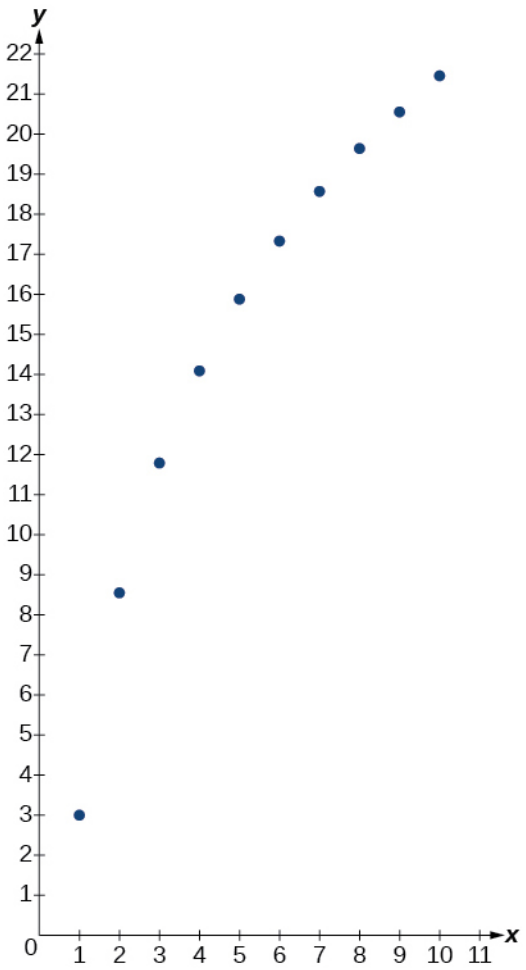
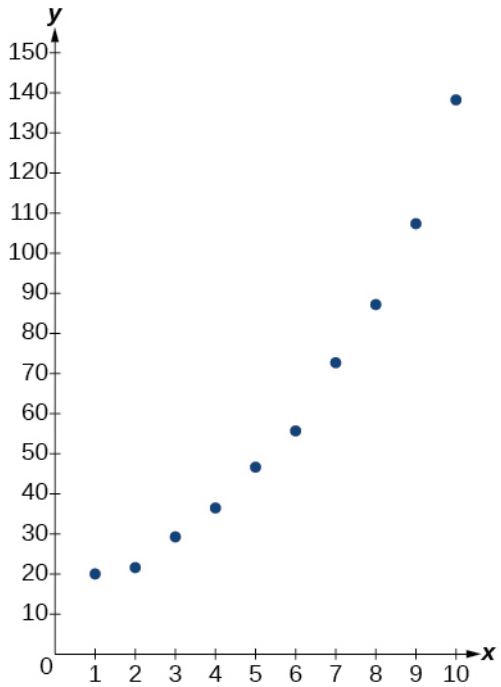
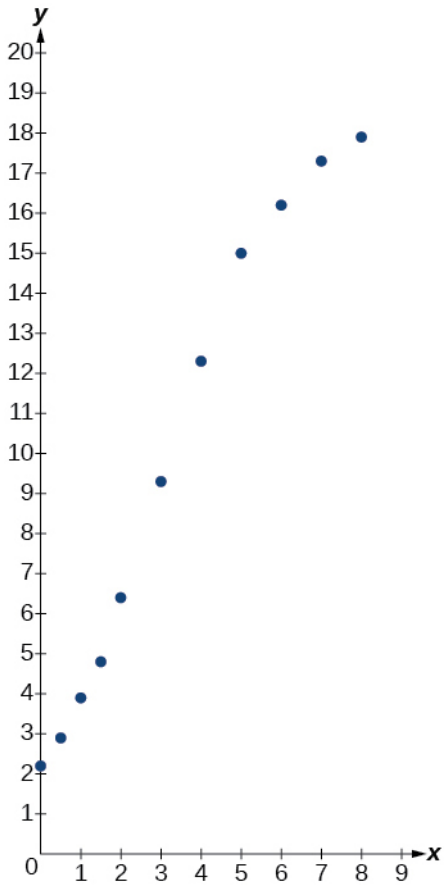



**588**. exponential;
$y = 15.10062(1.24621)^x$



**590**. logistic;

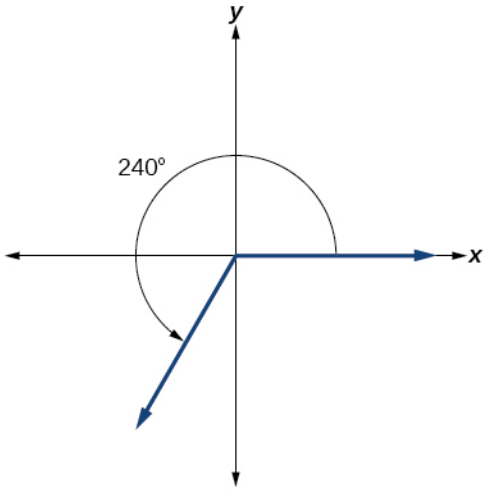
$$y = \frac{18.41659}{1 + 7.54644e^{-0.68375x}}$$




# Chapter 7

## Try It

**7.1**.




**7.2**.

$\frac{3\pi}{2}$

**7.3**.

$-135\ ^\circ$

**7.4**.
$\frac{7\pi}{10}$
**7.5**.
$\alpha = 150°$
**7.6**.
$\beta = 60°$
**7.7**.
$\frac{7\pi}{6}$
**7.8**.
$\frac{215\pi}{18} = 37.525$ units
**7.9**. 1.88
**7.10**.
$\frac{-3\pi}{2}$ rad/s
**7.11**. 1655 kilometers per hour
**7.12**.
$\frac{7}{25}$
**7.13**.
$$\sin t = \frac{33}{65}, \quad \cos t = \frac{56}{65}, \quad \tan t = \frac{33}{56},$$
$$\sec t = \frac{65}{56}, \quad \csc t = \frac{65}{33}, \quad \cot t = \frac{56}{33}$$
**7.14**.
$$\sin\left(\frac{\theta}{4}\right) = \frac{\sqrt{2}}{2}, \cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}, \tan\left(\frac{\pi}{4}\right) = 1,$$
$$\sec\left(\frac{\pi}{4}\right) = \sqrt{2}, \csc\left(\frac{\pi}{4}\right) = \sqrt{2}, \cot\left(\frac{\pi}{4}\right) = 1$$
**7.15**. 2
**7.16**.
adjacent $= 10$; opposite $= 10\sqrt{3}$; missing angle is $\frac{\pi}{6}$
**7.17**. About 52 ft
**7.18**.
$$\cos(t) = -\frac{\sqrt{2}}{2}, \sin(t) = \frac{\sqrt{2}}{2}$$
**7.19**.
$$\cos(\pi) = -1, \sin(\pi) = 0$$
**7.20**.
$$\sin(t) = -\frac{7}{25}$$
**7.21**. approximately 0.866025403
**7.22**.
$\frac{\pi}{3}$
**7.23**.
a.
$$\cos(315°) = \frac{\sqrt{2}}{2}, \quad \sin(315°) = \frac{-\sqrt{2}}{2}$$
b.
$$\cos\left(-\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}, \sin\left(-\frac{\pi}{6}\right) = -\frac{1}{2}$$
**7.24**.
$\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

**7.25**.
$\sin t = -\frac{\sqrt{2}}{2}$, $\cos t = \frac{\sqrt{2}}{2}$, $\tan t = -1$, $sec t = \sqrt{2}$, $\csc t = -\sqrt{2}$, $\cot t = -1$

**7.26**.
$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

$\cos\frac{\pi}{3} = \frac{1}{2}$

$\tan\frac{\pi}{3} = \sqrt{3}$

$\sec\frac{\pi}{3} = 2$

$\csc\frac{\pi}{3} = \frac{2\sqrt{3}}{3}$

$\cot\frac{\pi}{3} = \frac{\sqrt{3}}{3}$

**7.27**.
$\sin\left(\frac{-7\pi}{4}\right) = \frac{\sqrt{2}}{2}$, $\cos\left(\frac{-7\pi}{4}\right) = \frac{\sqrt{2}}{2}$, $\tan\left(\frac{-7\pi}{4}\right) = 1$,
$\sec\left(\frac{-7\pi}{4}\right) = \sqrt{2}$, $\csc\left(\frac{-7\pi}{4}\right) = \sqrt{2}$, $\cot\left(\frac{-7\pi}{4}\right) = 1$

**7.28**.
$-\sqrt{3}$

**7.29**.
$-2$

**7.30**.
$\sin t$

**7.31**.
$\cos t = -\frac{8}{17}$, $\sin t = \frac{15}{17}$, $\tan t = -\frac{15}{8}$

$\csc t = \frac{17}{15}$, $\cot t = -\frac{8}{15}$

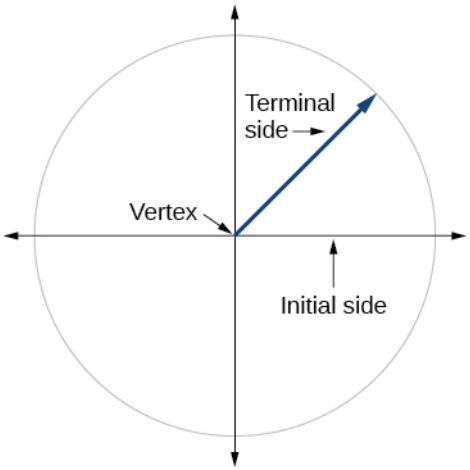
**7.32**.
$\sin t = -1$, $\cos t = 0$, $\tan t =$ Undefined
$\sec t =$ Undefined,$\csc t = -1$, $\cot t = 0$

**7.33**.
$\sec t = \sqrt{2}$, $\csc t = \sqrt{2}$, $\tan t = 1$, $\cot t = 1$

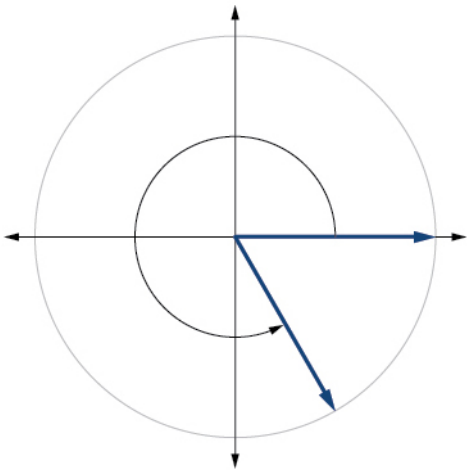
**7.34**.
$\approx -2.414$

## Section Exercises

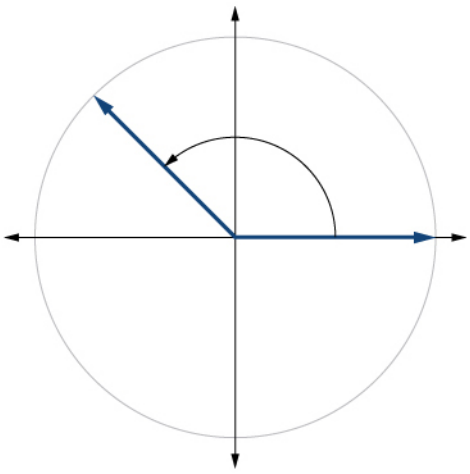
**1**.



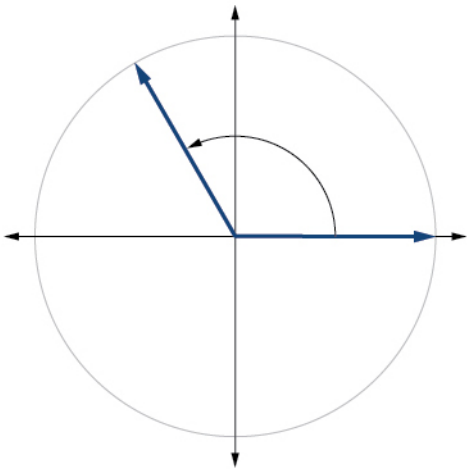

**3**. Whether the angle is positive or negative determines the direction. A positive angle is drawn in the counterclockwise direction, and a negative angle is drawn in the clockwise direction.
**5**. Linear speed is a measurement found by calculating distance of an arc compared to time. Angular speed is a measurement found by calculating the angle of an arc compared to time.
**7**.



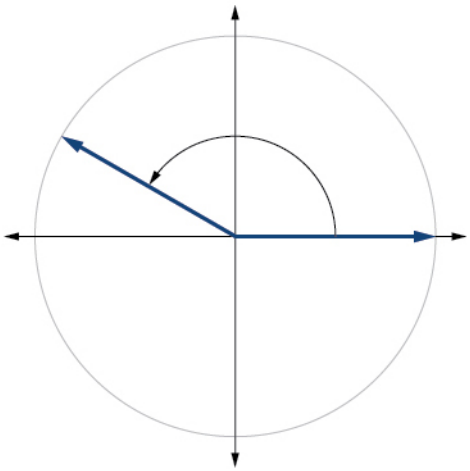
**9**.



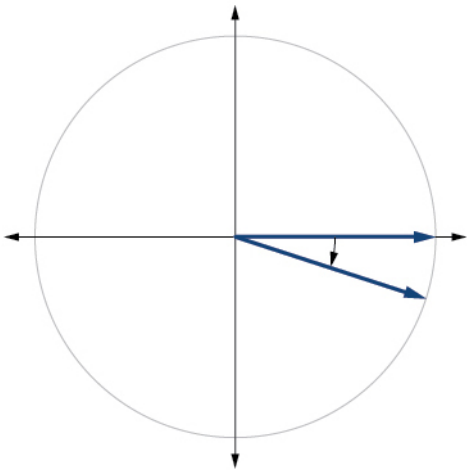
**11**.



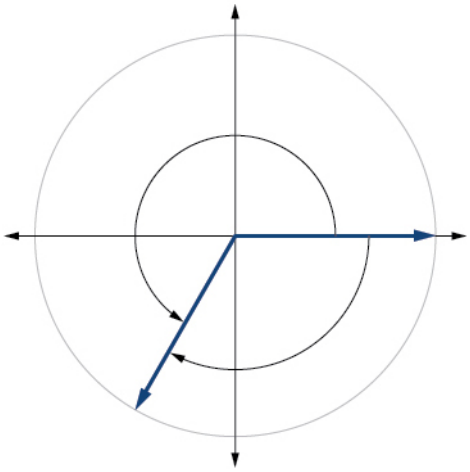
**13**.



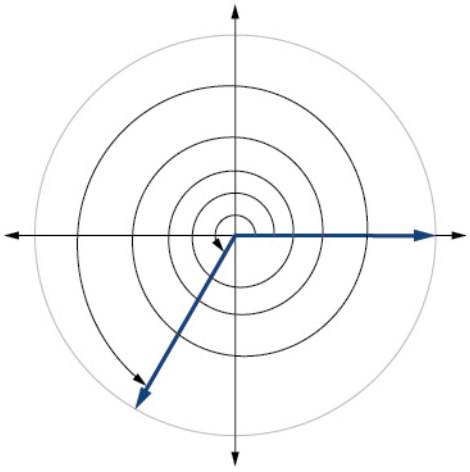
**15**.



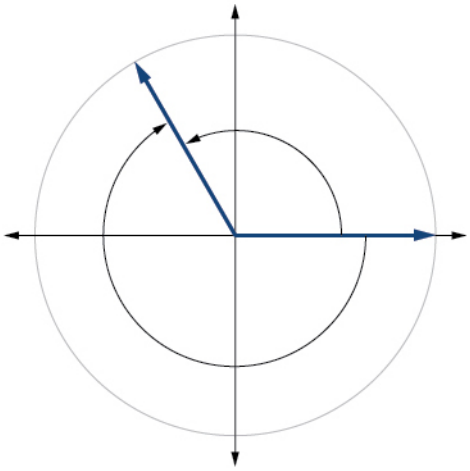
**17**.
$240°$



**19**.
$\frac{4\pi}{3}$

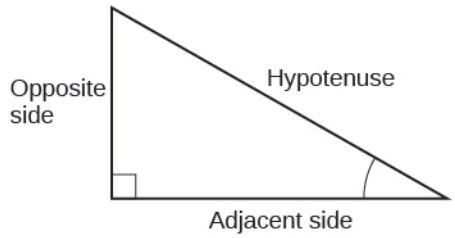


**21**.
$\frac{2\pi}{3}$



**23**.
$\frac{7\pi}{2} \approx 11.00 \text{ in}^2$
**25**.
$\frac{81\pi}{20} \approx 12.72 \text{ cm}^2$
**27**.
$20°$
**29**.
$60°$
**31**.
$-75°$
**33**.
$\frac{\pi}{2}$ radians
**35**.
$-3\pi$ radians
**37**.
$\pi$ radians
**39**.
$\frac{5\pi}{6}$ radians
**41**.
$\frac{5.02\pi}{3} \approx 5.26$ miles
**43**.
$\frac{25\pi}{9} \approx 8.73$ centimeters
**45**.
$\frac{21\pi}{10} \approx 6.60$ meters
**47**. $104.7198 \text{ cm}^2$
**49**. $0.7697 \text{ in}^2$
**51**.
$250°$
**53**.
$320°$

**55**.
$\frac{4\pi}{3}$
**57**.
$\frac{8\pi}{9}$
**59**.
$1320$ rad/min
$210.085$ RPM
**61**.
$7$ in./s, 4.77 RPM ,
$28.65$ deg/s
**63**.
$1,809,557.37$ mm/min $= 30.16$ m/s
**65**.
$5.76$ miles
**67**.
$120°$
**69**. 794 miles per hour
**71**. 2,234 miles per hour
**73**. 11.5 inches
**74**. 3361 revolutions
**75**.




**77**. The tangent of an angle is the ratio of the opposite side to the adjacent side.
**79**. For example, the sine of an angle is equal to the cosine of its complement; the cosine of an angle is equal to the sine of its complement.
**81**.
$\frac{\pi}{6}$
**83**.
$\frac{\pi}{4}$
**85**.
$b = \frac{20\sqrt{3}}{3}$, $c = \frac{40\sqrt{3}}{3}$
**87**.
$a = 10{,}000$, $c = 10{,}00.5$
**89**.
$b = \frac{5\sqrt{3}}{3}$, $c = \frac{10\sqrt{3}}{3}$
**91**.
$\frac{5\sqrt{29}}{29}$
**93**.
$\frac{5}{2}$
**95**.
$\frac{\sqrt{29}}{2}$

**97**.
$\frac{5\sqrt{41}}{41}$
**99**.
$\frac{5}{4}$
**101**.
$\frac{\sqrt{41}}{4}$
**103**.
$c = 14,\ b = 7\sqrt{3}$
**105**.
$a = 15,\ b = 15$
**107**.
$b = 9.9970,\ c = 12.2041$
**109**.
$a = 2.0838,\ b = 11.8177$
**111**.
$a = 55.9808,\ c = 57.9555$
**113**.
$a = 46.6790,\ b = 17.9184$
**115**.
$a = 16.4662,\ c = 16.8341$
**117**. 188.3159
**119**. 200.6737
**121**. 498.3471 ft
**123**. 1060.09 ft
**125**. 27.372 ft
**127**. 22.6506 ft
**129**. 368.7633 ft
**131**. The unit circle is a circle of radius 1 centered at the origin.
**133**. Coterminal angles are angles that share the same terminal side. A reference angle is the size of the smallest acute angle, $t$, formed by the terminal side of the angle $t$ and the horizontal axis.
**135**. The sine values are equal.
**137**. I
**139**. IV
**141**.
$\frac{\sqrt{3}}{2}$
**143**.
$\frac{1}{2}$
**145**.
$\frac{\sqrt{2}}{2}$
**147**. 0
**149**. -1
**151**.
$\frac{\sqrt{3}}{2}$
**153**.
$60°$
**155**.
$80\ °$
**157**.
$45°$

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**159**.
$\frac{\pi}{3}$

**161**.
$\frac{\pi}{3}$

**163**.
$\frac{\pi}{8}$

**165**.
$60°$, Quadrant IV,
$\sin(300°) = -\frac{\sqrt{3}}{2}, \cos(300°) = \frac{1}{2}$

**167**.
$45°$, Quadrant II,
$\sin(135°) = \frac{\sqrt{2}}{2}, \cos(135°) = -\frac{\sqrt{2}}{2}$

**169**.
$60°$, Quadrant II,
$\sin(120°) = \frac{\sqrt{3}}{2}, \cos(120°) = -\frac{1}{2}$

**171**.
$30°$, Quadrant II,
$\sin(150°) = \frac{1}{2}, \cos(150°) = -\frac{\sqrt{3}}{2}$

**173**.
$\frac{\pi}{6}$, Quadrant III,
$\sin\left(\frac{7\pi}{6}\right) = -\frac{1}{2}, \cos\left(\frac{7\pi}{6}\right) = -\frac{\sqrt{3}}{2}$

**175**.
$\frac{\pi}{4}$, Quadrant II,
$\sin\left(\frac{3\pi}{4}\right) = \frac{\sqrt{2}}{2}, \cos\left(\frac{4\pi}{3}\right) = -\frac{\sqrt{2}}{2}$

**177**.
$\frac{\pi}{3}$, Quadrant II,
$\sin\left(\frac{2\pi}{3}\right) = \frac{\sqrt{3}}{2}, \cos\left(\frac{2\pi}{3}\right) = -\frac{1}{2}$

**179**.
$\frac{\pi}{4}$, Quadrant IV,
$\sin\left(\frac{7\pi}{4}\right) = -\frac{\sqrt{2}}{2}, \cos\left(\frac{7\pi}{4}\right) = \frac{\sqrt{2}}{2}$

**181**.
$\frac{\sqrt{77}}{9}$

**183**.
$-\frac{\sqrt{15}}{4}$

**185**.
$\left(-10, \quad 10\sqrt{3}\right)$

**187**.
(–2.778, 15.757)

**189**.
[–1, 1]

**191**.
$\sin t = \frac{1}{2}, \cos t = -\frac{\sqrt{3}}{2}$
**193**.
$\sin t = -\frac{\sqrt{2}}{2}, \cos t = -\frac{\sqrt{2}}{2}$
**195**.
$\sin t = \frac{\sqrt{3}}{2}, \cos t = -\frac{1}{2}$
**197**.
$\sin t = -\frac{\sqrt{2}}{2}, \cos t = \frac{\sqrt{2}}{2}$
**199**.
$\sin t = 0, \quad \cos t = -1$
**201**.
$\sin t = -0.596, \quad \cos t = 0.803$
**203**.
$\sin t = \frac{1}{2}, \cos t = \frac{\sqrt{3}}{2}$
**205**.
$\sin t = -\frac{1}{2}, \cos t = \frac{\sqrt{3}}{2}$
**207**.
$\sin t = 0.761, \cos t = -0.649$
**209**.
$\sin t = 1, \cos t = 0$
**211**. −0.1736
**213**. 0.9511
**215**. −0.7071
**217**. −0.1392
**219**. −0.7660
**221**.
$\frac{\sqrt{2}}{4}$
**223**.
$-\frac{\sqrt{6}}{4}$
**225**.
$\frac{\sqrt{2}}{4}$
**227**.
$\frac{\sqrt{2}}{4}$
**229**. 0
**231**.
(0, –1)
**233**. 37.5 seconds, 97.5 seconds, 157.5 seconds, 217.5 seconds, 277.5 seconds, 337.5 seconds
**235**. Yes, when the reference angle is
$\frac{\pi}{4}$ and the terminal side of the angle is in quadrants I and III. Thus, a
$x = \frac{\pi}{4}, \frac{5\pi}{4},$ the sine and cosine values are equal.
**237**. Substitute the sine of the angle in for
$y$ in the Pythagorean Theorem
$x^2 + y^2 = 1.$ Solve for
$x$ and take the negative solution.

**239**. The outputs of tangent and cotangent will repeat every $\pi$ units.

**241**.
$\frac{2\sqrt{3}}{3}$

**243**.
$\sqrt{3}$

**245**.
$\sqrt{2}$

**247**. 1

**249**. 2

**251**.
$\frac{\sqrt{3}}{3}$

**253**.
$-\frac{2\sqrt{3}}{3}$

**255**.
$\sqrt{3}$

**257**.
$-\sqrt{2}$

**259**. –1

**261**. -2

**263**.
$-\frac{\sqrt{3}}{3}$

**265**. 2

**267**.
$\frac{\sqrt{3}}{3}$

**269**. –2

**271**. –1

**273**.
$\sin t = -\frac{2\sqrt{2}}{3}$, $\sec t = -3$, $\csc t = -\frac{3\sqrt{2}}{4}$, $\tan t = 2\sqrt{2}$, $\cot t = \frac{\sqrt{2}}{4}$

**275**.
$\sec t = 2$,

$\csc t = \frac{2\sqrt{3}}{3}$,

$\tan t = \sqrt{3}$,

$\cot t = \frac{\sqrt{3}}{3}$

**277**.
$-\frac{\sqrt{2}}{2}$

**279**. 3.1

**281**. 1.4

**283**.
$\sin t = \frac{\sqrt{2}}{2}$, $\cos t = \frac{\sqrt{2}}{2}$, $\tan t = 1$, $\cot t = 1$, $\sec t = \sqrt{2}$, $\csc t = \sqrt{2}$

**285**.
$\sin t = -\frac{\sqrt{3}}{2}$, $\cos t = -\frac{1}{2}$, $\tan t = \sqrt{3}$, $\cot t = \frac{\sqrt{3}}{3}$, $\sec t = -2$, $\csc t = -\frac{2\sqrt{3}}{3}$

**287**. –0.228

**289**. –2.414

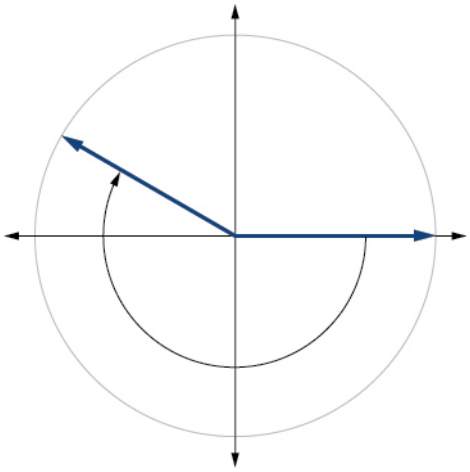
**291**. 1.414

**293**. 1.540

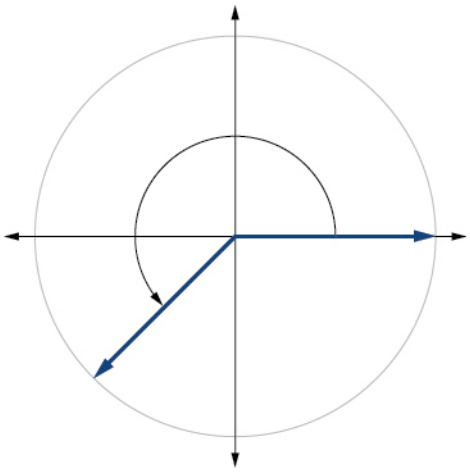
**295**. 1.556
**297**.
$\sin(t) \approx 0.79$
**299**.
$\csc t \approx 1.16$
**301**. even
**303**. even
**305**.
$\frac{\sin t}{\cos t} = \tan t$
**307**. 13.77 hours, period:
$1000\pi$
**309**. 7.73 inches

## Review Exercises

**311**.
$45°$
**313**.
$-\frac{7\pi}{6}$
**315**. 10.385 meters
**317**.
$60°$
**319**.
$\frac{2\pi}{11}$
**321**.



**323**.



**325**. 1036.73 miles per hour

**327**.

$$\frac{\sqrt{2}}{2}$$

**329**.

$$\frac{\sqrt{3}}{3}$$

**331**.

$72°$

**333**.

$$a = \frac{10}{3},\ c = \frac{2\sqrt{106}}{3}$$

**335**.

$$\frac{6}{11}$$

**337**.

$$a = \frac{5\sqrt{3}}{2},\ b = \frac{5}{2}$$

**339**. 369.2136 ft

**341**.

$$\frac{\sqrt{2}}{2}$$

**343**.

$60°$

**345**.

$$\frac{\sqrt{3}}{2}$$

**347**. all real numbers

**349**.

$$\frac{\sqrt{3}}{2}$$

**351**.

$$\frac{2\sqrt{3}}{3}$$

**353**. 2

**355**. –2.5

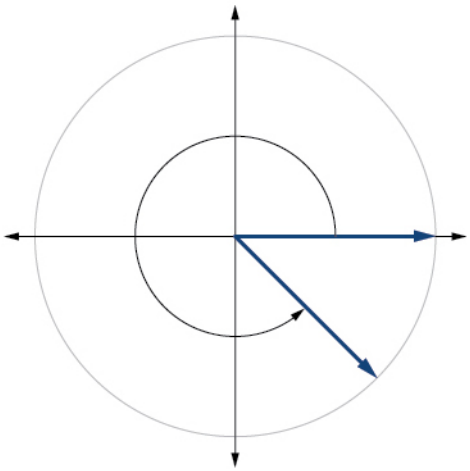
**357**.

$$\frac{1}{3}$$

**359**. cosine, secant

## Practice Test

**361**.
$150°$
**363**. 6.283 centimeters
**365**.
$15°$
**367**.



**369**. 3.351 feet per second,
$\frac{2\pi}{75}$ radians per second
**371**.
$a = \frac{9}{2}, b = \frac{9\sqrt{3}}{2}$
**373**.
$\frac{1}{2}$
**375**. real numbers
**377**. 1
**379**.
$-\sqrt{2}$
**381**. –0.68
**383**.
$\frac{\pi}{3}$

## Chapter 8

## Try It

**8.1**.
$6\pi$
**8.2**.
$\frac{1}{2}$ compressed
**8.3**.
$\frac{\pi}{2}$; right
**8.4**. 2 units up
**8.5**. midline:
$y = 0;$ amplitude:
$|A| = \frac{1}{2};$ period:

$P = \frac{2\pi}{|B|} = 6\pi$; phase shift:

$\frac{C}{B} = \pi$

**8.6**.

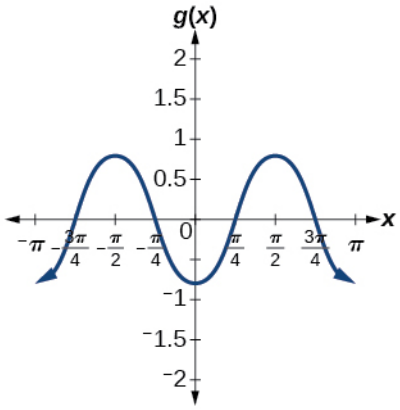
$f(x) = \sin(x) + 2$

**8.7**. two possibilities:

$y = 4\sin\left(\frac{\pi}{5}x - \frac{\pi}{5}\right) + 4$ or

$y = -4\sin\left(\frac{\pi}{5}x + \frac{4\pi}{5}\right) + 4$

**8.8**.




midline:

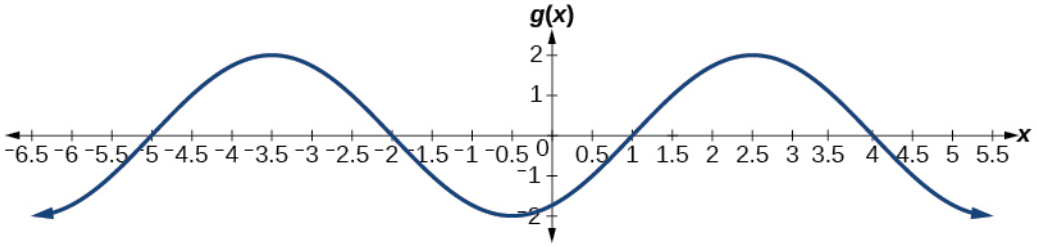
$y = 0$; amplitude:

$|A| = 0.8$; period:

$P = \frac{2\pi}{|B|} = \pi$; phase shift:

$\frac{C}{B} = 0$ or none

**8.9**.




midline:

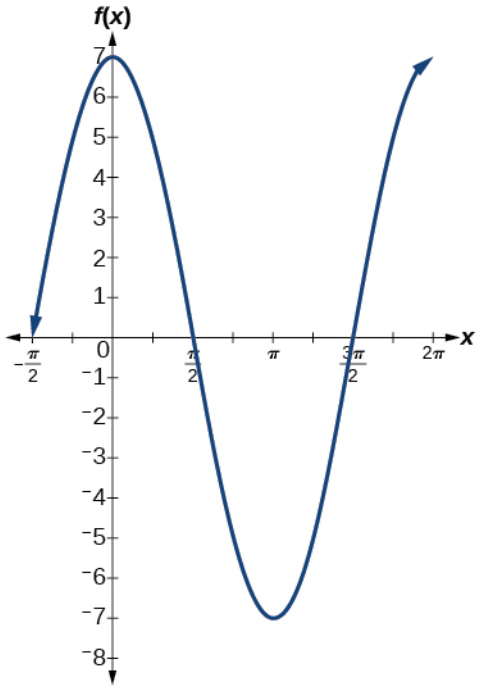
$y = 0$; amplitude:

$|A| = 2$; period:

$P = \frac{2\pi}{|B|} = 6$; phase shift:

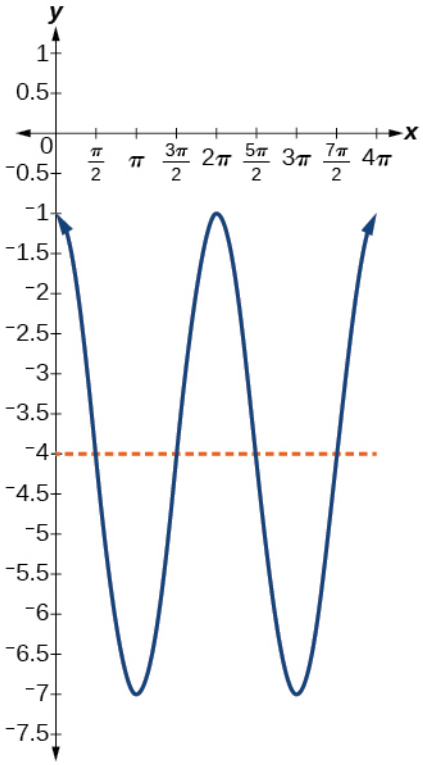
$\frac{C}{B} = -\frac{1}{2}$

**8.10**. 7

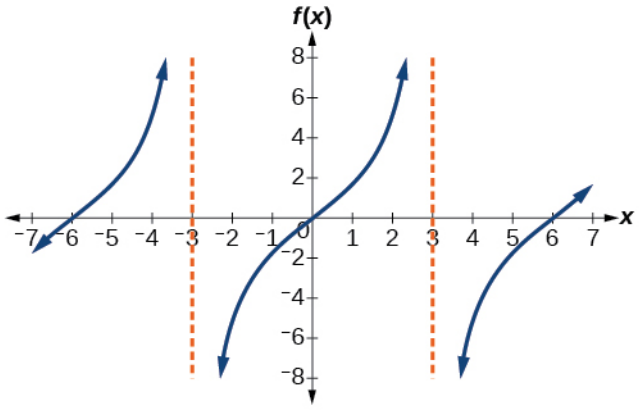



**8.11**.

$y = 3\cos(x) - 4$




**8.12**.

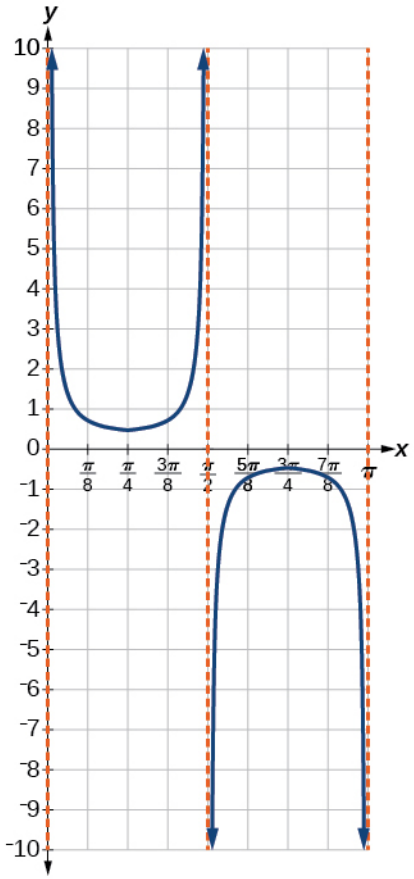



**8.13**. It would be reflected across the line $y = -1$, becoming an increasing function.

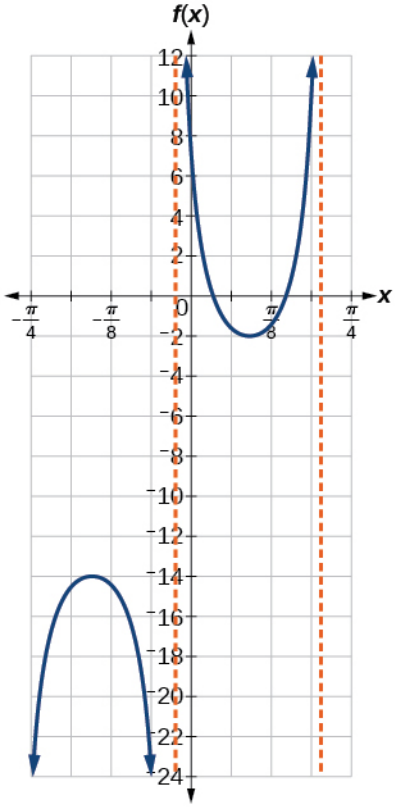
**8.14**.

$g(x) = 4\tan(2x)$

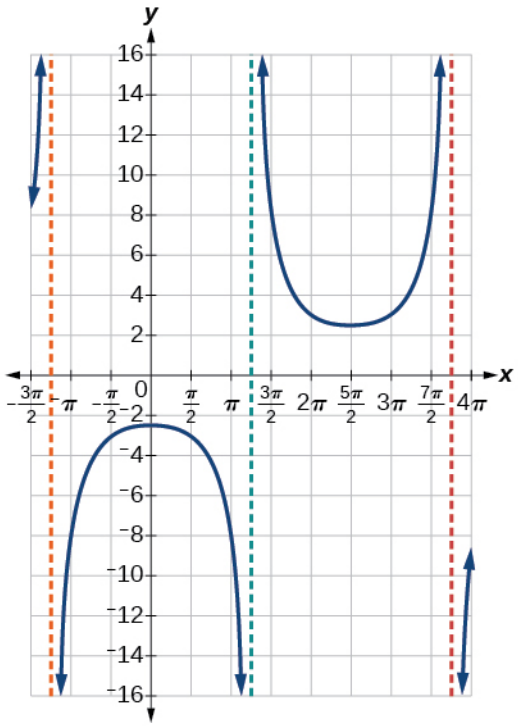
**8.15**. This is a vertical reflection of the preceding graph because $A$ is negative.



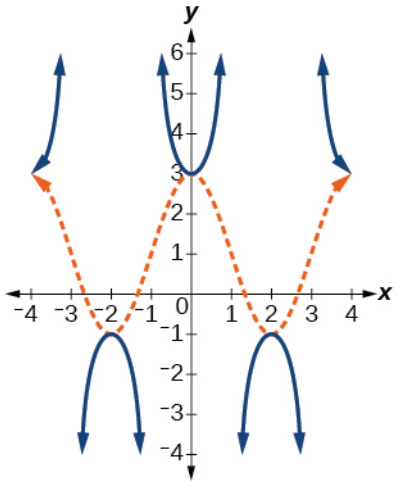

**8.16**.


f(x)
x
12
10
8
6
4
2
0
-2
-4
-6
-8
-10
-12
-14
-16
-18
-20
-22
-24
$-\frac{\pi}{4}$
$-\frac{\pi}{8}$
$\frac{\pi}{8}$
$\frac{\pi}{4}$

**8.17**.


y
x
16
14
12
10
8
6
4
2
0
-2
-4
-6
-8
-10
-12
-14
-16
$-\frac{3\pi}{2}$
$-\pi$
$-\frac{\pi}{2}$
$\frac{\pi}{2}$
$\pi$
$\frac{3\pi}{2}$
$2\pi$
$\frac{5\pi}{2}$
$3\pi$
$\frac{7\pi}{2}$
$4\pi$

**8.18**.




**8.19**.
$\arccos(0.8776) \approx 0.5$

**8.20**. a.
$-\frac{\pi}{2}$; b.
$-\frac{\pi}{4}$; c.
$\pi$; d.
$\frac{\pi}{3}$

**8.21**. 1.9823 or 113.578°

**8.22**.
$\sin^{-1}(0.6) = 36.87° = 0.6435$ radians

**8.23**.
$\frac{\pi}{8}, \frac{2\pi}{9}$

**8.24**.
$\frac{3\pi}{4}$

**8.25**.
$\frac{12}{13}$

**8.26**.
$\frac{4\sqrt{2}}{9}$

**8.27**.
$\frac{4x}{\sqrt{16x^2+1}}$

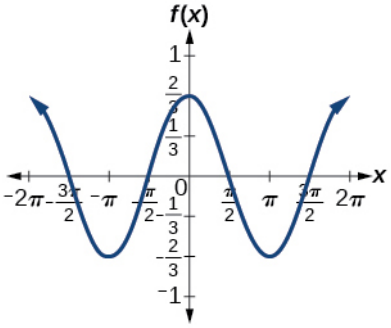
## Section Exercises

**1**. The sine and cosine functions have the property that $f(x+P) = f(x)$ for a certain $P$. This means that the function values repeat for every $P$ units on the $x$-axis.

**3**. The absolute value of the constant $A$ (amplitude) increases the total range and the constant $D$ (vertical shift) shifts the graph vertically.

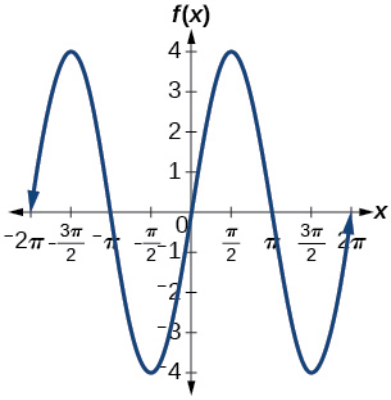
**5**. At the point where the terminal side of $t$ intersects the unit circle, you can determine that the $\sin t$ equals the $y$-coordinate of the point.

**7**.



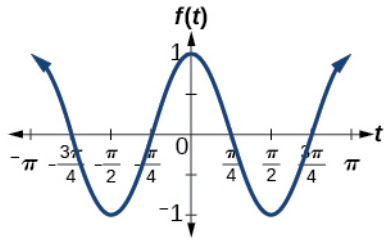

amplitude:
$\frac{2}{3}$; period:
$2\pi$; midline:
$y = 0$; maximum:
$y = \frac{2}{3}$ occurs at
$x = 0$; minimum:
$y = -\frac{2}{3}$ occurs at
$x = \pi$; for one period, the graph starts at 0 and ends at
$2\pi$

**9**.



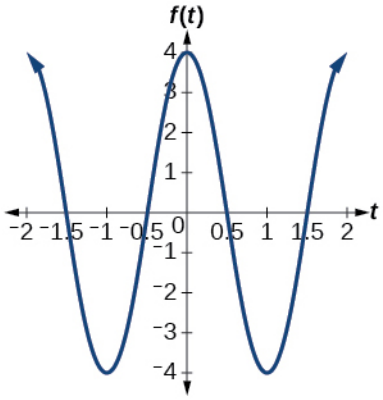

amplitude: 4; period:
$2\pi$; midline:
$y = 0$; maximum
$y = 4$ occurs at
$x = \frac{\pi}{2}$; minimum:
$y = -4$ occurs at
$x = \frac{3\pi}{2}$; one full period occurs from
$x = 0$ to
$x = 2\pi$

**11**.



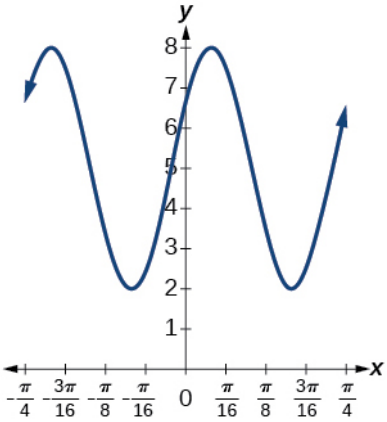
amplitude: 1; period: $\pi$; midline: $y = 0$; maximum: $y = 1$ occurs at $x = \pi$; minimum: $y = -1$ occurs at $x = \frac{\pi}{2}$; one full period is graphed from $x = 0$ to $x = \pi$

**13**.




amplitude: 4; period: 2; midline: $y = 0$; maximum: $y = 4$ occurs at $x = 0$; minimum: $y = -4$ occurs at $x = 1$

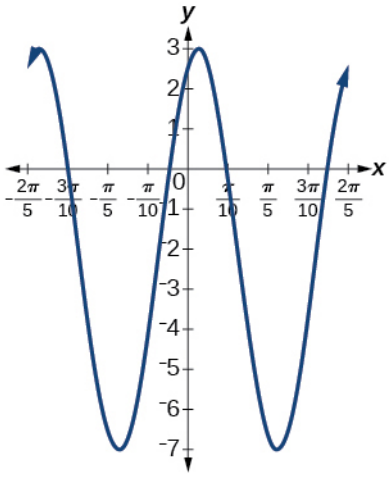
**15**.




amplitude: 3; period: $\frac{\pi}{4}$; midline: $y = 5$; maximum: $y = 8$ occurs at $x = 0.12$; minimum: $y = 2$ occurs at $x = 0.516$; horizontal shift: $-4$; vertical translation 5; one period occurs from

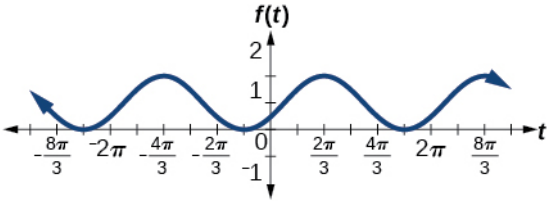
$x = 0$ to
$x = \frac{\pi}{4}$

**17**.



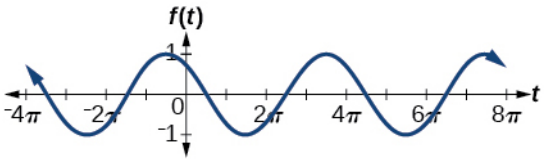

amplitude: 5; period:
$\frac{2\pi}{5}$; midline:
$y = -2$; maximum:
$y = 3$ occurs at
$x = 0.08$; minimum:
$y = -7$ occurs at
$x = 0.71$; phase shift:
$-4$; vertical translation:
$-2$; one full period can be graphed on
$x = 0$ to
$x = \frac{2\pi}{5}$

**19**.




amplitude: 1 ; period:
$2\pi$; midline:
$y = 1$; maximum:
$y = 2$ occurs at
$x = 2.09$; maximum:
$y = 2$ occurs at
$t = 2.09$; minimum:
$y = 0$ occurs at
$t = 5.24$; phase shift:
$-\frac{\pi}{3}$; vertical translation: 1; one full period is from
$t = 0$ to
$t = 2\pi$

**21**.




amplitude: 1; period: $4\pi$; midline: $y = 0$; maximum: $y = 1$ occurs at $t = 11.52$; minimum: $y = -1$ occurs at $t = 5.24$; phase shift: $-\frac{10\pi}{3}$; vertical shift: 0

**23**. amplitude: 2; midline: $y = -3$; period: 4; equation: $f(x) = 2\sin\left(\frac{\pi}{2}x\right) - 3$

**25**. amplitude: 2; period: 5; midline: $y = 3$; equation: $f(x) = -2\cos\left(\frac{2\pi}{5}x\right) + 3$

**27**. amplitude: 4; period: 2; midline: $y = 0$; equation: $f(x) = -4\cos\left(\pi\left(x - \frac{\pi}{2}\right)\right)$

**29**. amplitude: 2; period: 2; midline $y = 1$; equation: $f(x) = 2\cos(\pi x) + 1$

**32**. $\frac{\pi}{6}, \frac{5\pi}{6}$

**34**. $\frac{\pi}{4}, \frac{3\pi}{4}$

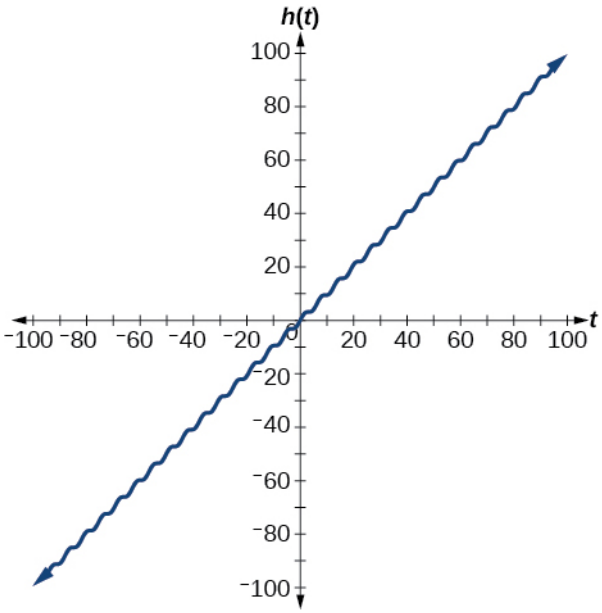
**36**. $\frac{3\pi}{2}$

**38**. $\frac{\pi}{2}, \frac{3\pi}{2}$

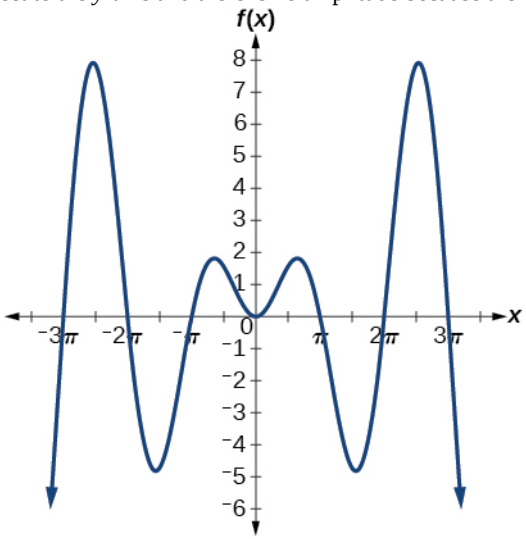
**40**. $\frac{\pi}{2}, \frac{3\pi}{2}$

**42**. $\frac{\pi}{6}, \frac{11\pi}{6}$

**44**. The graph appears linear. The linear functions dominate the shape of the graph for large values of $x$.




**46**. The graph is symmetric with respect to the *y*-axis and there is no amplitude because the function is not periodic.




**48**.

a. Amplitude: 12.5; period: 10; midline: $y = 13.5;$

b. $$h(t) = 12.5\sin\left(\frac{\pi}{5}(t - 2.5)\right) + 13.5;$$

c. 26 ft

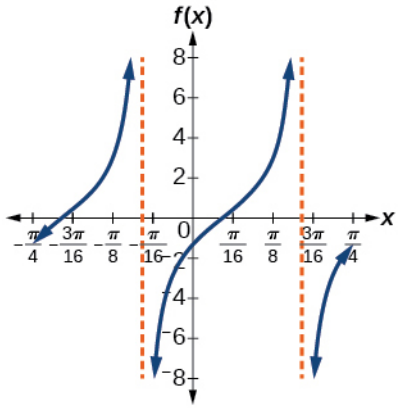
**49**. Since $y = \csc x$ is the reciprocal function of $y = \sin x,$ you can plot the reciprocal of the coordinates on the graph of $y = \sin x$ to obtain the *y*-coordinates of $y = \csc x.$ The *x*-intercepts of the graph $y = \sin x$ are the vertical asymptotes for the graph of $y = \csc x.$

**51**. Answers will vary. Using the unit circle, one can show that $\tan(x + \pi) = \tan x.$

**53**. The period is the same: $2\pi.$

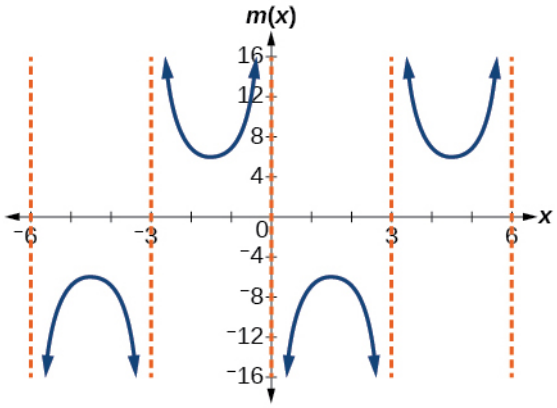
**55**. IV

**57**. III
**59**. period: 8; horizontal shift: 1 unit to left
**61**. 1.5
**63**. 5
**65**.
$-\cot x\cos x-\sin x$
**67**.



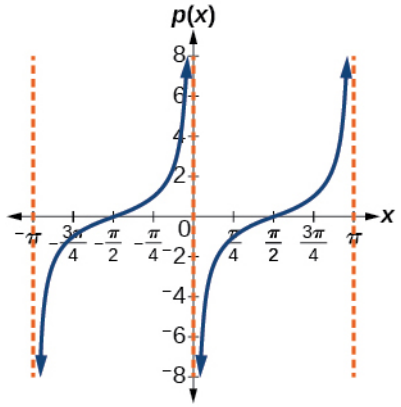

stretching factor: 2; period:
$\frac{\pi}{4}$;
asymptotes:
$x=\frac{1}{4}\left(\frac{\pi}{2}+\pi k\right)+8,$ where $k$ is an integer

**69**.



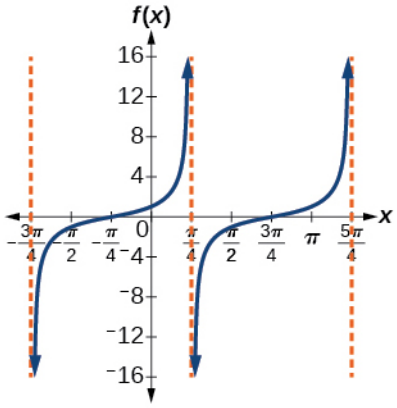

stretching factor: 6; period: 6; asymptotes:
$x=3k,$ where $k$ is an integer

**71**.



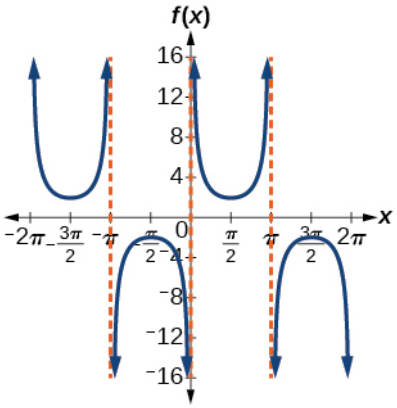
stretching factor: 1; period:
$\pi$; asymptotes:
$x = \pi k$, where $k$ is an integer

**73**.



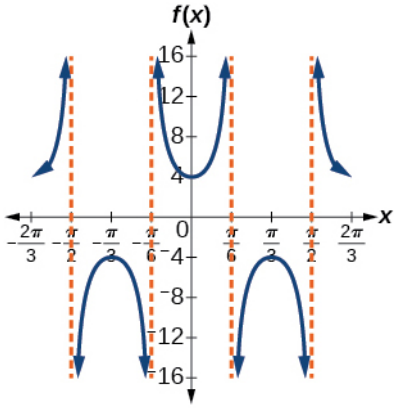

Stretching factor: 1; period:
$\pi$; asymptotes:
$x = \frac{\pi}{4} + \pi k$, where $k$ is an integer

**75**.



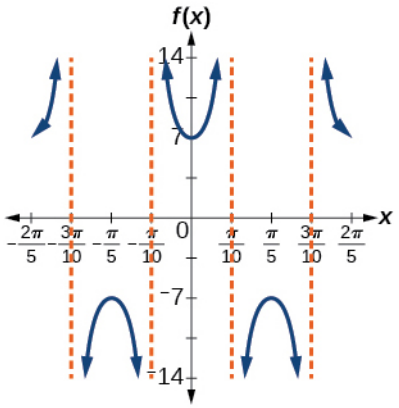

stretching factor: 2; period:
$2\pi$; asymptotes:
$x = \pi k$, where $k$ is an integer

**77**.



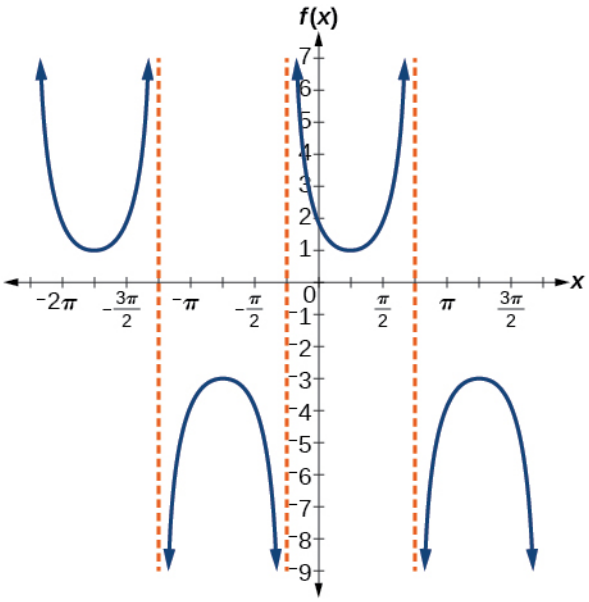

stretching factor: 4; period:
$\frac{2\pi}{3}$; asymptotes:
$x = \frac{\pi}{6}k$, where $k$ is an odd integer

**79**.



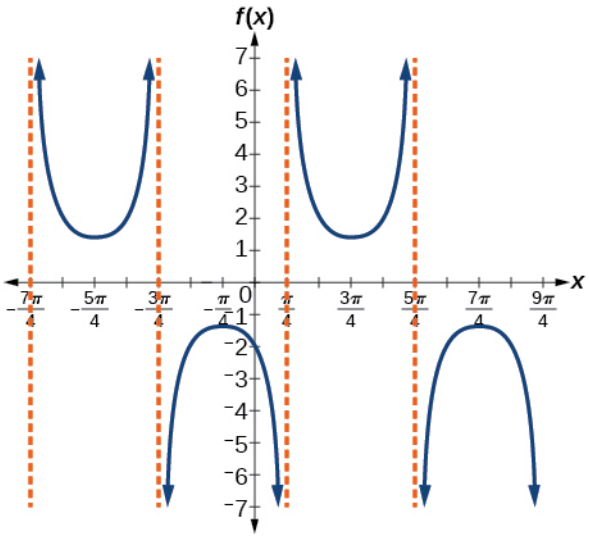

stretching factor: 7; period:
$\frac{2\pi}{5}$; asymptotes:
$x = \frac{\pi}{10}k$, where $k$ is an odd integer

**81**.



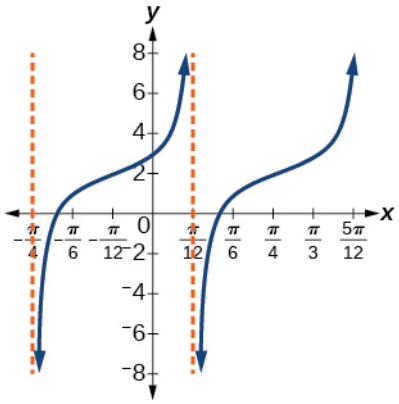

stretching factor: 2; period:
$2\pi$; asymptotes:
$x = -\frac{\pi}{4} + \pi k$, where $k$ is an integer

**83**.




stretching factor:
$\frac{7}{5}$; period:
$2\pi$; asymptotes:
$x = \frac{\pi}{4} + \pi k$, where $k$ is an integer

**85**.
$y = \tan\left(3\left(x - \frac{\pi}{4}\right)\right) + 2$




**87**.
$f(x) = \csc(2x)$

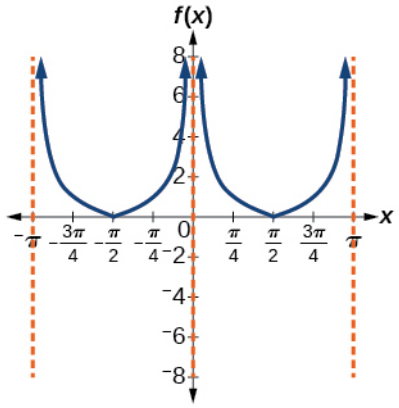
**89**.
$f(x) = \csc(4x)$

**91**.
$f(x) = 2\csc x$

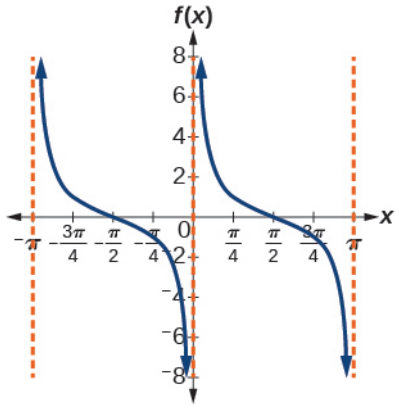
**93**.
$f(x) = \frac{1}{2}\tan(100\pi x)$

**95**.

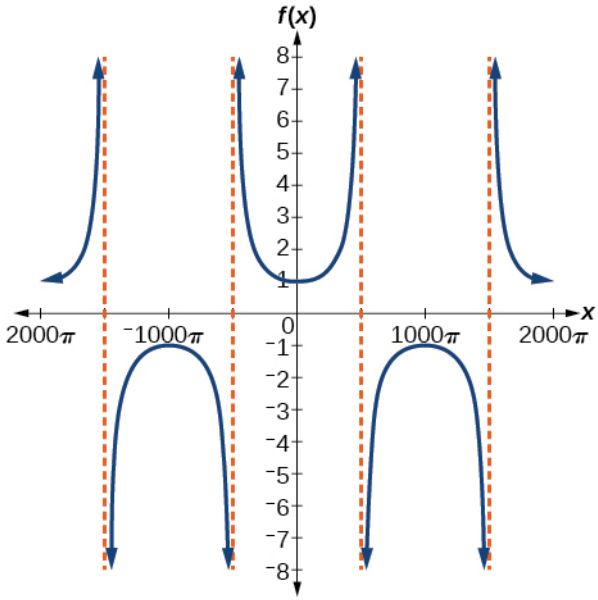
**97**.


f(x)
x
8
6
4
2
0
-2
-4
-6
-8
$-\pi$
$-\frac{3\pi}{4}$
$-\frac{\pi}{2}$
$-\frac{\pi}{4}$
$\frac{\pi}{4}$
$\frac{\pi}{2}$
$\frac{3\pi}{4}$
$\pi$

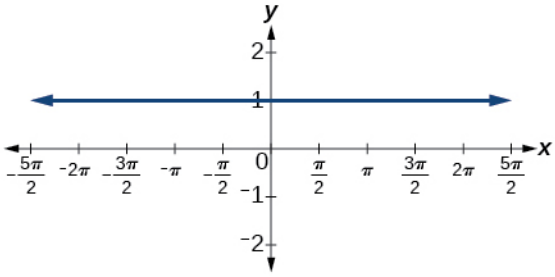
**99**.


f(x)
x
8
6
4
2
0
-2
-4
-6
-8
$-\pi$
$-\frac{3\pi}{4}$
$-\frac{\pi}{2}$
$-\frac{\pi}{4}$
$\frac{\pi}{4}$
$\frac{\pi}{2}$
$\frac{3\pi}{4}$
$\pi$

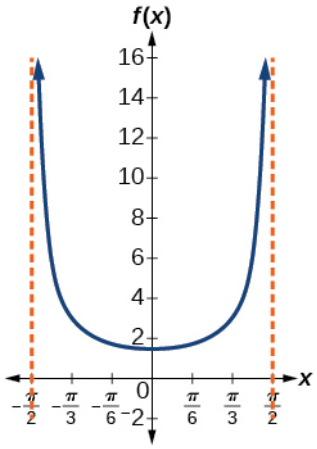
**101**.


f(x)
x
8
7
6
5
4
3
2
1
0
-1
-2
-3
-4
-5
-6
-7
-8
$2000\pi$
$-1000\pi$
$1000\pi$
$2000\pi$

**103**.


y
x
2
1
0
-1
-2
$-\frac{5\pi}{2}$
$-2\pi$
$-\frac{3\pi}{2}$
$-\pi$
$-\frac{\pi}{2}$
$\frac{\pi}{2}$
$\pi$
$\frac{3\pi}{2}$
$2\pi$
$\frac{5\pi}{2}$

a. $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$;

b. 


c. $x = -\frac{\pi}{2}$ and $x = \frac{\pi}{2}$; the distance grows without bound as $|x|$ approaches $\frac{\pi}{2}$—i.e., at right angles to the line representing due north, the boat would be so far away, the fisherman could not see it;

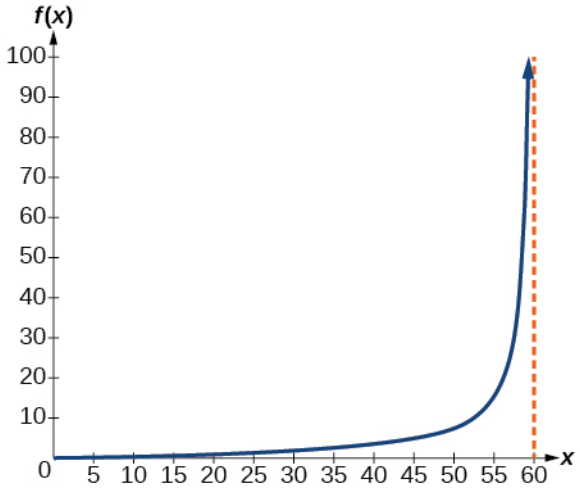
d. 3; when $x = -\frac{\pi}{3}$, the boat is 3 km away;

e. 1.73; when $x = \frac{\pi}{6}$, the boat is about 1.73 km away;

f. 1.5 km; when $x = 0$

**105**.

a. $h(x) = 2\tan\left(\frac{\pi}{120}x\right)$;

b. 


c. $h(0) = 0$ : after 0 seconds, the rocket is 0 mi above the ground;
$h(30) = 2$ : after 30 seconds, the rockets is 2 mi high;

d. As $x$ approaches 60 seconds, the values of $h(x)$ grow increasingly large. The distance to the rocket is growing so large that the camera can no longer track it.

**106**. The function $y = \sin x$ is one-to-one on $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$; thus, this interval is the range of the inverse function of $y = \sin x$, $f(x) = \sin^{-1} x$. The function $y = \cos x$ is one-to-one on $[0, \pi]$; thus, this interval is the range of the inverse function of $y = \cos x$, $f(x) = \cos^{-1} x$.

**108**. $\frac{\pi}{6}$ is the radian measure of an angle between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ whose sine is 0.5.

**110**. In order for any function to have an inverse, the function must be one-to-one and must pass the horizontal line test. The regular sine function is not one-to-one unless its domain is restricted in some way. Mathematicians have agreed to restrict the sine function to the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ so that it is one-to-one and possesses an inverse.

**112**. True . The angle, $\theta_1$ that equals $\arccos(-x)$, $x > 0$, will be a second quadrant angle with reference angle, $\theta_2$, where $\theta_2$ equals $\arccos x$, $x > 0$. Since $\theta_2$ is the reference angle for $\theta_1$, $\theta_2 = \pi - \theta_1$ and $\arccos(-x) = \pi - \arccos x$.

**114**. $-\frac{\pi}{6}$

**116**. $\frac{3\pi}{4}$

**118**. $-\frac{\pi}{3}$

**120**. $\frac{\pi}{3}$

**122**. 1.98

**124**. 0.93

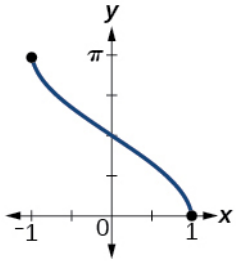
**126**. 1.41

**128**. 0.56 radians

**130**. 0

**132**. 0.71

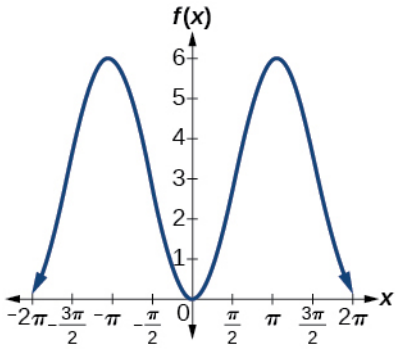
**134**. -0.71
**136**.
$-\frac{\pi}{4}$
**138**. 0.8
**140**.
$\frac{5}{13}$
**142**.
$\frac{x-1}{\sqrt{-x^2+2x}}$
**144**.
$\frac{\sqrt{x^2-1}}{x}$
**146**.
$\frac{x+0.5}{\sqrt{-x^2-x+\frac{3}{4}}}$
**148**.
$\frac{\sqrt{2x+1}}{x+1}$
**150**.
$\frac{\sqrt{2x+1}}{x}$
**152**.
$t$
**154**.



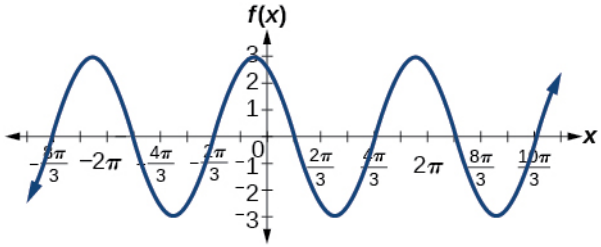

domain
$[-1,\ 1]$; range
$[0,\ \pi]$
**156**. approximately
$x = 0.00$
**158**. 0.395 radians
**160**. 1.11 radians
**162**. 1.25 radians
**164**. 0.405 radians
**166**. No. The angle the ladder makes with the horizontal is 60 degrees.

## Review Exercises

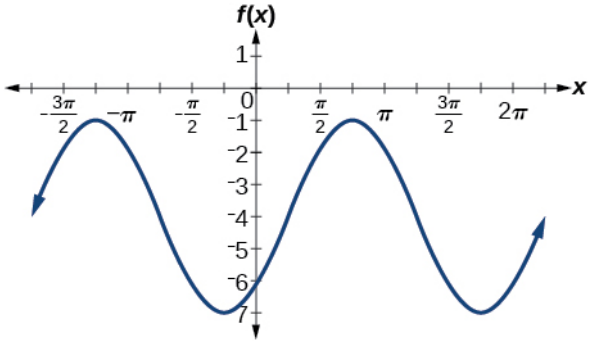
**168**. amplitude: 3; period:
$2\pi$; midline:
$y = 3$; no asymptotes



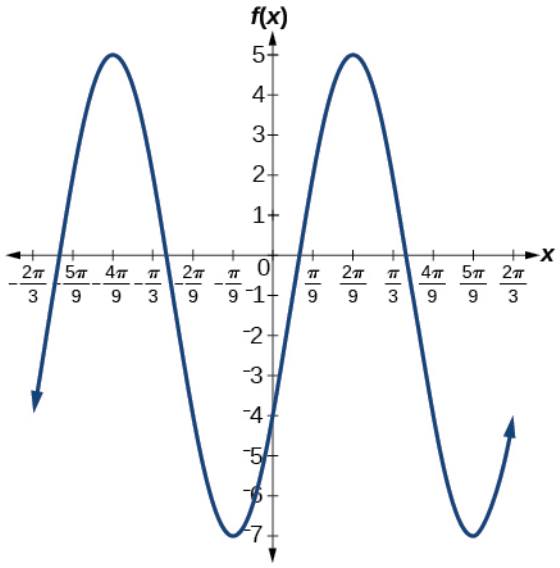

**170**. amplitude: 3; period: $2\pi$; midline: $y = 0$; no asymptotes



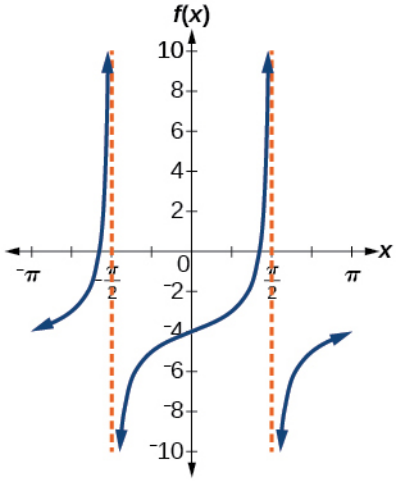

**172**. amplitude: 3; period: $2\pi$; midline: $y = -4$; no asymptotes



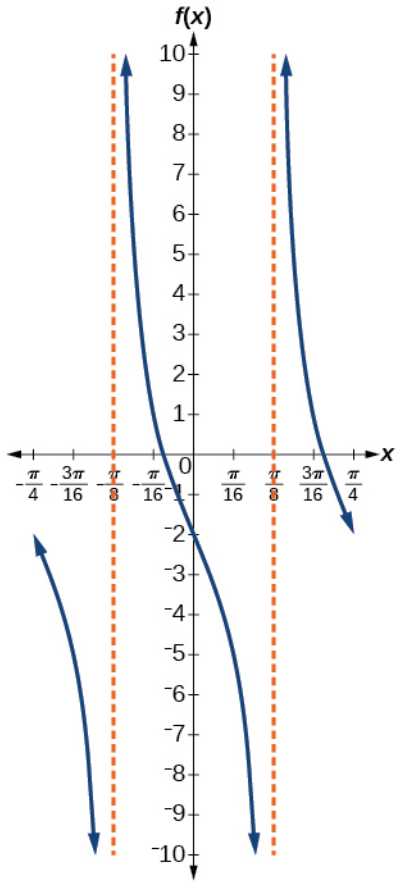

**174**. amplitude: 6; period: $\frac{2\pi}{3}$; midline: $y = -1$; no asymptotes



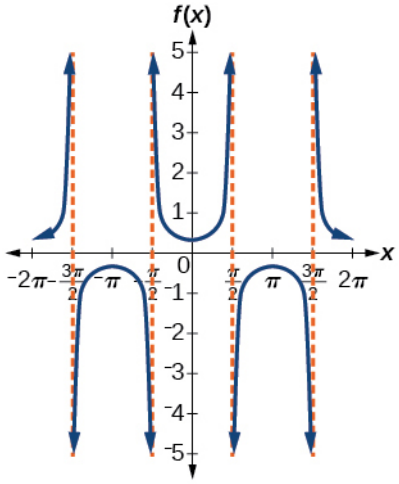

**176**. stretching factor: none; period: $\pi$; midline: $y = -4$; asymptotes: $x = \frac{\pi}{2} + \pi k$, where $k$ is an integer



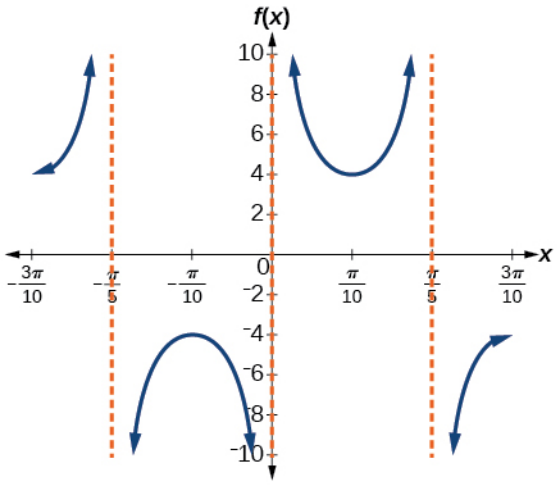

**178**. stretching factor: 3; period: $\frac{\pi}{4}$; midline: $y = -2$; asymptotes: $x = \frac{\pi}{8} + \frac{\pi}{4}k$, where $k$ is an integer



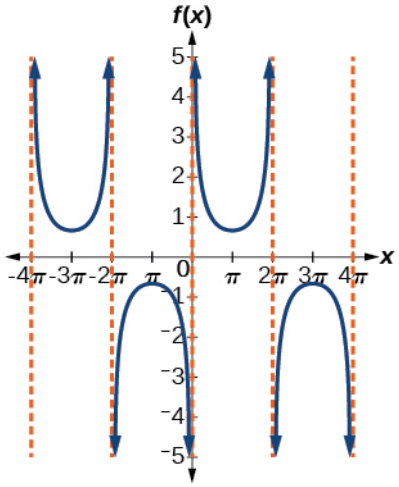

**180**. amplitude: none; period: $2\pi$; no phase shift; asymptotes: $x = \frac{\pi}{2}k$, where $k$ is an odd integer



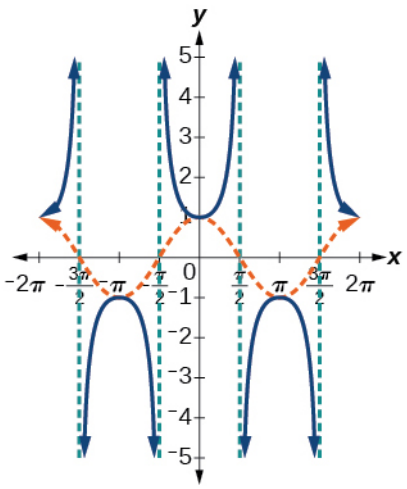

**182**. amplitude: none; period: $\frac{2\pi}{5}$; no phase shift; asymptotes: $x = \frac{\pi}{5}k$, where $k$ is an integer



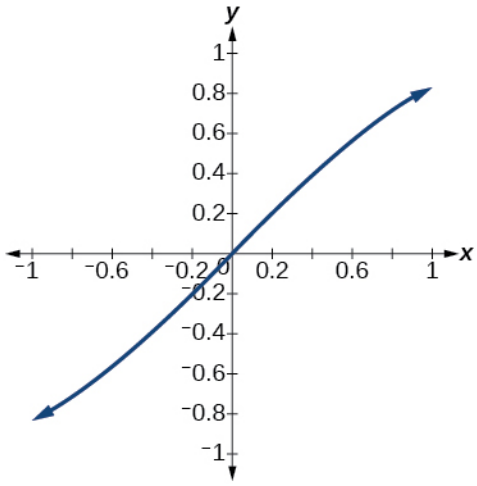

**184**. amplitude: none; period: $4\pi$; no phase shift; asymptotes: $x = 2\pi k$, where $k$ is an integer

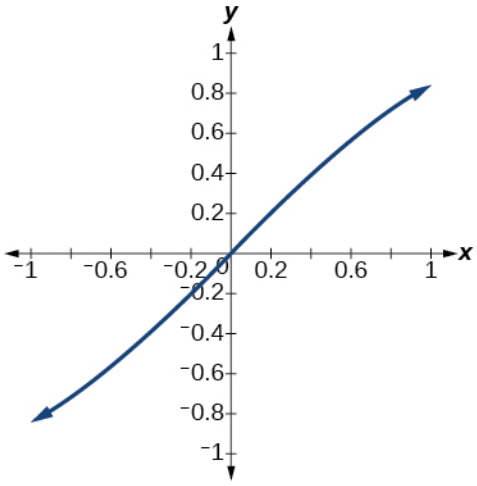



**186**. largest: 20,000; smallest: 4,000
**188**. amplitude: 8,000; period: 10; phase shift: 0
**190**. In 2007, the predicted population is 4,413. In 2010, the population will be 11,924.
**192**. 5 in.
**194**. 10 seconds
**196**. $\frac{\pi}{6}$
**198**. $\frac{\pi}{4}$
**200**. $\frac{\pi}{3}$
**202**. No solution
**204**. $\frac{12}{5}$
**206**. The graphs are not symmetrical with respect to the line $y = x$. They are symmetrical with respect to the $y$-axis.



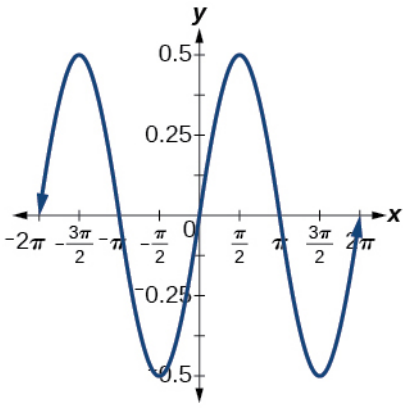

**208**. The graphs appear to be identical.



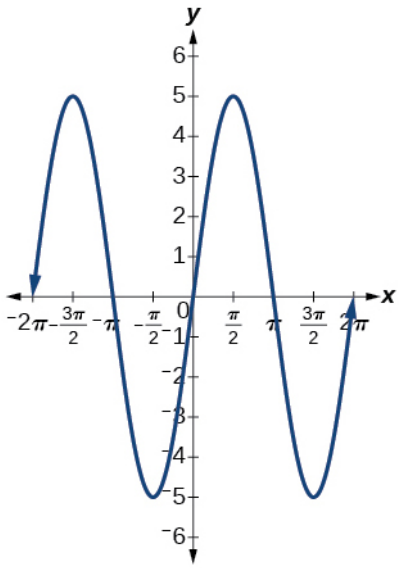




## Practice Test

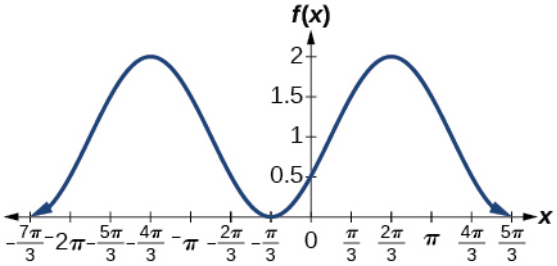
**209**. amplitude: 0.5; period: $2\pi$; midline $y = 0$



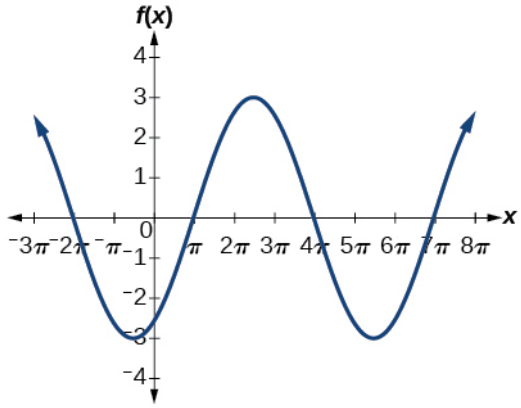

**211**. amplitude: 5; period: $2\pi$; midline: $y = 0$



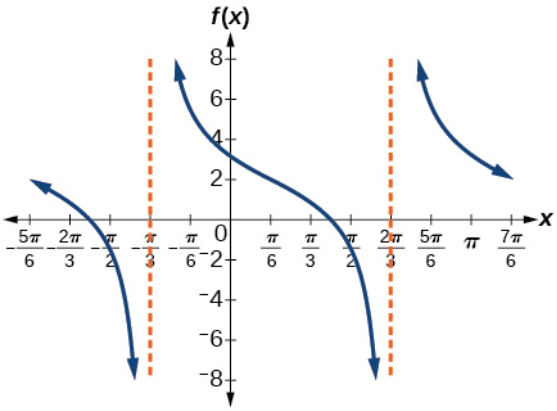

**213**. amplitude: 1; period: $2\pi$; midline: $y = 1$



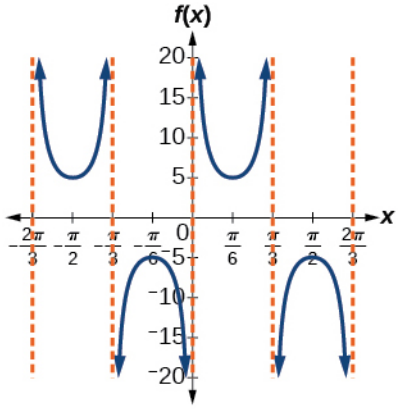

**215**. amplitude: 3; period: $6\pi$; midline: $y = 0$



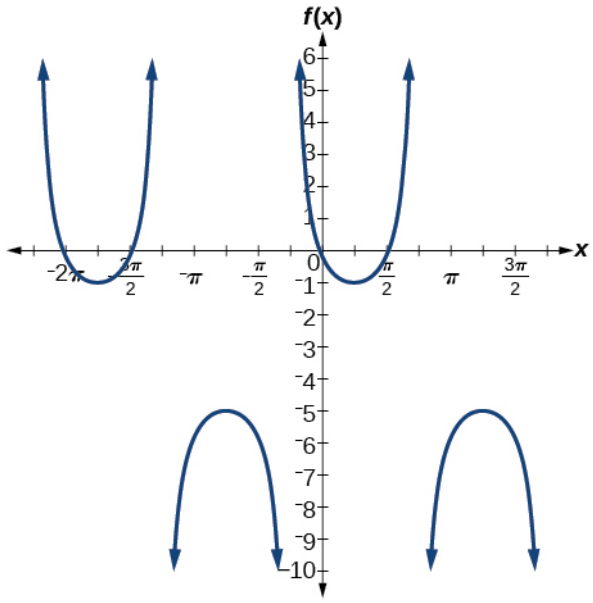

**217**. amplitude: none; period: $\pi$; midline: $y = 0$, asymptotes: $x = \frac{2\pi}{3} + \pi k$, where $k$ is an integer




**219**. amplitude: none; period: $\frac{2\pi}{3}$; midline: $y = 0$, asymptotes: $x = \frac{\pi}{3}k$, where $k$ is an integer




**221**. amplitude: none; period: $2\pi$; midline: $y = -3$




**223**. amplitude: 2; period: 2; midline:
$y = 0;$
$f(x) = 2\sin(\pi(x - 1))$

**225**. amplitude: 1; period: 12; phase shift:
$-6;$ midline
$y = -3$

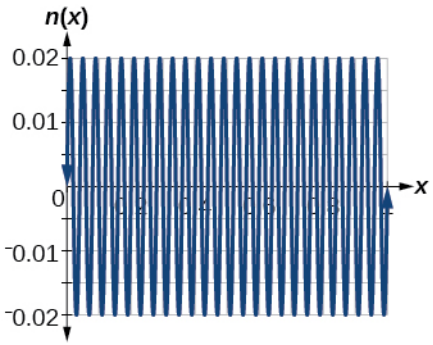
**227**.
$D(t) = 68 - 12\sin\left(\frac{\pi}{12}x\right)$

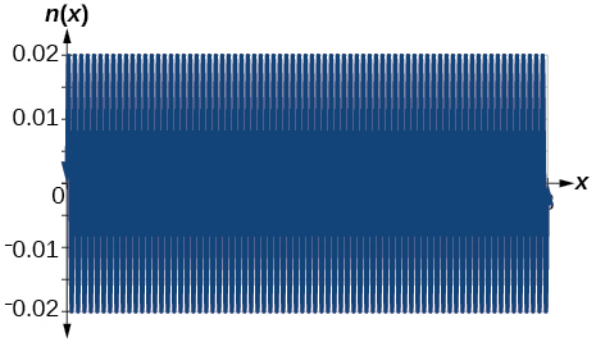
**229**. period:
$\frac{\pi}{6};$ horizontal shift:
$-7$

**231**.
$f(x) = \sec(\pi x);$ period: 2; phase shift: 0

**233**.
4

**235**. The views are different because the period of the wave is
$\frac{1}{25}.$ Over a bigger domain, there will be more cycles of the graph.




**237**.

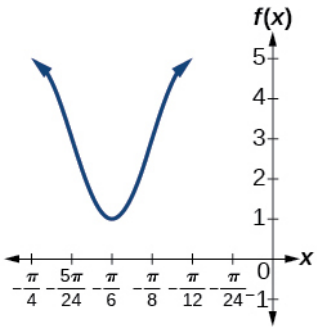
$\frac{3}{5}$

**239**. On the approximate intervals

$(0.5,\ 1),\ (1.6,\ 2.1),\ (2.6,\ 3.1),\ (3.7,\ 4.2),\ (4.7,\ 5.2),\ (5.6,\ 6.28)$

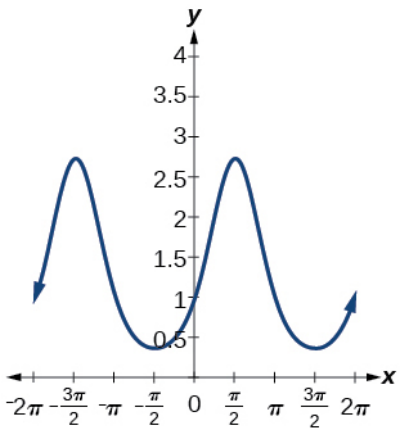
**241**.

$f(x) = 2\cos\left(12\left(x + \frac{\pi}{4}\right)\right) + 3$




**243**. This graph is periodic with a period of

$2\pi$.




**245**.
$\frac{\pi}{3}$

**247**.
$\frac{\pi}{2}$

**249**.
$\sqrt{1-(1-2x)^2}$

**251**.
$\frac{1}{\sqrt{1+x^4}}$

**253**.
$\frac{x+1}{x}$

**255**. False

**257**. approximately 0.07 radians

# Chapter 9

## Try It

**9.1**. (9.24)

$$\begin{aligned}\csc\theta\cos\theta\tan\theta &= \left(\frac{1}{\sin\theta}\right)\cos\theta\left(\frac{\sin\theta}{\cos\theta}\right)\\ &= \frac{\cos\theta}{\sin\theta}\left(\frac{\sin\theta}{\cos\theta}\right)\\ &= \frac{\sin\theta\cos\theta}{\sin\theta\cos\theta}\\ &= 1\end{aligned}$$

**9.2**. (9.29)

$$\begin{aligned}\frac{\cot\theta}{\csc\theta} &= \frac{\frac{\cos\theta}{\sin\theta}}{\frac{1}{\sin\theta}}\\ &= \frac{\cos\theta}{\sin\theta}\cdot\frac{\sin\theta}{1}\\ &= \cos\theta\end{aligned}$$

**9.3**.

$$\begin{aligned}\frac{\sin^2\theta-1}{\tan\theta\sin\theta-\tan\theta} &= \frac{(\sin\theta+1)(\sin\theta-1)}{\tan\theta(\sin\theta-1)}\\ &= \frac{\sin\theta+1}{\tan\theta}\end{aligned}$$

**9.4**. This is a difference of squares formula:
$25-9\sin^2\theta=(5-3\sin\theta)(5+3\sin\theta)$.

**9.5**. (9.40)

$$\begin{aligned}\frac{\cos\theta}{1+\sin\theta}\left(\frac{1-\sin\theta}{1-\sin\theta}\right) &= \frac{\cos\theta(1-\sin\theta)}{1-\sin^2\theta}\\ &= \frac{\cos\theta(1-\sin\theta)}{\cos^2\theta}\\ &= \frac{1-\sin\theta}{\cos\theta}\end{aligned}$$

**9.6**.
$\frac{\sqrt{2}+\sqrt{6}}{4}$

**9.7**.
$\frac{\sqrt{2}-\sqrt{6}}{4}$

**9.8**.
$\frac{1-\sqrt{3}}{1+\sqrt{3}}$

**9.9**.
$\cos\left(\frac{5\pi}{14}\right)$

**9.10**. (9.85)

$$\begin{aligned}\tan(\pi-\theta) &= \frac{\tan(\pi)-\tan\theta}{1+\tan(\pi)\tan\theta}\\ &= \frac{0-\tan\theta}{1+0\cdot\tan\theta}\\ &= -\tan\theta\end{aligned}$$

**9.11**.
$\cos(2\alpha)=\frac{7}{32}$

**9.12**.
$\cos^4\theta-\sin^4\theta=\left(\cos^2\theta+\sin^2\theta\right)\left(\cos^2\theta-\sin^2\theta\right)=\cos(2\theta)$

**9.13**.
$\cos(2\theta)\cos\theta=\left(\cos^2\theta-\sin^2\theta\right)\cos\theta=\cos^3\theta-\cos\theta\sin^2\theta$

**9.14**.

$$\begin{aligned}10\cos^4 x &= 10\left(\cos^2 x\right)^2\\ &= 10\left[\frac{1+\cos(2x)}{2}\right]^2 && \text{Substitute reduction formula for } \cos^2 x.\\ &= \frac{10}{4}\left[1+2\cos(2x)+\cos^2(2x)\right]\\ &= \frac{10}{4}+\frac{10}{2}\cos(2x)+\frac{10}{4}\left(\frac{1+\cos 2(2x)}{2}\right) && \text{Substitute reduction formula for } \cos^2 x.\\ &= \frac{10}{4}+\frac{10}{2}\cos(2x)+\frac{10}{8}+\frac{10}{8}\cos(4x)\\ &= \frac{30}{8}+5\cos(2x)+\frac{10}{8}\cos(4x)\\ &= \frac{15}{4}+5\cos(2x)+\frac{5}{4}\cos(4x)\end{aligned}$$

**9.15**.
$-\frac{2}{\sqrt{5}}$

**9.16**.
$\frac{1}{2}(\cos 6\theta+\cos 2\theta)$

**9.17**.
$\frac{1}{2}(\sin 2x + \sin 2y)$

**9.18**.
$\frac{-2-\sqrt{3}}{4}$

**9.19**.
$2\sin(2\theta)\cos(\theta)$

**9.20**.
$$\begin{aligned}\tan\theta\cot\theta - \cos^2\theta &= \left(\frac{\sin\theta}{\cos\theta}\right)\left(\frac{\cos\theta}{\sin\theta}\right) - \cos^2\theta \\ &= 1 - \cos^2\theta \\ &= \sin^2\theta\end{aligned}$$

**9.21**.
$x = \frac{7\pi}{6}, \frac{11\pi}{6}$

**9.22**.
$\frac{\pi}{3} \pm \pi k$

**9.23**.
$\theta \approx 1.7722 \pm 2\pi k$ and
$\theta \approx 4.5110 \pm 2\pi k$

**9.24**.
$\cos\theta = -1,\ \theta = \pi$

**9.25**.
$\frac{\pi}{2}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{3\pi}{2}$

## Section Exercises

**1**. All three functions,
$F,\ G,$ and
$H,$ are even. This is because
$F(-x) = \sin(-x)\sin(-x) = (-\sin x)(-\sin x) = \sin^2 x = F(x),\ G(-x) = \cos(-x)\cos(-x) = \cos x\cos x = \cos^2 x = G(x)$
and
$H(-x) = \tan(-x)\tan(-x) = (-\tan x)(-\tan x) = \tan^2 x = H(x).$

**3**. When
$\cos t = 0,$ then
$\sec t = \frac{1}{0},$ which is undefined.

**5**.
$\sin x$

**7**.
$\sec x$

**9**.
$\csc t$

**11**.
$-1$

**13**.
$\sec^2 x$

**15**.
$\sin^2 x + 1$

**17**.
$\frac{1}{\sin x}$

**19**.
$\frac{1}{\cot x}$
**21**.
$\tan x$
**23**.
$-4\sec x\tan x$
**25**.
$\pm\sqrt{\frac{1}{\cot^2 x}+1}$
**27**.
$\frac{\pm\sqrt{1-\sin^2 x}}{\sin x}$
**29**. Answers will vary. Sample proof:

$$\begin{aligned}\cos x-\cos^3 x &= \cos x\left(1-\cos^2 x\right)\\ &= \cos x\sin^2 x\end{aligned}$$

**31**. Answers will vary. Sample proof:

$$\frac{1+\sin^2 x}{\cos^2 x}=\frac{1}{\cos^2 x}+\frac{\sin^2 x}{\cos^2 x}=\sec^2 x+\tan^2 x=\tan^2 x+1+\tan^2 x=1+2\tan^2 x$$

**33**. Answers will vary. Sample proof:

$$\cos^2 x-\tan^2 x=1-\sin^2 x-\left(\sec^2 x-1\right)=1-\sin^2 x-\sec^2 x+1=2-\sin^2 x-\sec^2 x$$

**35**. False
**37**. False
**39**. Proved with negative and Pythagorean identities
**41**. True

$$3\sin^2\theta+4\cos^2\theta=3\sin^2\theta+3\cos^2\theta+\cos^2\theta=3\left(\sin^2\theta+\cos^2\theta\right)+\cos^2\theta=3+\cos^2\theta$$

**43**. The cofunction identities apply to complementary angles. Viewing the two acute angles of a right triangle, if one of those angles measures $x$, the second angle measures $\frac{\pi}{2}-x$. Then $\sin x=\cos\left(\frac{\pi}{2}-x\right)$. The same holds for the other cofunction identities. The key is that the angles are complementary.
**45**.
$\sin(-x)=-\sin x$, so $\sin x$ is odd.
$\cos(-x)=\cos(0-x)=\cos x$, so $\cos x$ is even.
**47**.
$\frac{\sqrt{2}+\sqrt{6}}{4}$
**49**.
$\frac{\sqrt{6}-\sqrt{2}}{4}$
**51**.
$-2-\sqrt{3}$
**53**.
$-\frac{\sqrt{2}}{2}\sin x-\frac{\sqrt{2}}{2}\cos x$
**55**.
$-\frac{1}{2}\cos x-\frac{\sqrt{3}}{2}\sin x$

**57**.
$\csc\theta$

**59**.
$\cot x$

**61**.
$\tan\left(\frac{x}{10}\right)$

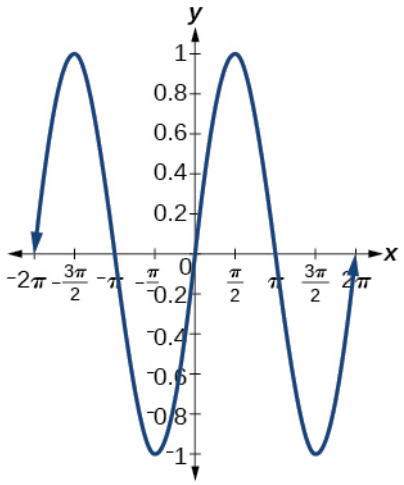
**63**.

$$\sin(a-b) = \left(\frac{4}{5}\right)\left(\frac{1}{3}\right) - \left(\frac{3}{5}\right)\left(\frac{2\sqrt{2}}{3}\right) = \frac{4-6\sqrt{2}}{15}$$

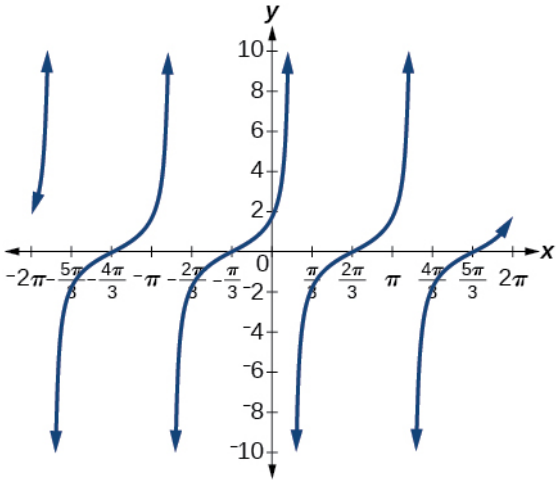
$$\cos(a+b) = \left(\frac{3}{5}\right)\left(\frac{1}{3}\right) - \left(\frac{4}{5}\right)\left(\frac{2\sqrt{2}}{3}\right) = \frac{3-8\sqrt{2}}{15}$$

**65**.
$\frac{\sqrt{2}-\sqrt{6}}{4}$

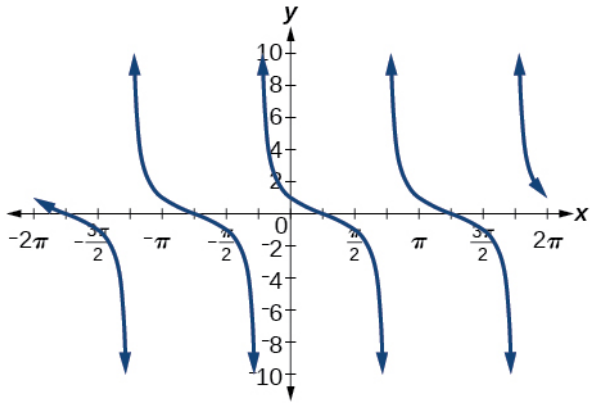
**67**.
$\sin x$




**69**.
$\cot\left(\frac{\pi}{6}-x\right)$

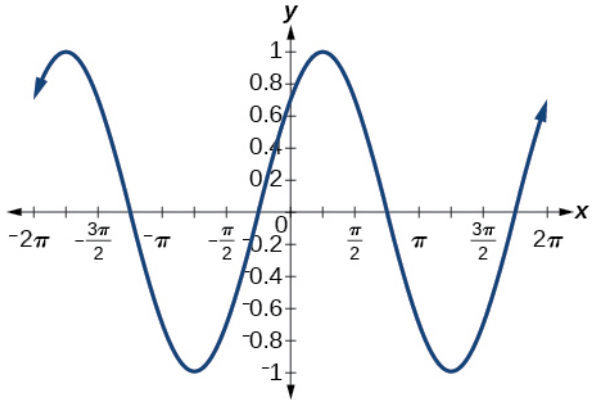



**71**.
$\cot\left(\frac{\pi}{4}+x\right)$




**73**.

$$\frac{\sin x}{\sqrt{2}} + \frac{\cos x}{\sqrt{2}}$$




**75**. They are the same.

**77**. They are the different, try

$$g(x) = \sin(9x) - \cos(3x)\sin(6x).$$

**79**. They are the same.

**81**. They are the different, try

$$g(\theta) = \frac{2\tan\theta}{1 - \tan^2\theta}.$$

**83**. They are different, try

$$g(x) = \frac{\tan x - \tan(2x)}{1 + \tan x \tan(2x)}.$$

**85**.

$$-\frac{\sqrt{3} - 1}{2\sqrt{2}}, \text{ or } -0.2588$$

**87**.

$$\frac{1 + \sqrt{3}}{2\sqrt{2}}, \text{ or } 0.9659$$

**89**.

$$\tan\left(x + \frac{\pi}{4}\right) =$$

$$\frac{\tan x + \tan\left(\frac{\pi}{4}\right)}{1 - \tan x \tan\left(\frac{\pi}{4}\right)} =$$

$$\frac{\tan x + 1}{1 - \tan x(1)} = \frac{\tan x + 1}{1 - \tan x}$$

**91**.

$$\frac{\cos(a+b)}{\cos a\cos b} =$$

$$\frac{\cos a\cos b}{\cos a\cos b} - \frac{\sin a\sin b}{\cos a\cos b} = 1 - \tan a\tan b$$

**93**.

$$\frac{\cos(x+h)-\cos x}{h} =$$

$$\frac{\cos x\cosh - \sin x\sinh - \cos x}{h} =$$

$$\frac{\cos x(\cosh - 1) - \sin x\sinh}{h} = \cos x\frac{\cos h - 1}{h} - \sin x\frac{\sin h}{h}$$

**95**. True

**97**. True. Note that $\sin(\alpha+\beta) = \sin(\pi-\gamma)$ and expand the right hand side.

**99**. Use the Pythagorean identities and isolate the squared term.

**101**. $\frac{1-\cos x}{\sin x}, \frac{\sin x}{1+\cos x},$ multiplying the top and bottom by $\sqrt{1-\cos x}$ and $\sqrt{1+\cos x},$ respectively.

**103**. a) $\frac{3\sqrt{7}}{32}$ b) $\frac{31}{32}$ c) $\frac{3\sqrt{7}}{31}$

**105**. a) $\frac{\sqrt{3}}{2}$ b) $-\frac{1}{2}$ c) $-\sqrt{3}$

**107**. $\cos\theta = -\frac{2\sqrt{5}}{5}, \sin\theta = \frac{\sqrt{5}}{5}, \tan\theta = -\frac{1}{2}, \csc\theta = \sqrt{5}, \sec\theta = -\frac{\sqrt{5}}{2}, \cot\theta = -2$

**109**. $2\sin\left(\frac{\pi}{2}\right)$

**111**. $\frac{\sqrt{2-\sqrt{2}}}{2}$

**113**. $\frac{\sqrt{2-\sqrt{3}}}{2}$

**115**. $2+\sqrt{3}$

**117**. $-1-\sqrt{2}$

**119**. a) $\frac{3\sqrt{13}}{13}$ b)

$-\frac{2\sqrt{13}}{13}$ c)

$-\frac{3}{2}$

**121**. a)

$\frac{\sqrt{10}}{4}$ b)

$\frac{\sqrt{6}}{4}$ c)

$\frac{\sqrt{15}}{3}$

**123**.

$\frac{120}{169}, \ -\frac{119}{169}, \ -\frac{120}{119}$

**125**.

$\frac{2\sqrt{13}}{13}, \frac{3\sqrt{13}}{13}, \frac{2}{3}$

**127**.

$\cos(74°)$

**129**.

$\cos(18x)$

**131**.

$3\sin(10x)$

**133**.

$-2\sin(-x)\cos(-x) = -2(-\sin(x)\cos(x)) = \sin(2x)$

**135**.

$$\frac{\sin(2\theta)}{1+\cos(2\theta)}\tan^2\theta = \frac{2\sin(\theta)\cos(\theta)}{1+\cos^2\theta-\sin^2\theta}\tan^2\theta =$$
$$\frac{2\sin(\theta)\cos(\theta)}{2\cos^2\theta}\tan^2\theta = \frac{\sin(\theta)}{\cos\theta}\tan^2\theta =$$
$$\cot(\theta)\tan^2\theta = \tan\theta$$

**137**.

$\frac{1+\cos(12x)}{2}$

**139**.

$\frac{3+\cos(12x)-4\cos(6x)}{8}$

**141**.

$\frac{2+\cos(2x)-2\cos(4x)-\cos(6x)}{32}$

**143**.

$\frac{3+\cos(4x)-4\cos(2x)}{3+\cos(4x)+4\cos(2x)}$

**145**.

$\frac{1-\cos(4x)}{8}$

**147**.

$\frac{3+\cos(4x)-4\cos(2x)}{4(\cos(2x)+1)}$

**149**.

$\frac{(1+\cos(4x))\sin x}{2}$

**151**.

$4\sin x\cos x\left(\cos^2 x-\sin^2 x\right)$

**153**.

$$\frac{2\tan x}{1+\tan^2 x} = \frac{\frac{2\sin x}{\cos x}}{1+\frac{\sin^2 x}{\cos^2 x}} = \frac{\frac{2\sin x}{\cos x}}{\frac{\cos^2 x+\sin^2 x}{\cos^2 x}} =$$

$$\frac{2\sin x}{\cos x}.\frac{\cos^2 x}{1} = 2\sin x\cos x = \sin(2x)$$

**155**.

$$\frac{2\sin x\cos x}{2\cos^2 x-1} = \frac{\sin(2x)}{\cos(2x)} = \tan(2x)$$

**157**.

$$\begin{aligned}\sin(x+2x) &= \sin x\cos(2x)+\sin(2x)\cos x\\ &= \sin x\left(\cos^2 x-\sin^2 x\right)+2\sin x\cos x\cos x\\ &= \sin x\cos^2 x-\sin^3 x+2\sin x\cos^2 x\\ &= 3\sin x\cos^2 x-\sin^3 x\end{aligned}$$

**159**.

$$\begin{aligned}\frac{1+\cos(2t)}{\sin(2t)-\cos t} &= \frac{1+2\cos^2 t-1}{2\sin t\cos t-\cos t}\\ &= \frac{2\cos^2 t}{\cos t(2\sin t-1)}\\ &= \frac{2\cos t}{2\sin t-1}\end{aligned}$$

**161**.

$$\begin{aligned}\left(\cos^2(4x)-\sin^2(4x)-\sin(8x))(\cos^2(4x)-\sin^2(4x)+\sin(8x)\right) &=\\ &= (\cos(8x)-\sin(8x))(\cos(8x)+\sin(8x))\\ &= \cos^2(8x)-\sin^2(8x)\\ &= \cos(16x)\end{aligned}$$

**162**. Substitute
$\alpha$ into cosine and
$\beta$ into sine and evaluate.

**164**. Answers will vary. There are some equations that involve a sum of two trig expressions where when converted to a product are easier to solve. For example:
$\frac{\sin(3x)+\sin x}{\cos x} = 1.$ When converting the numerator to a product the equation becomes:
$\frac{2\sin(2x)\cos x}{\cos x} = 1$

**166**.

$8(\cos(5x)-\cos(27x))$

**168**.

$\sin(2x)+\sin(8x)$

**170**.

$\frac{1}{2}(\cos(6x)-\cos(4x))$

**172**.

$2\cos(5t)\cos t$

**174**.

$2\cos(7x)$

**176**.

$2\cos(6x)\cos(3x)$

**178**.

$\frac{1}{4}\left(1+\sqrt{3}\right)$

**180**.
$\frac{1}{4}(\sqrt{3}-2)$

**182**.
$\frac{1}{4}(\sqrt{3}-1)$

**184**.
$\cos(80°)-\cos(120°)$

**186**.
$\frac{1}{2}(\sin(221°)+\sin(205°))$

**188**.
$\sqrt{2}\cos(31°)$

**190**.
$2\cos(66.5°)\sin(34.5°)$

**192**.
$2\sin(-1.5°)\cos(0.5°)$

**194**.
$2\sin(7x)-2\sin x = 2\sin(4x+3x)-2\sin(4x-3x) =$
$2(\sin(4x)\cos(3x)+\sin(3x)\cos(4x))-2(\sin(4x)\cos(3x)-\sin(3x)\cos(4x)) =$
$2\sin(4x)\cos(3x)+2\sin(3x)\cos(4x))-2\sin(4x)\cos(3x)+2\sin(3x)\cos(4x)) =$
$4\sin(3x)\cos(4x)$

**196**.
$\sin x+\sin(3x) = 2\sin\left(\frac{4x}{2}\right)\cos\left(\frac{-2x}{2}\right) =$
$2\sin(2x)\cos x = 2(2\sin x\cos x)\cos x =$
$4\sin x\cos^2 x$

**198**.
$2\tan x\cos(3x) = \frac{2\sin x\cos(3x)}{\cos x} = \frac{2(.5(\sin(4x)-\sin(2x)))}{\cos x} =$

$\frac{1}{\cos x}(\sin(4x)-\sin(2x)) = \sec x(\sin(4x)-\sin(2x))$

**200**.
$2\cos(35°)\cos(23°),\ 1.5081$

**202**.
$-2\sin(33°)\sin(11°),\ -0.2078$

**204**.
$\frac{1}{2}(\cos(99°)-\cos(71°)),\ -0.2410$

**206**. It is an identity.
**208**. It is not an identity, but
$2\cos^3 x$ is.

**210**.
$\tan(3t)$

**212**.
$2\cos(2x)$

**214**.
$-\sin(14x)$

**216**. Start with
$\cos x+\cos y$. Make a substitution and let
$x=\alpha+\beta$ and let
$y=\alpha-\beta$, so
$\cos x+\cos y$ becomes

$\cos(\alpha+\beta)+\cos(\alpha-\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta+\cos\alpha\cos\beta+\sin\alpha\sin\beta=$
$2\cos\alpha\cos\beta$ Since
$x=\alpha+\beta$ and
$y=\alpha-\beta,$ we can solve for
$\alpha$ and
$\beta$ in terms of $x$ and $y$ and substitute in for
$2\cos\alpha\cos\beta$ and get
$2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right).$

**218**.
$\frac{\cos(3x)+\cos x}{\cos(3x)-\cos x}=\frac{2\cos(2x)\cos x}{-2\sin(2x)\sin x}=-\cot(2x)\cot x$

**220**.
$\frac{\cos(2y)-\cos(4y)}{\sin(2y)+\sin(4y)}=\frac{-2\sin(3y)\sin(-y)}{2\sin(3y)\cos y}=$
$\frac{2\sin(3y)\sin(y)}{2\sin(3y)\cos y}=\tan y$

**222**.
$\cos x-\cos(3x)=-2\sin(2x)\sin(-x)=$
$2(2\sin x\cos x)\sin x=4\sin^2 x\cos x$

**224**.
$\tan\left(\frac{\pi}{4}-t\right)=\frac{\tan\left(\frac{\pi}{4}\right)-\tan t}{1+\tan\left(\frac{\pi}{4}\right)\tan(t)}=\frac{1-\tan t}{1+\tan t}$

**225**. There will not always be solutions to trigonometric function equations. For a basic example, $\cos(x)=-5$.

**227**. If the sine or cosine function has a coefficient of one, isolate the term on one side of the equals sign. If the number it is set equal to has an absolute value less than or equal to one, the equation has solutions, otherwise it does not. If the sine or cosine does not have a coefficient equal to one, still isolate the term but then divide both sides of the equation by the leading coefficient. Then, if the number it is set equal to has an absolute value greater than one, the equation has no solution.

**229**.
$\frac{\pi}{3},\frac{2\pi}{3}$

**231**.
$\frac{3\pi}{4},\frac{5\pi}{4}$

**233**.
$\frac{\pi}{4},\frac{5\pi}{4}$

**235**.
$\frac{\pi}{4},\frac{3\pi}{4},\frac{5\pi}{4},\frac{7\pi}{4}$

**237**.
$\frac{\pi}{4},\frac{7\pi}{4}$

**239**.
$\frac{7\pi}{6},\frac{11\pi}{6}$

**241**.
$\frac{\pi}{18},\frac{5\pi}{18},\frac{13\pi}{18},\frac{17\pi}{18},\frac{25\pi}{18},\frac{29\pi}{18}$

**243**.
$\frac{3\pi}{12},\frac{5\pi}{12},\frac{11\pi}{12},\frac{13\pi}{12},\frac{19\pi}{12},\frac{21\pi}{12}$

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**245**.
$\frac{1}{6}, \frac{5}{6}, \frac{13}{6}, \frac{17}{6}, \frac{25}{6}, \frac{29}{6}, \frac{37}{6}$

**247**.
$0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}$

**249**.
$\frac{\pi}{3}, \pi, \frac{5\pi}{3}$

**251**.
$\frac{\pi}{3}, \frac{3\pi}{2}, \frac{5\pi}{3}$

**253**.
$0, \pi$

**255**.
$\pi - \sin^{-1}\left(-\frac{1}{4}\right), \frac{7\pi}{6}, \frac{11\pi}{6}, 2\pi + \sin^{-1}\left(-\frac{1}{4}\right)$

**257**.
$\frac{1}{3}\left(\sin^{-1}\left(\frac{9}{10}\right)\right), \frac{\pi}{3} - \frac{1}{3}\left(\sin^{-1}\left(\frac{9}{10}\right)\right), \frac{2\pi}{3} + \frac{1}{3}\left(\sin^{-1}\left(\frac{9}{10}\right)\right), \pi - \frac{1}{3}\left(\sin^{-1}\left(\frac{9}{10}\right)\right), \frac{4\pi}{3} + \frac{1}{3}\left(\sin^{-1}\left(\frac{9}{10}\right)\right), \frac{5\pi}{3} - \frac{1}{3}\left(\sin^{-1}\left(\frac{9}{10}\right)\right)$

**259**.
$0$

**261**.
$\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

**263**.
$\frac{3\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}$

**265**.
$0, \frac{\pi}{3}, \pi, \frac{4\pi}{3}$

**267**. There are no solutions.

**269**.
$\cos^{-1}\left(\frac{1}{3}(1-\sqrt{7})\right), 2\pi - \cos^{-1}\left(\frac{1}{3}(1-\sqrt{7})\right)$

**271**.
$\tan^{-1}\left(\frac{1}{2}(\sqrt{29}-5)\right), \pi + \tan^{-1}\left(\frac{1}{2}(-\sqrt{29}-5)\right), \pi + \tan^{-1}\left(\frac{1}{2}(\sqrt{29}-5)\right), 2\pi + \tan^{-1}\left(\frac{1}{2}(-\sqrt{29}-5)\right)$

**273**. There are no solutions.

**275**. There are no solutions.

**277**.
$0, \frac{2\pi}{3}, \frac{4\pi}{3}$

**279**.
$\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

**281**.
$\sin^{-1}\left(\frac{3}{5}\right), \frac{\pi}{2}, \pi - \sin^{-1}\left(\frac{3}{5}\right), \frac{3\pi}{2}$

**283**.
$\cos^{-1}\left(-\frac{1}{4}\right), 2\pi - \cos^{-1}\left(-\frac{1}{4}\right)$

**285**.
$\frac{\pi}{3}, \cos^{-1}\left(-\frac{3}{4}\right), 2\pi - \cos^{-1}\left(-\frac{3}{4}\right), \frac{5\pi}{3}$

**287**.
$\cos^{-1}\left(\frac{3}{4}\right), \cos^{-1}\left(-\frac{2}{3}\right), 2\pi - \cos^{-1}\left(-\frac{2}{3}\right), 2\pi - \cos^{-1}\left(\frac{3}{4}\right)$

**289**.
$0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$

**291**.
$\frac{\pi}{3}, \cos^{-1}\left(-\frac{1}{4}\right), 2\pi - \cos^{-1}\left(-\frac{1}{4}\right), \frac{5\pi}{3}$

**293**. There are no solutions.

**295**.
$\pi + \tan^{-1}(-2), \pi + \tan^{-1}\left(-\frac{3}{2}\right), 2\pi + \tan^{-1}(-2), 2\pi + \tan^{-1}\left(-\frac{3}{2}\right)$

**297**.
$2\pi k + 0.2734, 2\pi k + 2.8682$

**299**.
$\pi k - 0.3277$

**301**.
$0.6694, 1.8287, 3.8110, 4.9703$

**303**.
$1.0472, 3.1416, 5.2360$

**305**.
$0.5326, 1.7648, 3.6742, 4.9064$

**307**.
$\sin^{-1}\left(\frac{1}{4}\right), \pi - \sin^{-1}\left(\frac{1}{4}\right), \frac{3\pi}{2}$

**309**.
$\frac{\pi}{2}, \frac{3\pi}{2}$

**311**. There are no solutions.

**313**.
$0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$

**315**. There are no solutions.

**317**.
$7.2^\circ$

**319**.
$5.7^\circ$

**321**.
$82.4^\circ$

**323**.
$31.0^\circ$

**325**.
$88.7^\circ$

**327**.
$59.0^\circ$

**329**.
$36.9^\circ$

## Review Exercises

**331**.
$\sin^{-1}\left(\frac{\sqrt{3}}{3}\right), \pi - \sin^{-1}\left(\frac{\sqrt{3}}{3}\right), \pi + \sin^{-1}\left(\frac{\sqrt{3}}{3}\right), 2\pi - \sin^{-1}\left(\frac{\sqrt{3}}{3}\right)$

**333**.
$\frac{7\pi}{6}, \frac{11\pi}{6}$

**335**.
$\sin^{-1}\left(\frac{1}{4}\right), \pi - \sin^{-1}\left(\frac{1}{4}\right)$

**337**.
1
**339**. Yes
**341**.
$-2-\sqrt{3}$
**343**.
$\frac{\sqrt{2}}{2}$
**345**.

$$\begin{aligned}\cos(4x)-\cos(3x)\cos x &= \cos(2x+2x)-\cos(x+2x)\cos x\\ &= \cos(2x)\cos(2x)-\sin(2x)\sin(2x)-\cos x\cos(2x)\cos x+\sin x\sin(2x)\cos x\\ &= \left(\cos^2 x-\sin^2 x\right)^2-4\cos^2 x\sin^2 x-\cos^2 x\left(\cos^2 x-\sin^2 x\right)+\sin x(2)\sin x\cos x\cos x\\ &= \left(\cos^2 x-\sin^2 x\right)^2-4\cos^2 x\sin^2 x-\cos^2 x\left(\cos^2 x-\sin^2 x\right)+2\sin^2 x\cos^2 x\\ &= \cos^4 x-2\cos^2 x\sin^2 x+\sin^4 x-4\cos^2 x\sin^2 x-\cos^4 x+\cos^2 x\sin^2 x+2\sin^2 x\cos^2 x\\ &= \sin^4 x-4\cos^2 x\sin^2 x+\cos^2 x\sin^2 x\\ &= \sin^2 x\left(\sin^2 x+\cos^2 x\right)-4\cos^2 x\sin^2 x\\ &= \sin^2 x-4\cos^2 x\sin^2 x\end{aligned}$$

**347**.
$\tan\left(\frac{5}{8}x\right)$
**349**.
$\frac{\sqrt{3}}{3}$
**351**.
$-\frac{24}{25},\ -\frac{7}{25},\ \frac{24}{7}$
**353**.
$\sqrt{2(2+\sqrt{2})}$
**355**.
$\frac{\sqrt{2}}{10},\ \frac{7\sqrt{2}}{10},\ \frac{1}{7},\ \frac{3}{5},\ \frac{4}{5},\ \frac{3}{4}$
**357**.

$$\begin{aligned}\cot x\cos(2x) &= \cot x\left(1-2\sin^2 x\right)\\ &= \cot x-\frac{\cos x}{\sin x}(2)\sin^2 x\\ &= -2\sin x\cos x+\cot x\\ &= -\sin(2x)+\cot x\end{aligned}$$

**359**.
$\frac{10\sin x-5\sin(3x)+\sin(5x)}{8(\cos(2x)+1)}$
**361**.
$\frac{\sqrt{3}}{2}$
**363**.
$-\frac{\sqrt{2}}{2}$
**365**.
$\frac{1}{2}(\sin(6x)+\sin(12x))$

**367**.
$2\sin\left(\frac{13}{2}x\right)\cos\left(\frac{9}{2}x\right)$

**369**.
$\frac{3\pi}{4}, \frac{7\pi}{4}$

**371**.
$0, \frac{\pi}{6}, \frac{5\pi}{6}, \pi$

**373**.
$\frac{3\pi}{2}$

**375**. No solution

**377**.
$0.2527, 2.8889, 4.7124$

**379**.
$1.3694, 1.9106, 4.3726, 4.9137$

## Practice Test

**380**. 1

**382**.
$\sec(\theta)$

**383**. 1

**384**.
$\frac{\sqrt{2}-\sqrt{6}}{4}$

**386**.
$-\sqrt{2}-\sqrt{3}$

**388**.
$-\frac{1}{2}\cos(\theta)-\frac{\sqrt{3}}{2}\sin(\theta)$

**389**.
$\frac{-1+\tan(\theta)}{1+\tan(\theta)}$

**390**.
$\frac{1-\cos(64^\circ)}{2}$

**391**.
$\pm\sqrt{\frac{1+\cos(\theta)}{1-\cos(\theta)}}$

**392**.
$0, \pi$

**393**.
$\sin^{-1}\left(\frac{1}{4}(\sqrt{13}-1)\right), \pi-\sin^{-1}\left(\frac{1}{4}(\sqrt{13}-1)\right)$

**395**.
$0, \frac{\pi}{6}, \frac{5\pi}{6}, \pi$

**397**.
$4[\sin(18x)-\sin(12x)]$

**398**.
$4\sin(2\theta)\cos(6\theta)$

**399**.
$\frac{\pi}{3}+k\pi$

**401**.
$120^\circ$

**402**.
$60^\circ$

**403**.
$-\frac{24}{25},\ -\frac{7}{25},\ \frac{24}{7}$

**405**.
$\frac{1}{8}(3+\cos(4x)-4\cos(2x))$

**407**.

$$\begin{aligned}\sin(3x)-\cos x\sin(2x) &= \\ \sin(x+2x)-\cos x(2\sin x\cos x) &= \\ \sin x\cos(2x)+\sin(2x)\cos x-2\sin x\cos^2 x &= \\ \sin x(\cos^2 x-\sin^2 x)+2\sin x\cos x\cos x-2\sin x\cos^2 x &= \\ \sin x\cos^2 x-\sin^3+0 &= \\ \cos^2 x\sin x-\sin^3 x &= \cos^2 x\sin x-\sin^3 x\end{aligned}$$

**409**.
$y=2\cos(\pi x+\pi)$

**411**.
$81.5°,\ 78.7°$

**413**.
$6+5\cos\left(\frac{\pi}{6}(1-x)\right)$. From November 23 to February 6.

**415**.
$D(t)=2\cos\left(\frac{\pi}{6}t\right)+108+\frac{1}{4}t,$ 93.5855 months (or 7.8 years) from now

## Chapter 10

## Try It

**10.1**.
$\alpha=98^\circ \quad a=34.6$
$\beta=39^\circ \quad b=22$
$\gamma=43^\circ \quad c=23.8$

**10.2**. Solution 1 (10.16)

$$\begin{aligned}\alpha &= 80^\circ & a &= 120\\ \beta &\approx 83.2^\circ & b &= 121\\ \gamma &\approx 16.8^\circ & c &\approx 35.2\end{aligned}$$

Solution 2 (10.17)

$$\begin{aligned}\alpha' &= 80^\circ & a' &= 120\\ \beta' &\approx 96.8^\circ & b' &= 121\\ \gamma' &\approx 3.2^\circ & c' &\approx 6.8\end{aligned}$$

**10.3**.
$\beta\approx 5.7^\circ,\ \gamma\approx 94.3^\circ,\ c\approx 101.3$

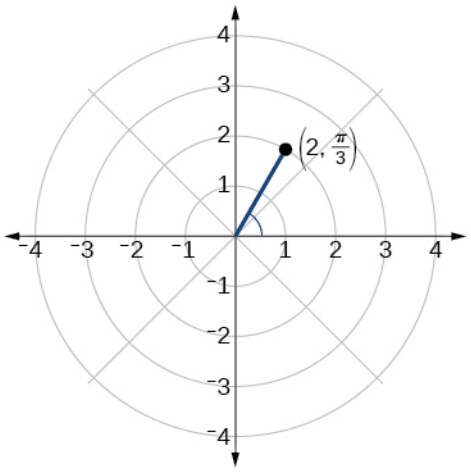
**10.4**. two
**10.5**. about
8.2 square feet

**10.6**. 161.9 yd.
**10.7**.
$a\approx 14.9,\ \ \beta\approx 23.8^\circ,\ \ \gamma\approx 126.2^\circ.$

**10.8**.
$\alpha\approx 27.7^\circ,\ \ \beta\approx 40.5^\circ,\ \ \gamma\approx 111.8^\circ$

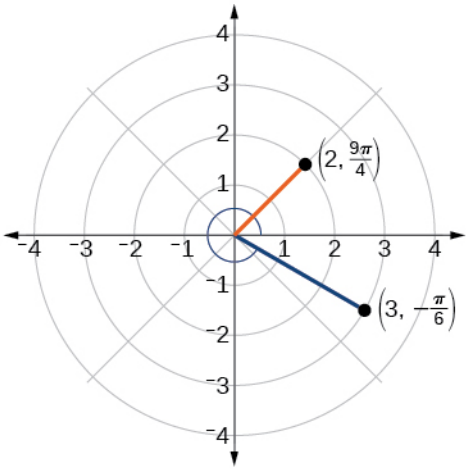
**10.9**. Area = 552 square feet

**10.10**. about 8.15 square feet




**10.11**.

**10.12**.




**10.13**.

$(x, y) = \left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

**10.14**.

$r = \sqrt{3}$

**10.15**.

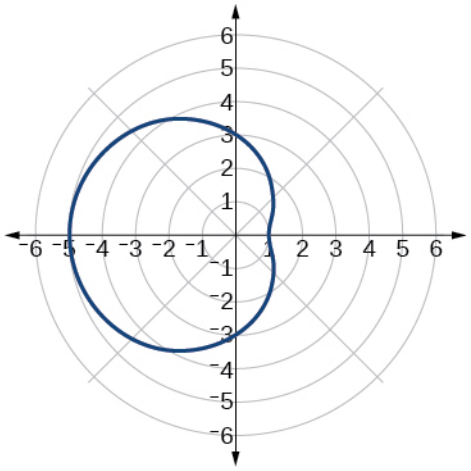
$x^2 + y^2 = 2y$ or, in the standard form for a circle,

$x^2 + (y - 1)^2 = 1$

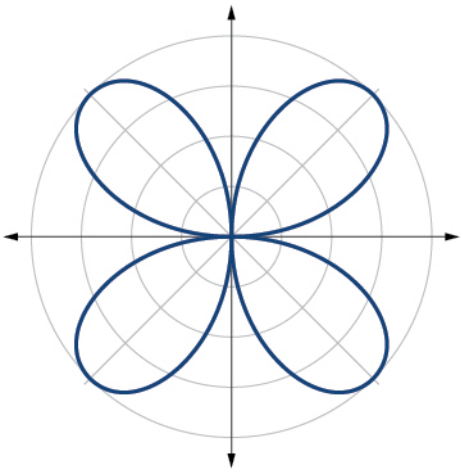
**10.16**. The equation fails the symmetry test with respect to the line $\theta = \frac{\pi}{2}$ and with respect to the pole. It passes the polar axis symmetry test.

**10.17**. Tests will reveal symmetry about the polar axis. The zero is $\left(0, \frac{\pi}{2}\right)$, and the maximum value is $(3, 0)$.

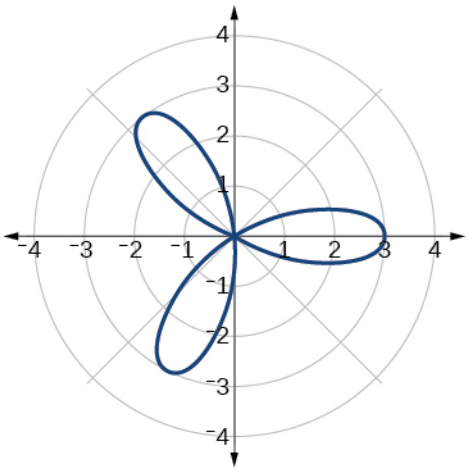
**10.18**.



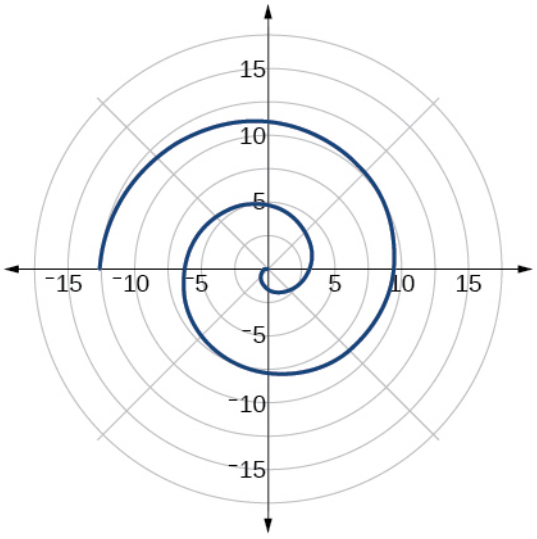

**10.19**. The graph is a rose curve,
$n$ even



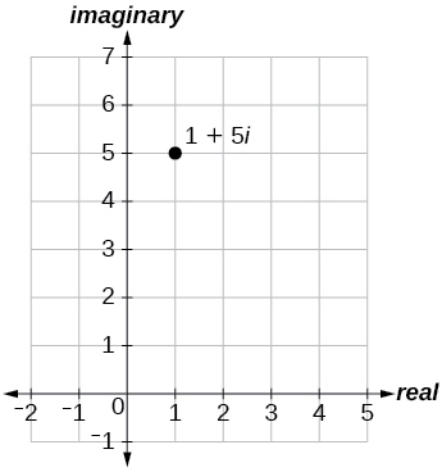
**10.20**.




Rose curve,
$n$ odd
**10.21**.




**10.22**.




**10.23**. 13

**10.24**.

$|z| = \sqrt{50} = 5\sqrt{2}$

**10.25**.

$z = 3\left(\cos\left(\frac{\pi}{2}\right) + i\sin\left(\frac{\pi}{2}\right)\right)$

**10.26**.

$z = 2\left(\cos\left(\frac{\pi}{6}\right) + i\sin\left(\frac{\pi}{6}\right)\right)$

**10.27**.

$z = 2\sqrt{3} - 2i$

**10.28**.

$z_1 z_2 = -4\sqrt{3}; \frac{z_1}{z_2} = -\frac{\sqrt{3}}{2} + \frac{3}{2}i$

**10.29**.

$z_0 = 2(\cos(30°) + i\sin(30°))$

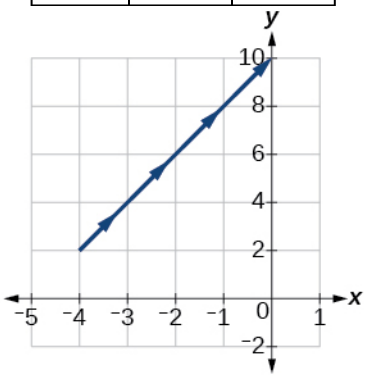
$z_1 = 2(\cos(120°) + i\sin(120°))$

$z_2 = 2(\cos(210°) + i\sin(210°))$

$z_3 = 2(\cos(300°) + i\sin(300°))$

**10.30**.

| $t$ | $x(t)$ | $y(t)$ |
|---|---|---|
| $-1$ | $-4$ | 2 |
| 0 | $-3$ | 4 |
| 1 | $-2$ | 6 |
| 2 | $-1$ | 8 |




**10.31**.
$x(t) = t^3 - 2t$

$y(t) = t$

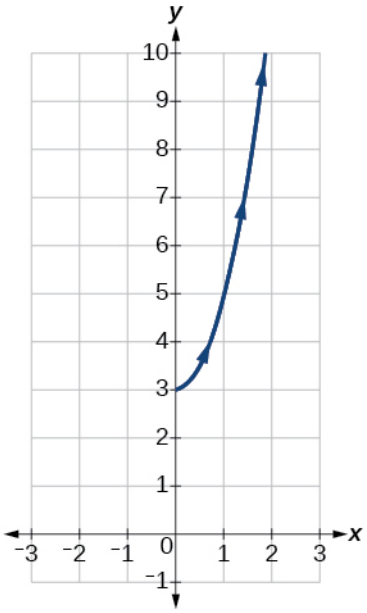
**10.32**.
$y = 5 - \sqrt{\frac{1}{2}x - 3}$

**10.33**.
$y = \ln\sqrt{x}$

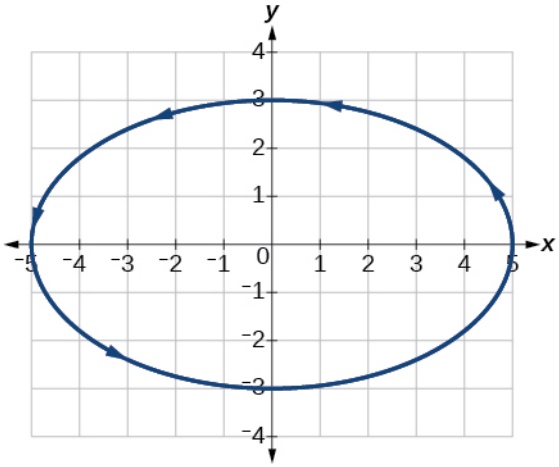
**10.34**.
$\frac{x^2}{4} + \frac{y^2}{9} = 1$

**10.35**.
$y = x^2$

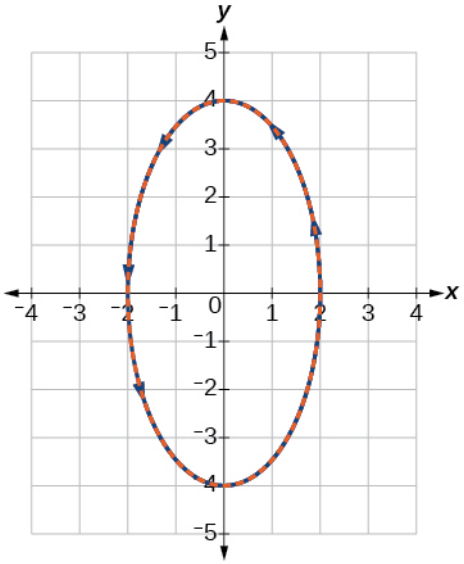
**10.36**.



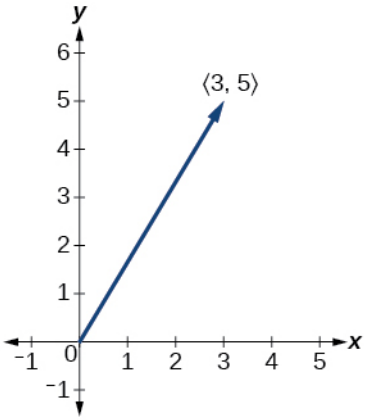

**10.37**.




**10.38**. The graph of the parametric equations is in red and the graph of the rectangular equation is drawn in blue dots on top of the parametric equations.




**10.39**.




**10.40**.
$3u = \langle 15, 12 \rangle$

**10.41**.
$u = 8i - 11j$

**10.42**.
$v = \sqrt{34}\cos(59°)i + \sqrt{34}\sin(59°)j$ Magnitude = $\sqrt{34}$
$\theta = \tan^{-1}\left(\frac{5}{3}\right) = 59.04°$

## Section Exercises

**1**. The altitude extends from any vertex to the opposite side or to the line containing the opposite side at a 90° angle.
**3**. When the known values are the side opposite the missing angle and another side and its opposite angle.
**5**. A triangle with two given sides and a non-included angle.
**7**.
$\beta = 72°$, $a \approx 12.0$, $b \approx 19.9$

**9**.
$\gamma = 20°$, $b \approx 4.5$, $c \approx 1.6$

**11**.
$b \approx 3.78$

**13**.
$c \approx 13.70$

**15**. one triangle,
$\alpha \approx 50.3°$, $\beta \approx 16.7°$, $a \approx 26.7$

**17**. two triangles,
$\gamma \approx 54.3°$, $\beta \approx 90.7°$, $b \approx 20.9$ or
$\gamma' \approx 125.7°$, $\beta' \approx 19.3°$, $b' \approx 6.9$

**19**. two triangles,
$\beta \approx 75.7°$, $\gamma \approx 61.3°$, $b \approx 9.9$ or
$\beta' \approx 18.3°$, $\gamma' \approx 118.7°$, $b' \approx 3.2$

**21**. two triangles,
$\alpha \approx 143.2°$, $\beta \approx 26.8°$, $a \approx 17.3$ or
$\alpha' \approx 16.8°$, $\beta' \approx 153.2°$, $a' \approx 8.3$

**23**. no triangle possible
**25**.
$A \approx 47.8°$ or
$A' \approx 132.2°$

**27**.
8.6

**29**.
370.9

**31**.
12.3
**33**.
12.2
**35**.
16.0
**37**.
29.7°
**39**.
$x = 76.9°$or $x = 103.1°$
**41**.
110.6°
**43**.
$A \approx 39.4,\ C \approx 47.6,\ BC \approx 20.7$
**45**.
57.1
**47**.
42.0
**49**.
430.2
**51**.
10.1
**53**.
$AD \approx\ 13.8$
**55**.
$AB \approx 2.8$
**57**.
$L \approx 49.7,\ N \approx 56.3,\ LN \approx 5.8$
**59**. 51.4 feet
**61**. The distance from the satellite to station
$A$ is approximately 1716 miles. The satellite is approximately 1706 miles above the ground.
**63**. 2.6 ft
**65**. 5.6 km
**67**. 371 ft
**69**. 5936 ft
**71**. 24.1 ft
**73**. 19,056 $ft^2$
**75**. 445,624 square miles
**77**. 8.65 $ft^2$
**78**. two sides and the angle opposite the missing side.
**80**.
$s$ is the semi-perimeter, which is half the perimeter of the triangle.
**82**. The Law of Cosines must be used for any oblique (non-right) triangle.
**84**. 11.3
**86**. 34.7
**88**. 26.7
**90**. 257.4
**92**. not possible
**94**. 95.5°
**96**. 26.9°
**98**.
$B \approx 45.9°,\ C \approx 99.1°,\ a \approx 6.4$
**100**.
$A \approx 20.6°,\ B \approx 38.4°,\ c \approx 51.1$
**102**.
$A \approx 37.8°,\ B \approx 43.8,\ C \approx 98.4°$

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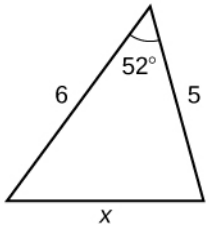
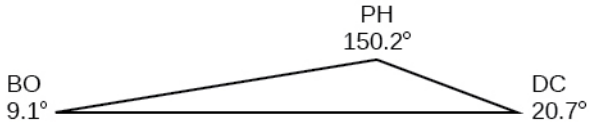
**104**. 177.56 $in^2$
**106**. 0.04 $m^2$
**108**. 0.91 $yd^2$
**110**. 3.0
**112**. 29.1
**114**. 0.5
**116**. 70.7°
**118**. 77.4°
**120**. 25.0
**122**. 9.3
**124**. 43.52
**126**. 1.41
**128**. 0.14
**130**. 18.3
**132**. 48.98
**134**.




**136**. 7.62
**138**. 85.1
**140**. 24.0 km
**142**. 99.9 ft
**144**. 37.3 miles
**146**. 2371 miles
**148**.




**150**. 599.8 miles
**152**. 65.4 $cm^2$
**154**. 468 $ft^2$
**156**. For polar coordinates, the point in the plane depends on the angle from the positive *x*-axis and distance from the origin, while in Cartesian coordinates, the point represents the horizontal and vertical distances from the origin. For each point in the coordinate plane, there is one representation, but for each point in the polar plane, there are infinite representations.
**158**. Determine
$\theta$ for the point, then move
$r$ units from the pole to plot the point. If
$r$ is negative, move
$r$ units from the pole in the opposite direction but along the same angle. The point is a distance of
$r$ away from the origin at an angle of
$\theta$ from the polar axis.
**160**. The point
$\left(-3, \frac{\pi}{2}\right)$ has a positive angle but a negative radius and is plotted by moving to an angle of
$\frac{\pi}{2}$ and then moving 3 units in the negative direction. This places the point 3 units down the negative *y*-axis. The point
$\left(3, -\frac{\pi}{2}\right)$ has a negative angle and a positive radius and is plotted by first moving to an angle of
$-\frac{\pi}{2}$ and then moving 3 units down, which is the positive direction for a negative angle. The point is also 3 units down the negative *y*-axis.

**162**.
$(-5,\ 0)$

**164**.
$\left(-\frac{3\sqrt{3}}{2},\ -\frac{3}{2}\right)$

**166**.
$\left(2\sqrt{5},\quad 0.464\right)$

**168**.
$\left(\sqrt{34},\ 5.253\right)$

**170**.
$\left(8\sqrt{2},\ \frac{\pi}{4}\right)$

**172**.
$r = 4\csc\theta$

**174**.
$r = \sqrt[3]{\frac{sin\theta}{2cos^4\theta}}$

**176**.
$r = 3\cos\theta$

**178**.
$r = \frac{3\sin\theta}{\cos(2\theta)}$

**180**.
$r = \frac{9\sin\theta}{\cos^2\theta}$

**182**.
$r = \sqrt{\frac{1}{9\cos\theta\sin\theta}}$

**184**.
$x^2 + y^2 = 4x$ or
$\frac{(x-2)^2}{4} + \frac{y^2}{4} = 1;$ circle

**186**.
$3y + x = 6;$ line

**188**.
$y = 3;$ line

**190**.
$xy = 4;$ hyperbola

**192**.
$x^2 + y^2 = 4;$ circle

**194**.
$x - 5y = 3;$ line

**196**.
$\left(3,\ \frac{3\pi}{4}\right)$

**198**.
$(5,\ \pi)$

**200**.

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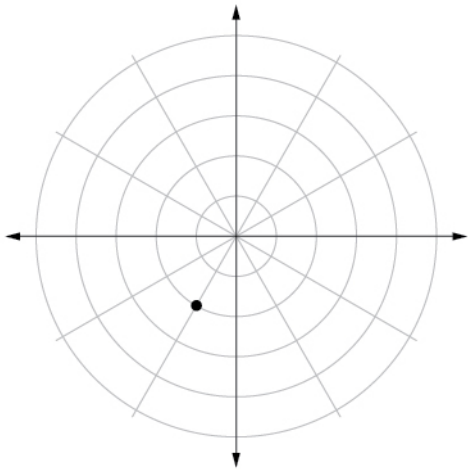
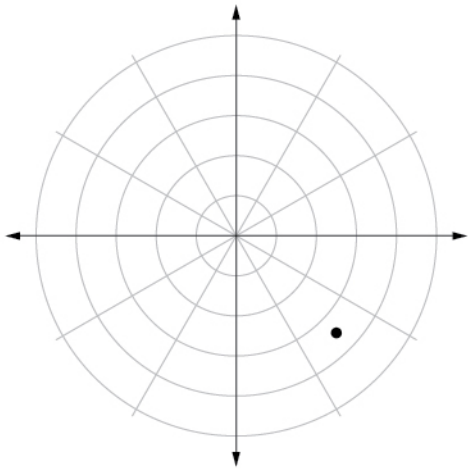
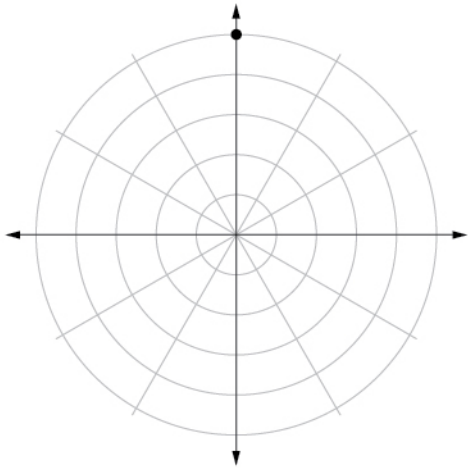


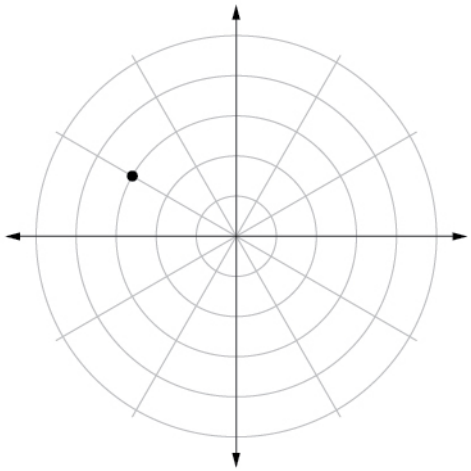
**202**.



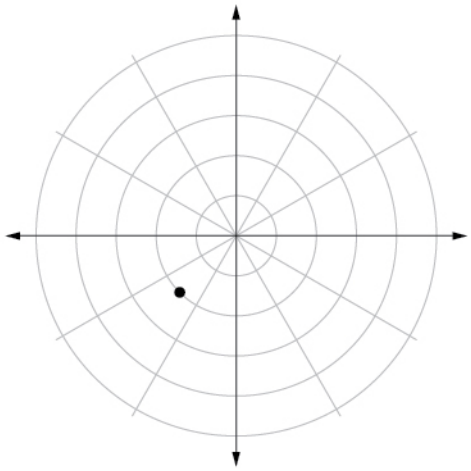
**204**.



**206**.

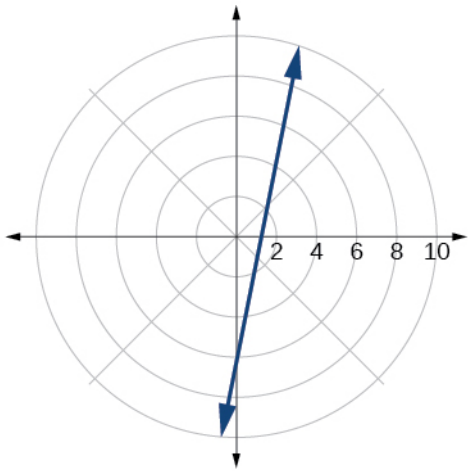


**208**.



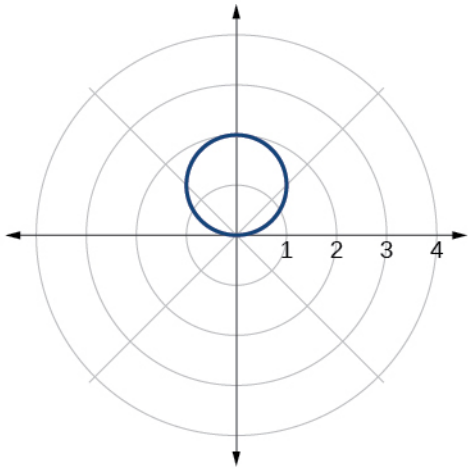
**210**.

$r = \frac{6}{5\cos\theta - \sin\theta}$



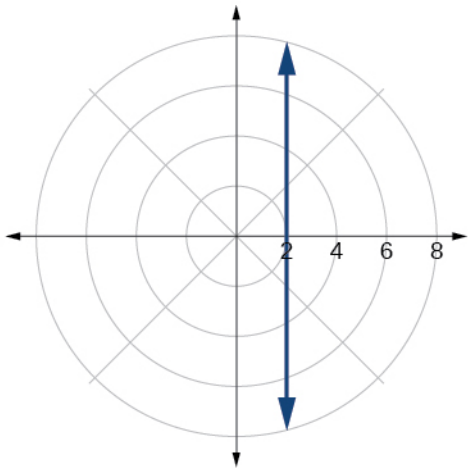

**212**.

$r = 2\sin\theta$



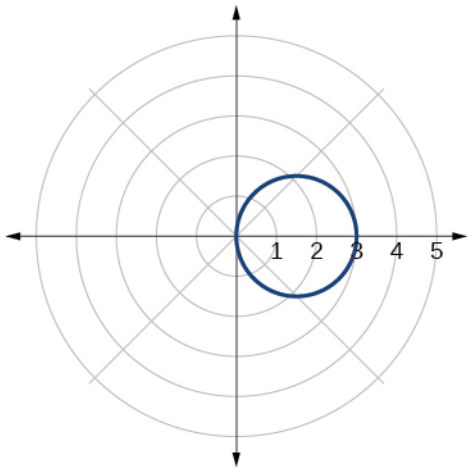

**214**.

$r = \frac{2}{\cos\theta}$



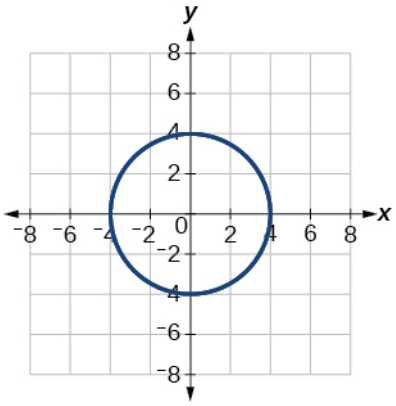

**216**.

$r = 3\cos\theta$

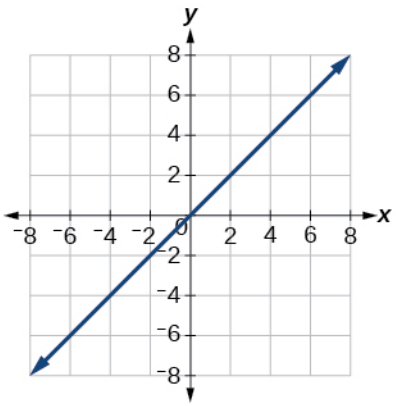



**218**.

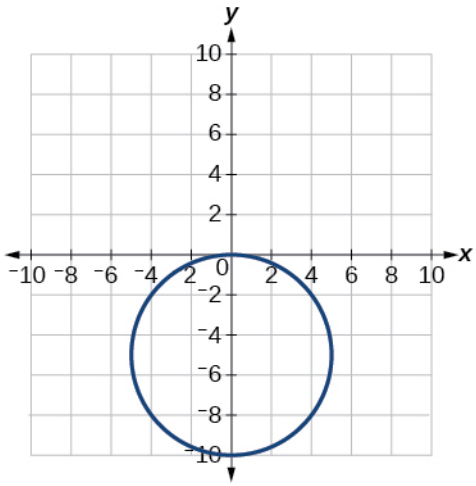
$x^2 + y^2 = 16$




**220**.
$y = x$




**222**.
$x^2 + (y + 5)^2 = 25$




**224**.
$(1.618, -1.176)$

**226**.
$(10.630, 131.186°)$

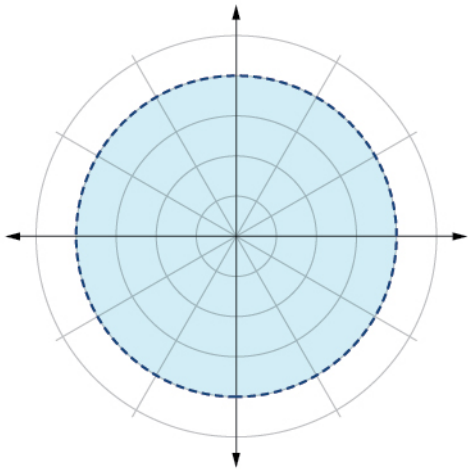
**228**.
$(2, 3.14) or (2, \pi)$

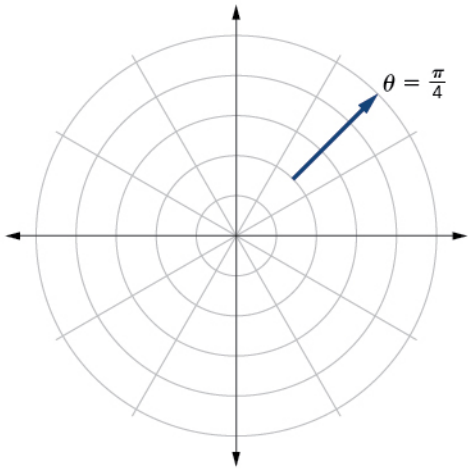
**230**. A vertical line with
$a$ units left of the $y$-axis.

**232**. A horizontal line with
$a$ units below the $x$-axis.

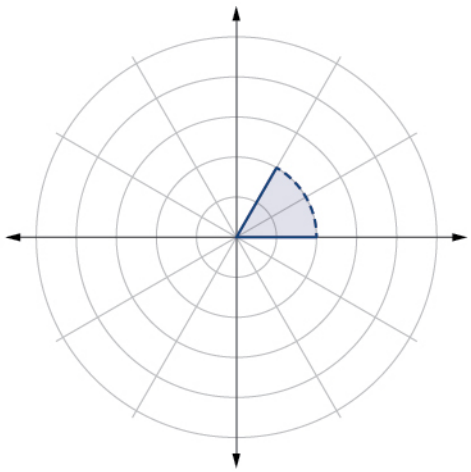
**234**.

**236**.






**238**.

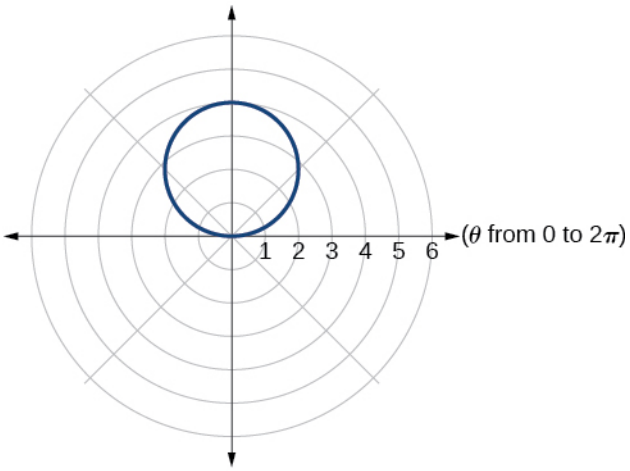


**240**. Symmetry with respect to the polar axis is similar to symmetry about the $x$-axis, symmetry with respect to the pole is similar to symmetry about the origin, and symmetric with respect to the line $\theta = \frac{\pi}{2}$ is similar to symmetry about the $y$-axis.

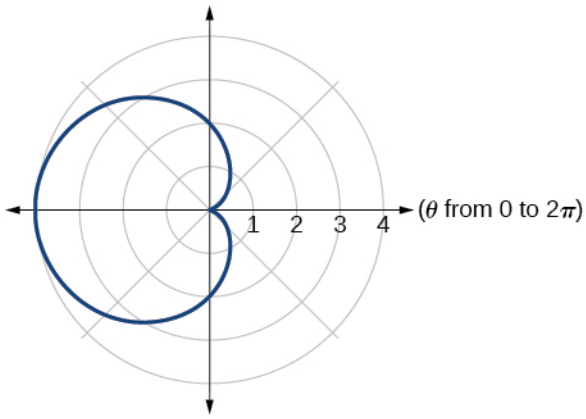
**242**. Test for symmetry; find zeros, intercepts, and maxima; make a table of values. Decide the general type of graph, cardioid, limaçon, lemniscate, etc., then plot points at $\theta = 0, \frac{\pi}{2}, \pi$ and $\frac{3\pi}{2}$, and sketch the graph.

**244**. The shape of the polar graph is determined by whether or not it includes a sine, a cosine, and constants in the equation.

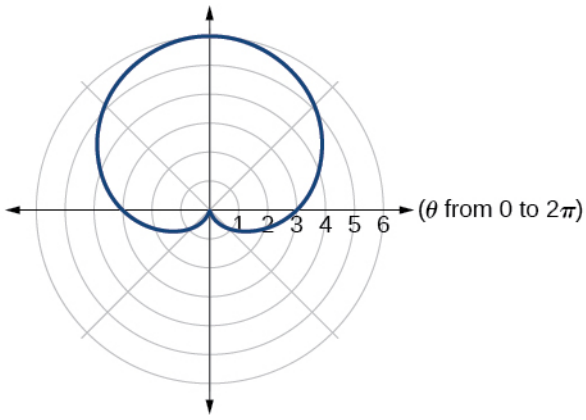
**246**. symmetric with respect to the polar axis
**248**. symmetric with respect to the polar axis, symmetric with respect to the line $\theta = \frac{\pi}{2}$, symmetric with respect to the pole
**250**. no symmetry
**252**. no symmetry
**254**. symmetric with respect to the pole
**256**. circle



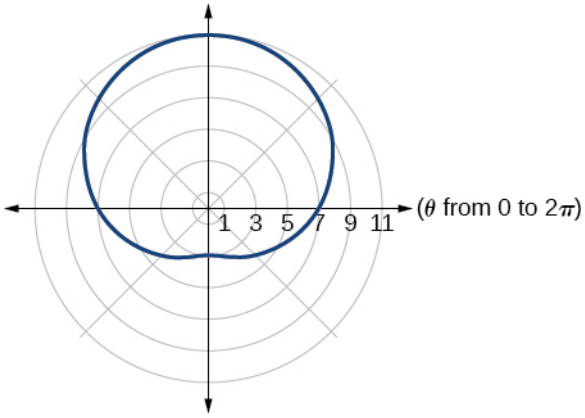

**258**. cardioid



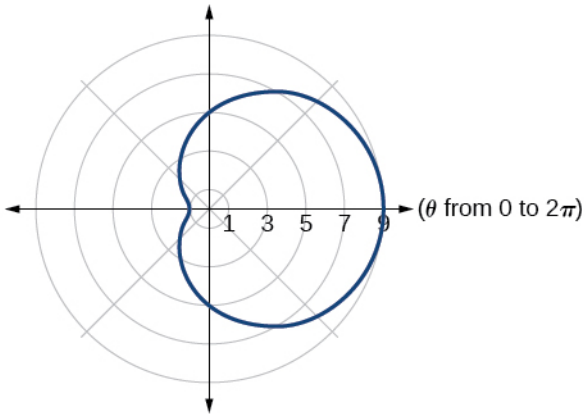

**260**. cardioid



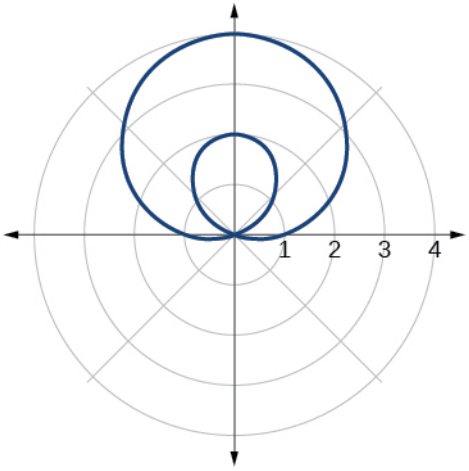

**262**. one-loop/dimpled limaçon



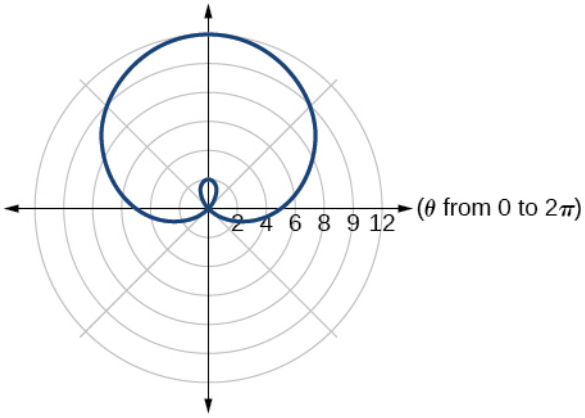

**264**. one-loop/dimpled limaçon



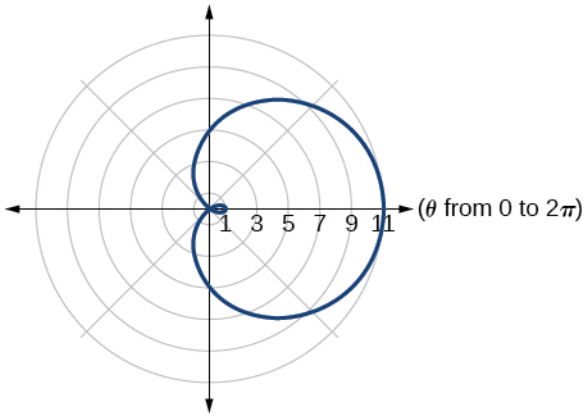

**266**. inner loop/two-loop limaçon



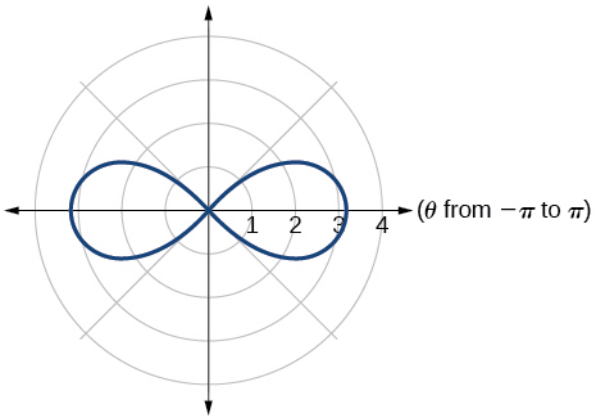

**268**. inner loop/two-loop limaçon



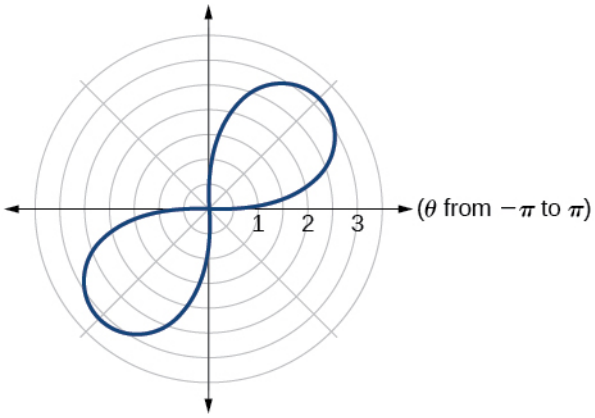

**270**. inner loop/two-loop limaçon



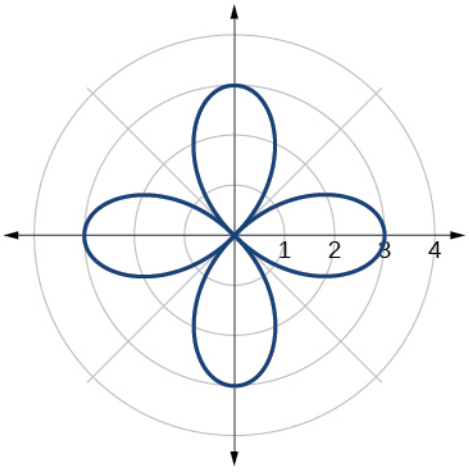

**272**. lemniscate



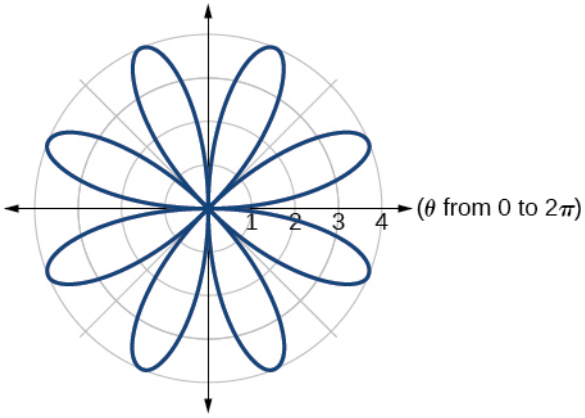

**274**. lemniscate



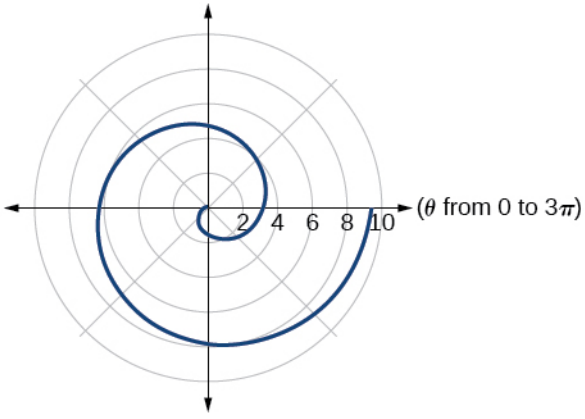
**276**. rose curve




**278**. rose curve

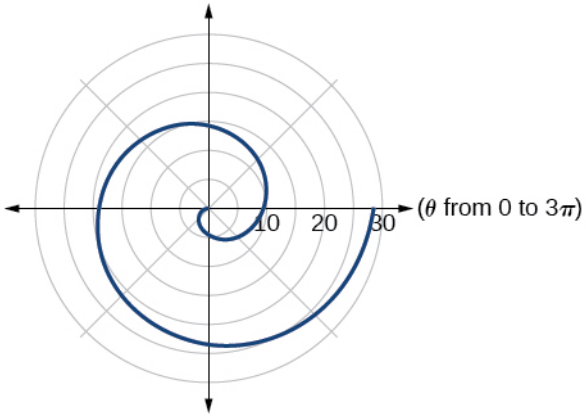



**280**. Archimedes' spiral

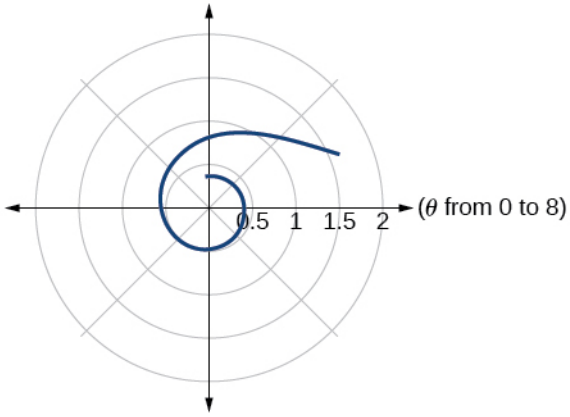



**282**. Archimedes' spiral

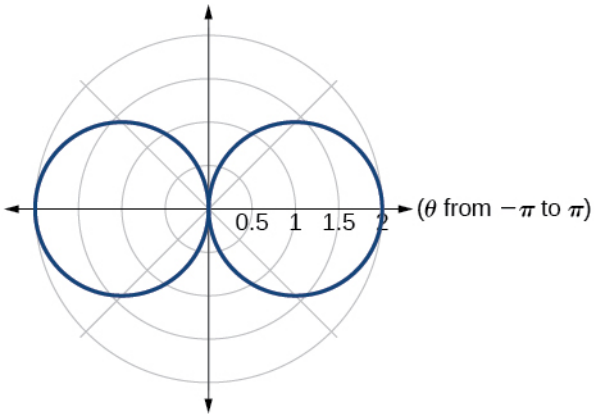
**284**.



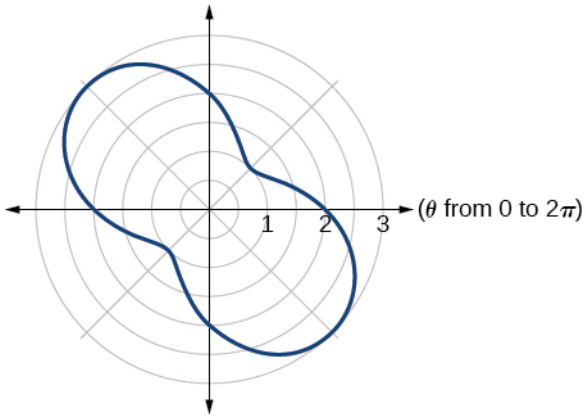

**286**.

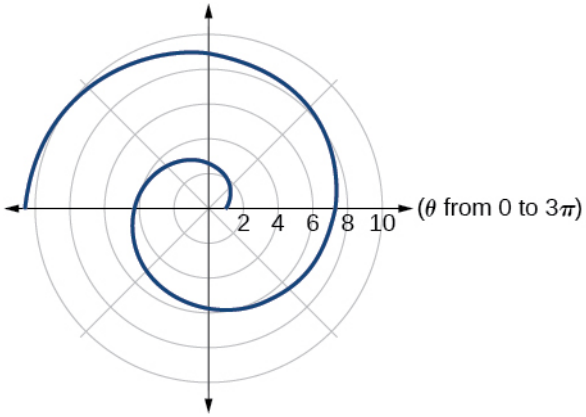



**288**.

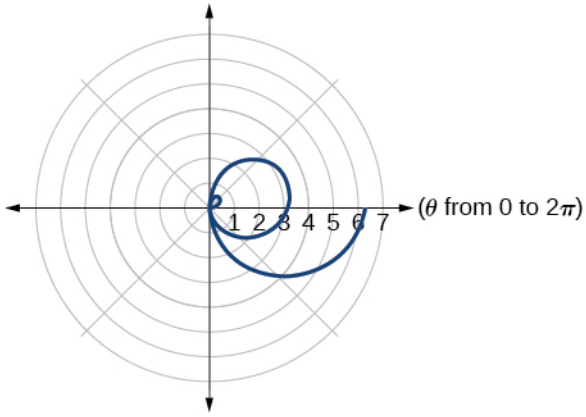



**290**.



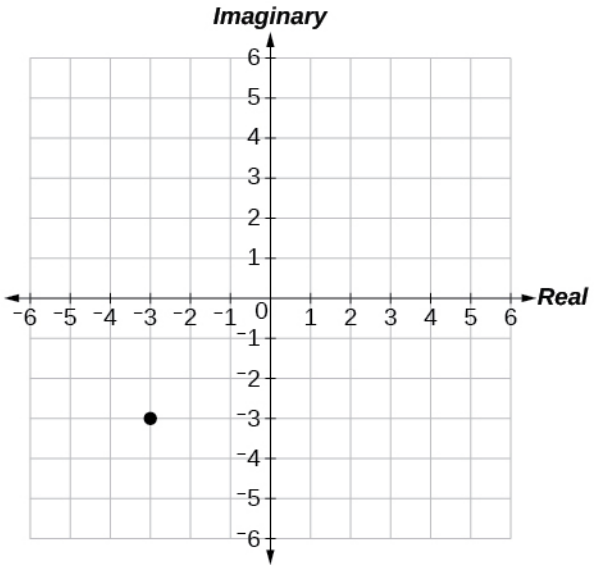



**292**.

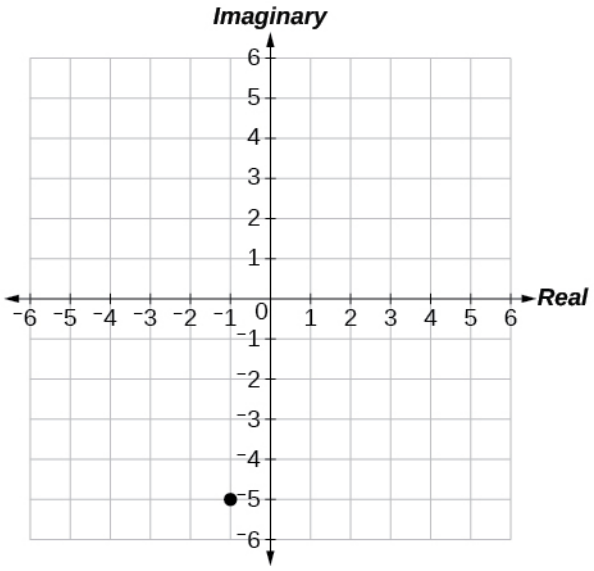



**294**. They are both spirals, but not quite the same.
**296**. Both graphs are curves with 2 loops. The equation with a coefficient of $\theta$ has two loops on the left, the equation with a coefficient of 2 has two loops side by side. Graph these from 0 to $4\pi$ to get a better picture.
**298**. When the width of the domain is increased, more petals of the flower are visible.
**300**. The graphs are three-petal, rose curves. The larger the coefficient, the greater the curve's distance from the pole.
**302**. The graphs are spirals. The smaller the coefficient, the tighter the spiral.
**304**.
$\left(4, \frac{\pi}{3}\right), \left(4, \frac{5\pi}{3}\right)$
**306**.
$\left(\frac{3}{2}, \frac{\pi}{3}\right), \left(\frac{3}{2}, \frac{5\pi}{3}\right)$
**308**.
$\left(0, \frac{\pi}{2}\right), (0, \pi), \left(0, \frac{3\pi}{2}\right), (0, 2\pi)$
**310**.
$\left(\frac{\sqrt[4]{8}}{2}, \frac{\pi}{4}\right), \left(\frac{\sqrt[4]{8}}{2}, \frac{5\pi}{4}\right)$ and at
$\theta = \frac{3\pi}{4}, \frac{7\pi}{4}$ since
$r$ is squared
**312**. $a$ is the real part, $b$ is the imaginary part, and
$i = \sqrt{-1}$
**314**. Polar form converts the real and imaginary part of the complex number in polar form using
$x = r\cos\theta$ and
$y = r\sin\theta$.
**316**.
$z^n = r^n(\cos(n\theta) + i\sin(n\theta))$ It is used to simplify polar form when a number has been raised to a power.

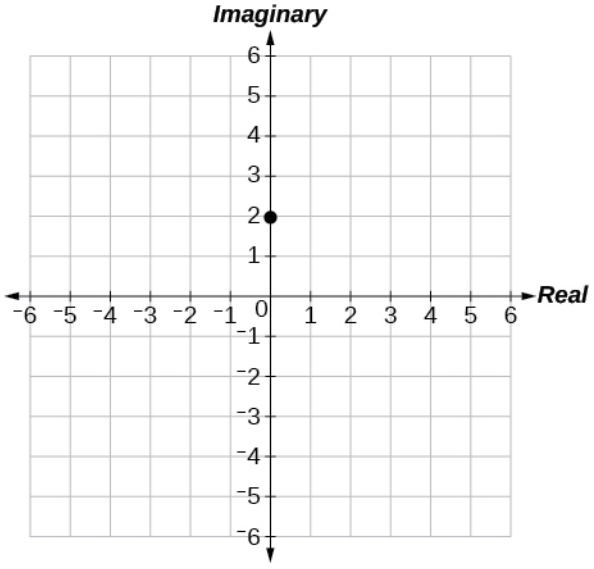
**318**.
$5\sqrt{2}$
**320**.
$\sqrt{38}$
**322**.
$\sqrt{14.45}$
**324**.
$4\sqrt{5}\text{cis}(333.4°)$
**326**.
$2\text{cis}\left(\frac{\pi}{6}\right)$
**328**.
$\frac{7\sqrt{3}}{2} + i\frac{7}{2}$
**330**.
$-2\sqrt{3} - 2i$
**332**.
$-1.5 - i\frac{3\sqrt{3}}{2}$
**334**.
$4\sqrt{3}\text{cis}(198°)$
**336**.
$\frac{3}{4}\text{cis}(180°)$
**338**.
$5\sqrt{3}\text{cis}\left(\frac{17\pi}{24}\right)$
**340**.
$7\text{cis}(70°)$
**342**.
$5\text{cis}(80°)$
**344**.
$5\text{cis}\left(\frac{\pi}{3}\right)$
**346**.
$125\text{cis}(135°)$
**348**.
$9\text{cis}(240°)$
**350**.
$\text{cis}\left(\frac{3\pi}{4}\right)$
**352**.
$3\text{cis}(80°),\ 3\text{cis}(200°),\ 3\text{cis}(320°)$
**354**.
$2\sqrt[3]{4}\text{cis}\left(\frac{2\pi}{9}\right),\ 2\sqrt[3]{4}\text{cis}\left(\frac{8\pi}{9}\right),\ 2\sqrt[3]{4}\text{cis}\left(\frac{14\pi}{9}\right)$
**356**.
$2\sqrt{2}\text{cis}\left(\frac{7\pi}{8}\right),\ 2\sqrt{2}\text{cis}\left(\frac{15\pi}{8}\right)$
**358**.


Imaginary
Real
6
5
4
3
2
1
0
-1
-2
-3
-4
-5
-6
-6 -5 -4 -3 -2 -1 1 2 3 4 5 6

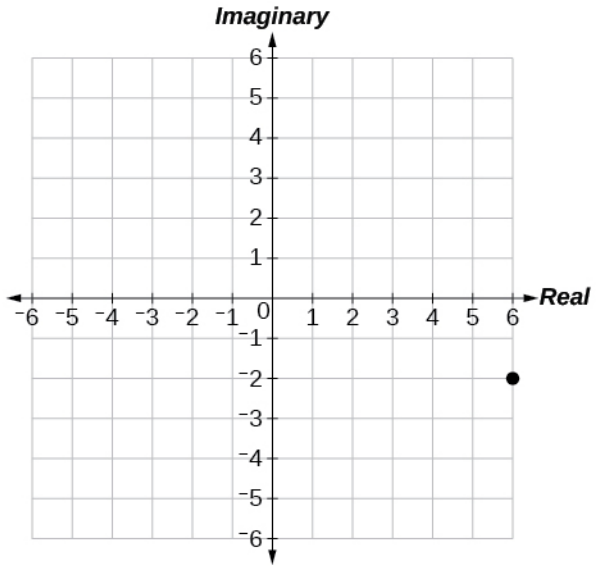
**360**.


Imaginary
Real
6
5
4
3
2
1
0
-1
-2
-3
-4
-5
-6
-6 -5 -4 -3 -2 -1 1 2 3 4 5 6

**362**.


Imaginary
Real
6
5
4
3
2
1
0
-1
-2
-3
-4
-5
-6
-6 -5 -4 -3 -2 -1 1 2 3 4 5 6

**364**.




**366**.

Plot of 1-4i in the complex plane (1 along the real axis, -4 along the imaginary axis).

**368**.

$3.61e^{-0.59i}$

**370**.

$-2+3.46i$

**372**.

$-4.33-2.50i$

**373**. A pair of functions that is dependent on an external factor. The two functions are written in terms of the same parameter. For example,

$x=f(t)$ and

$y=f(t)$.

**375**. Choose one equation to solve for

$t,$ substitute into the other equation and simplify.

**377**. Some equations cannot be written as functions, like a circle. However, when written as two parametric equations, separately the equations are functions.

**379**.

$y=-2+2x$

**381**.

$y=3\sqrt{\frac{x-1}{2}}$

**383**.

$x=2e^{\frac{1-y}{5}}$ or

$y=1-5ln\left(\frac{x}{2}\right)$

**385**.

$x=4\log\left(\frac{y-3}{2}\right)$

**387**.

$x=\left(\frac{y}{2}\right)^3-\frac{y}{2}$

**389**.

$y=x^3$

**391**.

$\left(\frac{x}{4}\right)^2+\left(\frac{y}{5}\right)^2=1$

**393**.
$y^2 = 1 - \frac{1}{2}x$

**395**.
$y = x^2 + 2x + 1$

**397**.
$y = \left(\frac{x+1}{2}\right)^3 - 2$

**399**.
$y = -3x + 14$

**401**.
$y = x + 3$

**403**.
$\begin{cases} x(t) = t \\ y(t) = 2\sin t + 1 \end{cases}$

**405**.
$\begin{cases} x(t) = \sqrt{t} + 2t \\ y(t) = t \end{cases}$

**407**.
$\begin{cases} x(t) = 4\cos t \\ y(t) = 6\sin t \end{cases}$; Ellipse

**409**.
$\begin{cases} x(t) = \sqrt{10}\cos t \\ y(t) = \sqrt{10}\sin t \end{cases}$; Circle

**411**.
$\begin{cases} x(t) = -1 + 4t \\ y(t) = -2t \end{cases}$

**413**.
$\begin{cases} x(t) = 4 + 2t \\ y(t) = 1 - 3t \end{cases}$

**415**. yes, at
$t = 2$

**417**.

| $t$ | $x$ | $y$ |
|---|---|---|
| 1 | -3 | 1 |
| 2 | 0 | 7 |
| 3 | 5 | 17 |

**419**. answers may vary:
$\begin{cases} x(t) = t - 1 \\ y(t) = t^2 \end{cases}$ and $\begin{cases} x(t) = t + 1 \\ y(t) = (t+2)^2 \end{cases}$

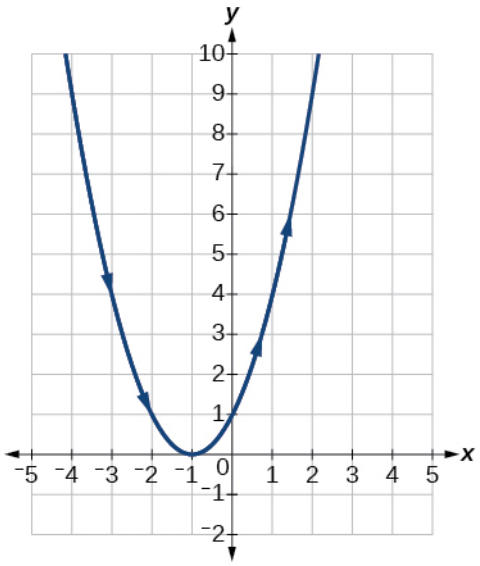
**421**. answers may vary: ,
$\begin{cases} x(t) = t \\ y(t) = t^2 - 4t + 4 \end{cases}$ and $\begin{cases} x(t) = t + 2 \\ y(t) = t^2 \end{cases}$

**422**. plotting points with the orientation arrow and a graphing calculator

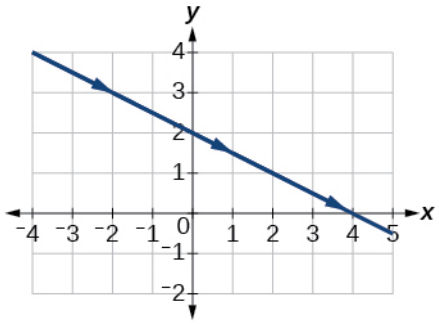
**424**. The arrows show the orientation, the direction of motion according to increasing values of $t$.

**426**. The parametric equations show the different vertical and horizontal motions over time.

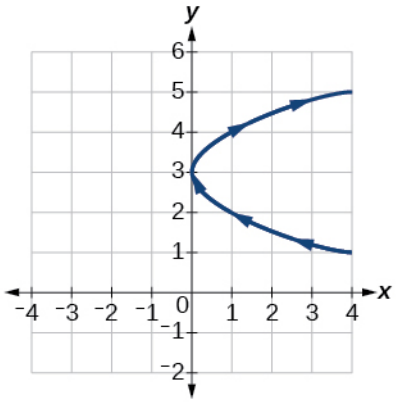
**428**.



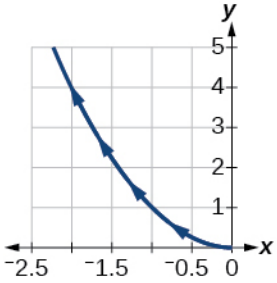

**430**.




**432**.



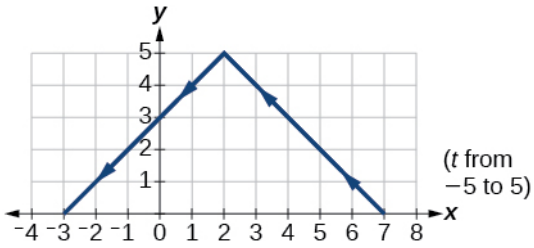

**434**.



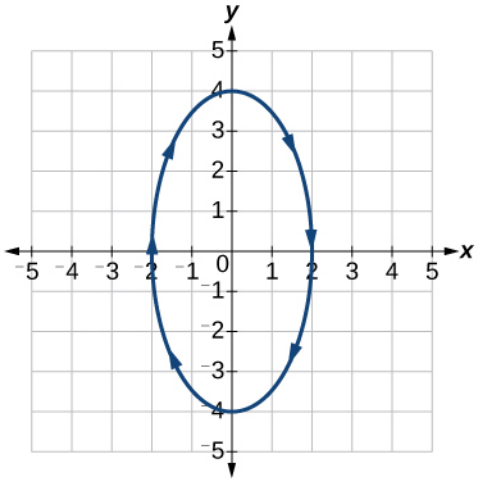

($t$ from $-1$ to 5)

**436**.

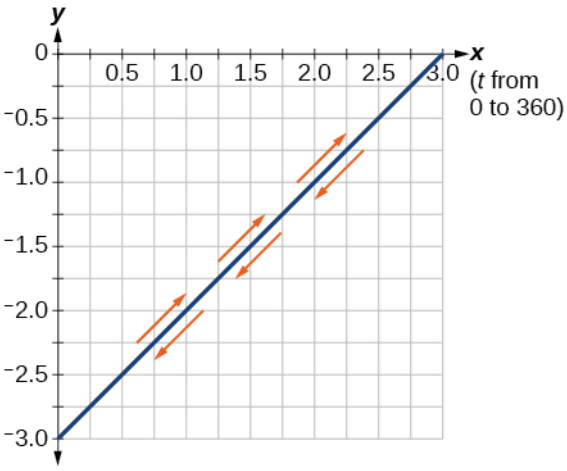
**438**.



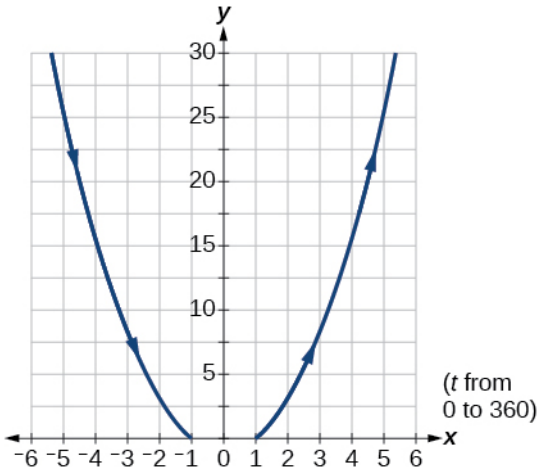

**440**.

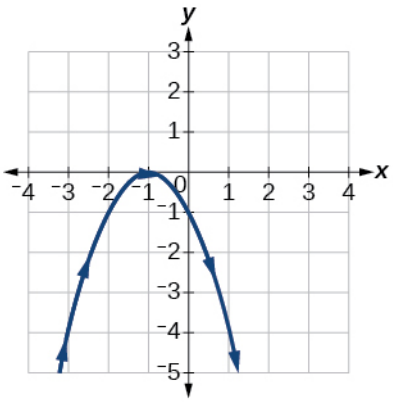


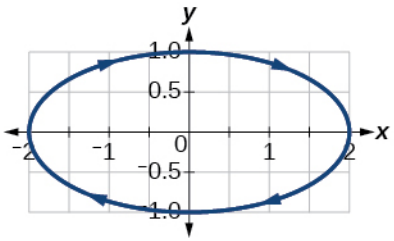

**442**.




**444**.



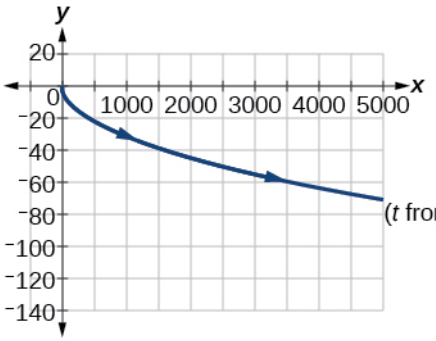



($t$ from 0 to $2\pi$)

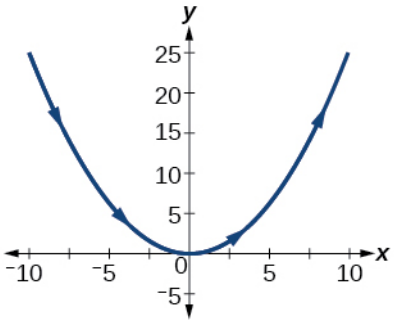
**446**.

**448**.




($t$ from $-1$ to 5)

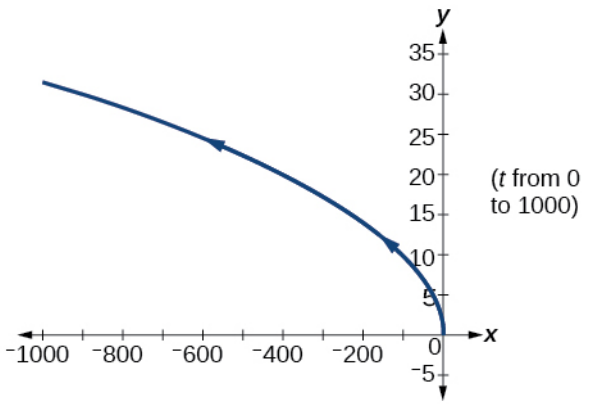
**450**.



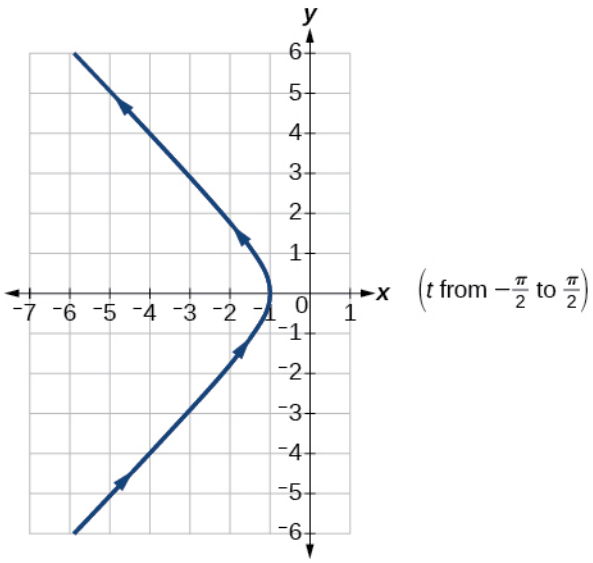

($t$ from $-5$ to 5)

**452**.

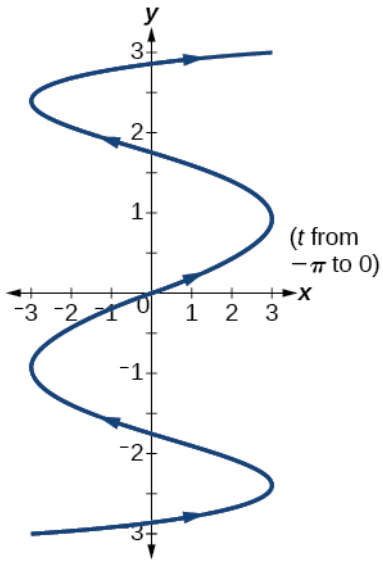
**454**.

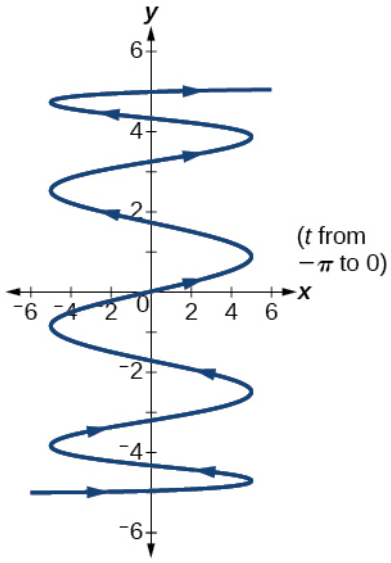

y
35
30
25
20
15
10
5
x
−1000
−800
−600
−400
−200
0
−5
(t from 0 to 1000)

**456**.


y
6
5
4
3
2
1
x
−7
−6
−5
−4
−3
−2
−1
0
1
−1
−2
−3
−4
−5
−6
$\left(t \text{ from } -\frac{\pi}{2} \text{ to } \frac{\pi}{2}\right)$

**458**.


y
3
2
1
x
−3
−2
−1
0
1
2
3
−1
−2
−3
(t from −π to 0)




**460**. There will be 100 back-and-forth motions.

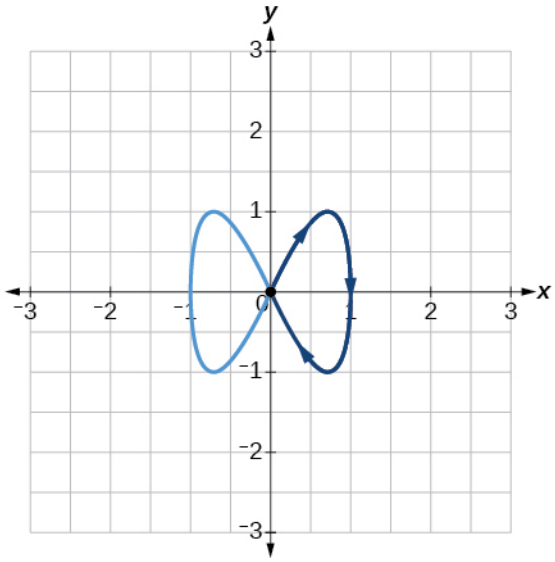
**462**. Take the opposite of the $x(t)$ equation.

**464**. The parabola opens up.

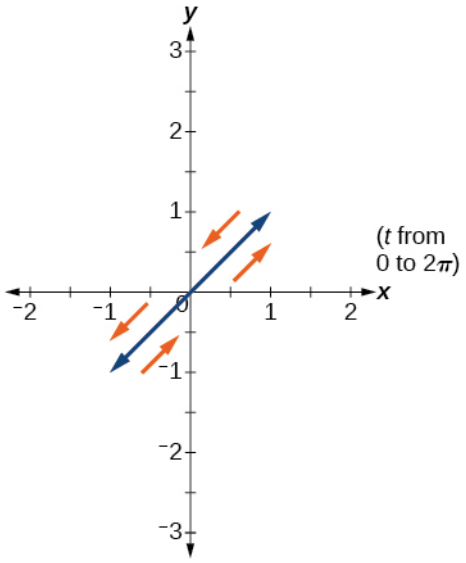
**466**.

$$\begin{cases} x(t) = 5\cos t \\ y(t) = 5\sin t \end{cases}$$

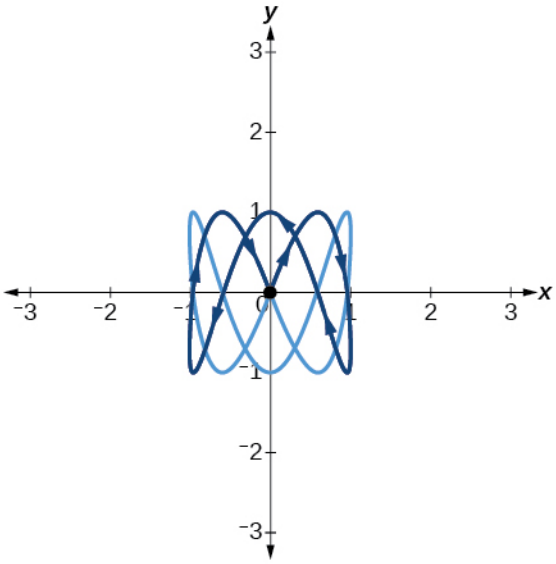
**468**.




**470**.




**472**.

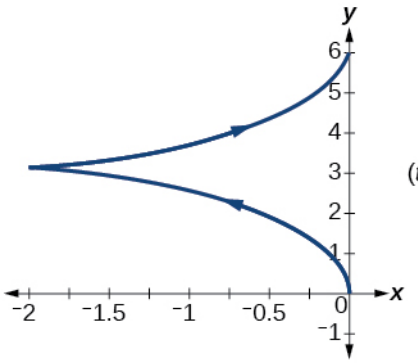



**474**.

$a = 4,\ b = 3,\ c = 6,\ d = 1$

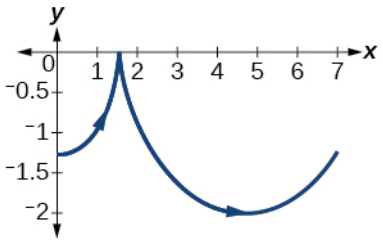
**476**.

$a = 4,\ b = 2,\ c = 3,\ d = 3$

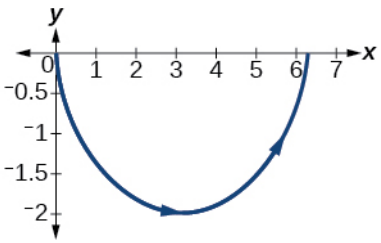



($t$ from 0 to $2\pi$)

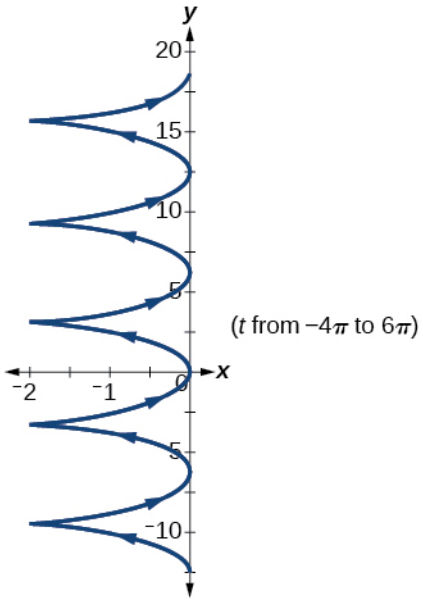
**478**.

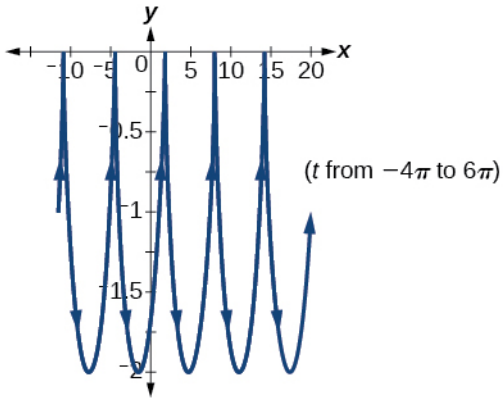



($t$ from 0 to $2\pi$)

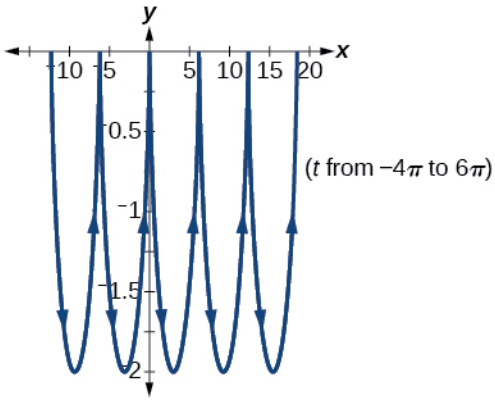



($t$ from 0 to $2\pi$)




**480**.




**482**. The $y$-intercept changes.

**484**.

$$y(x) = -16\left(\frac{x}{15}\right)^2 + 20\left(\frac{x}{15}\right)$$

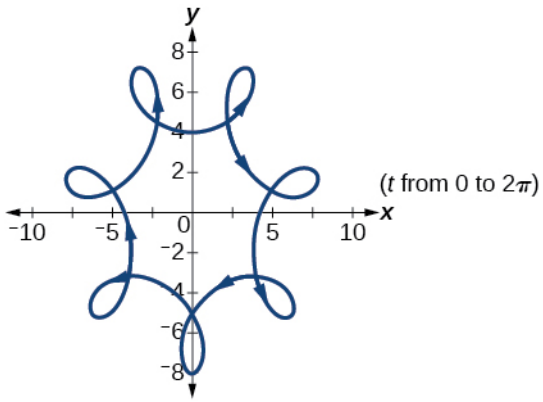
**486**.

$$\begin{cases} x(t) = 64t\cos(52^\circ) \\ y(t) = -16t^2 + 64t\sin(52^\circ) \end{cases}$$

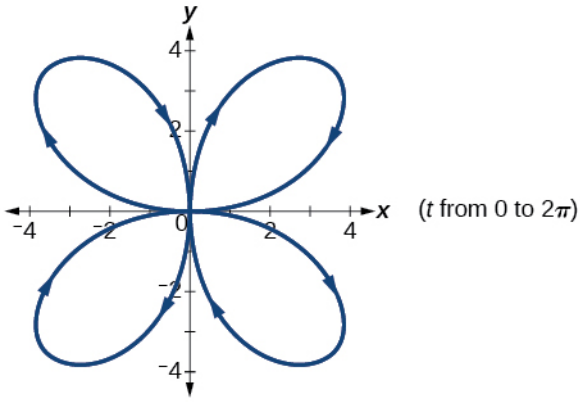
**488**. approximately 3.2 seconds

**490**. 1.6 seconds

**492**.




**494**.




**495**. lowercase, bold letter, usually
$u, v, w$

**497**. They are unit vectors. They are used to represent the horizontal and vertical components of a vector. They each have a magnitude of 1.

**499**. The first number always represents the coefficient of the
$i$, and the second represents the
$j$.

**501**.
$\langle 7, -5 \rangle$

**503**. not equal

**505**. equal

**507**. equal

**509**.
$7i - 3j$

**511**.
$-6i - 2j$

**513**.
$u + v = \langle -5, 5 \rangle$ , $u - v = \langle -1, 3 \rangle$ , $2u - 3v = \langle 0, 5 \rangle$

**515**.
$-10i - 4j$

**517**.
$-\frac{2\sqrt{29}}{29}i + \frac{5\sqrt{29}}{29}j$

**519**.
$-\frac{2\sqrt{229}}{229}i + \frac{15\sqrt{229}}{229}j$

**521**.
$-\frac{7\sqrt{2}}{10}i + \frac{\sqrt{2}}{10}j$

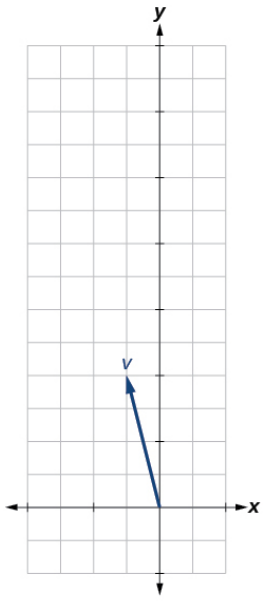
**523**.
$|v| = 7.810,\ \theta = 39.806\ ^\circ$

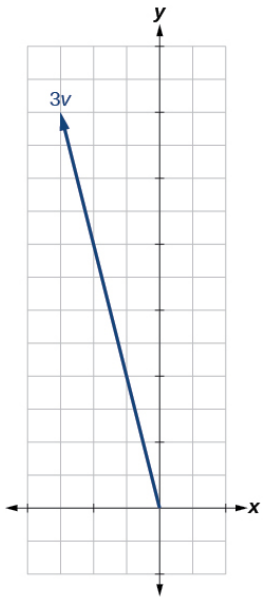
**525**.
$|v| = 7.211,\ \theta = 236.310^\circ$

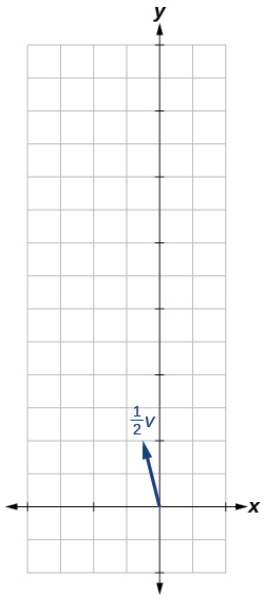
**527**.
$-6$

**529**.
$-12$

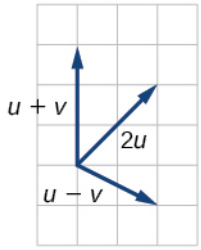
**531**.


y
v
x

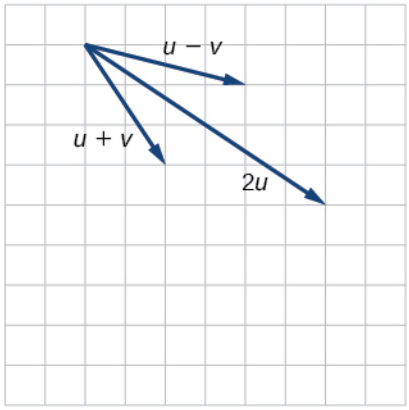

y
3v
x


y
$\frac{1}{2}v$
x

**533**.



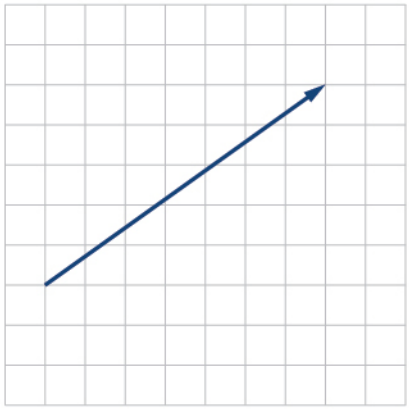

**535**.




**537**.



**539**.

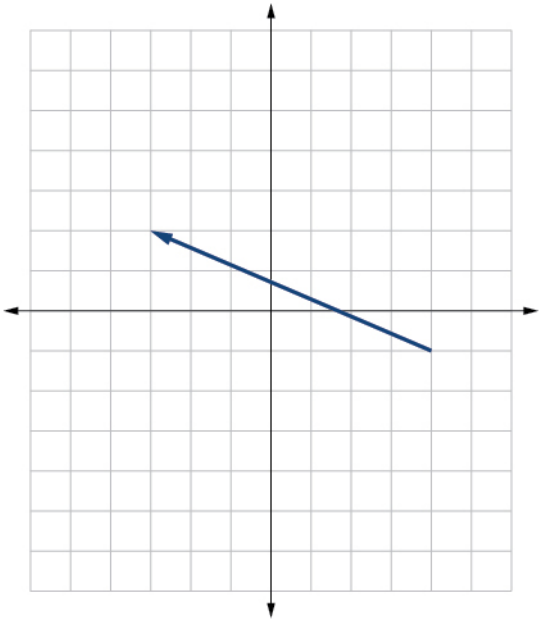


**541**.

? 4, 1 ?

**543**.

$v = -7i + 3j$



**545**.
$3\sqrt{2}i + 3\sqrt{2}j$

**547**.
$i - \sqrt{3}j$

**549**. a. 58.7; b. 12.5

**551**.
$x = 7.13$ pounds,
$y = 3.63$ pounds

**553**.
$x = 2.87$ pounds,
$y = 4.10$ pounds

**555**. 4.635 miles, 17.764° N of E

**557**. 17 miles. 10.318 miles

**559**. Distance: 2.868. Direction: 86.474° North of West, or 3.526° West of North

**561**. 4.924°. 659 km/hr

**563**. 4.424°

**565**.
$(0.081,\ 8.602)$

**567**. 21.801°, relative to the car's forward direction

**569**. parallel: 16.28, perpendicular: 47.28 pounds

**571**. 19.35 pounds, 231.54° from the horizontal

**573**. 5.1583 pounds, 75.8° from the horizontal

## Review Exercises

**574**. Not possible

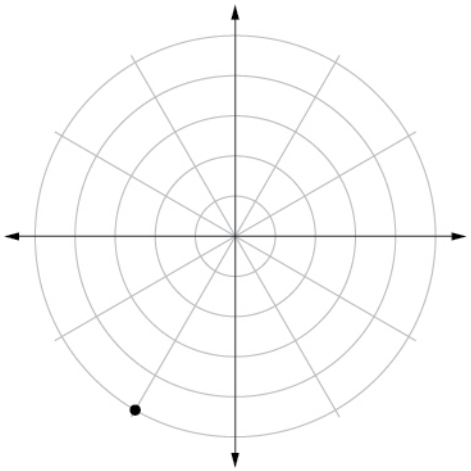
**576**.
$C = 120°,\ a = 23.1,\ c = 34.1$

**578**. distance of the plane from point
$A$ : 2.2 km, elevation of the plane: 1.6 km

**580**.
$B = 71.0°,\ C = 55.0°,\ a = 12.8$

**582**. 40.6 km

**584**.



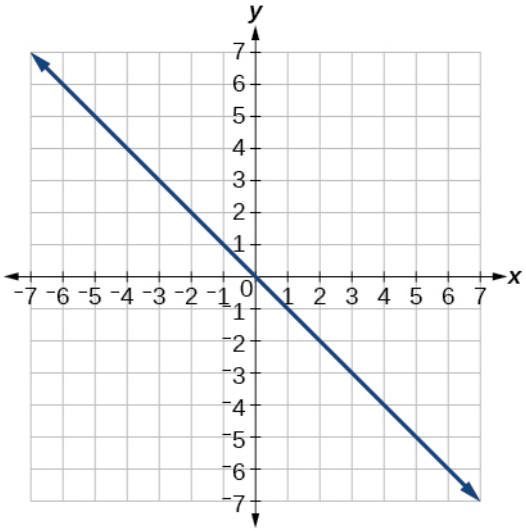
**586**.
(0, 2)

**588**.
(9.8489, 203.96°)

**590**.
$r = 8$

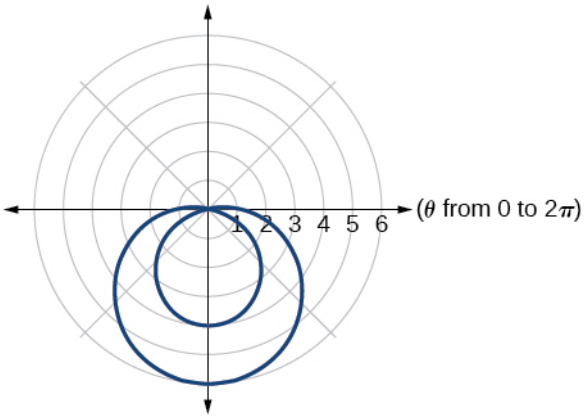
**592**.
$x^2 + y^2 = 7x$

**594**.
$y = -x$

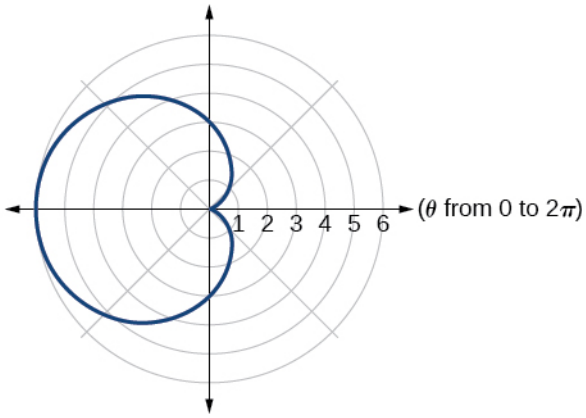



**596**. symmetric with respect to the line
$\theta = \frac{\pi}{2}$

**598**.




**600**.




**602**. 5

**604**.

$\text{cis}\left(-\frac{\pi}{3}\right)$

**606**.

$2.3 + 1.9i$

**608**.

$60\text{cis}\left(\frac{\pi}{2}\right)$

**610**.

$3\text{cis}\left(\frac{4\pi}{3}\right)$

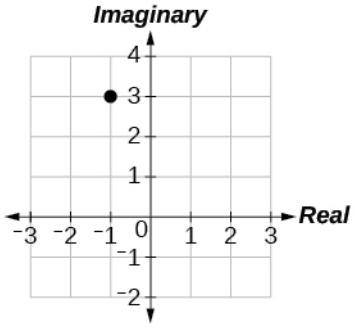
**612**.

$25\text{cis}\left(\frac{3\pi}{2}\right)$

**614**.

$5\text{cis}\left(\frac{3\pi}{4}\right),\ 5\text{cis}\left(\frac{7\pi}{4}\right)$

**616**.




**618**.

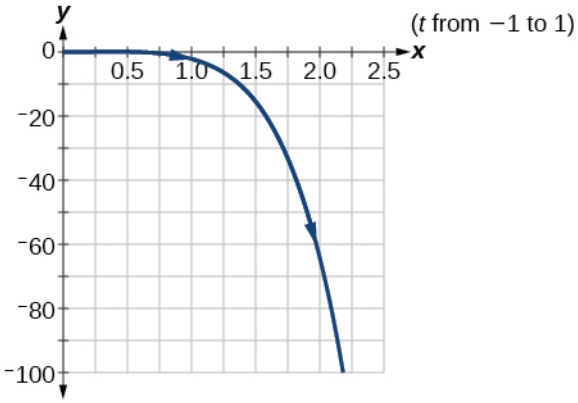
$x^2 + \frac{1}{2}y = 1$

**620**.

$\begin{cases} x(t) = -2 + 6t \\ y(t) = 3 + 4t \end{cases}$

**622**.

$y = -2x^5$




**624**.

a.

$\begin{cases} x(t) = (80\cos(40°))t \\ y(t) = -16t^2 + (80\sin(40°))t + 4 \end{cases}$

b. The ball is 14 feet high and 184 feet from where it was launched.

c. 3.3 seconds

**626**. not equal

**628**. $4\boldsymbol{i}$

**630**.

$-\frac{3\sqrt{10}}{10}\boldsymbol{i}$

$-\frac{\sqrt{10}}{10}\boldsymbol{j}$

**632**. Magnitude:

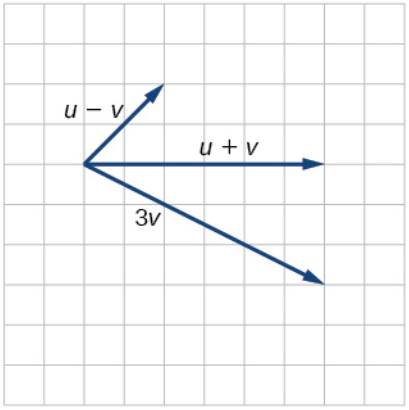
$3\sqrt{2}$, Direction:

$225°$

**634**.

16

**636**.



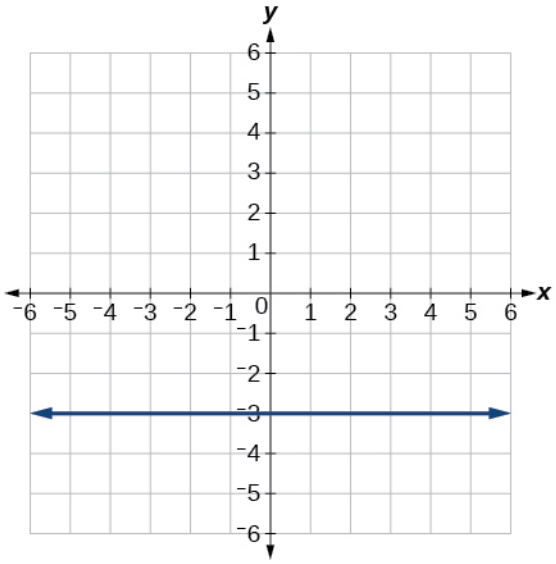

## Practice Test

**638**.
$\alpha = 67.1°, \gamma = 44.9°, a = 20.9$

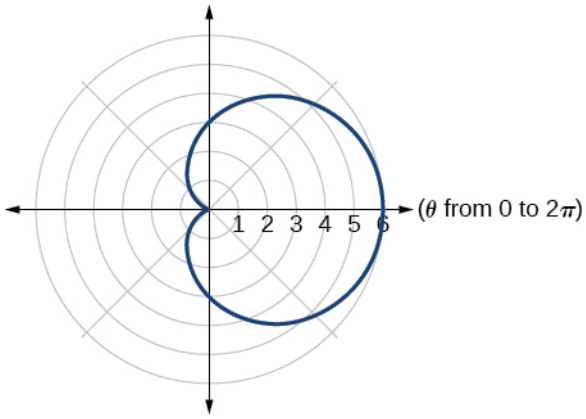
**640**.
1712 miles

**642**.
$(1, \sqrt{3})$

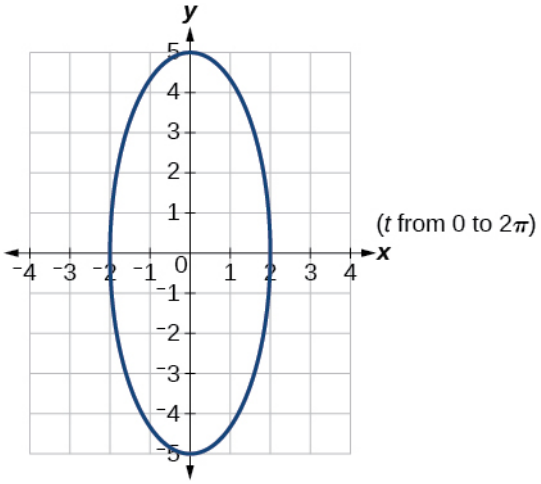
**644**.
$y = -3$



**646**.




**648**.
$\sqrt{106}$
**650**.
$\frac{-5}{2} + i\frac{5\sqrt{3}}{2}$
**652**.
$4\text{cis}(21°)$
**654**.
$2\sqrt{2}\text{cis}(18°),\ 2\sqrt{2}\text{cis}(198°)$
**656**.
$y = 2(x-1)^2$
**658**.

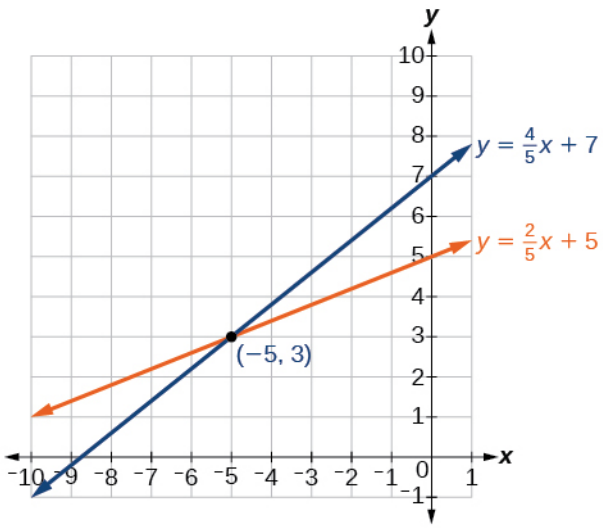



**660**. $-4\boldsymbol{i} - 15\boldsymbol{j}$
**662**.
$\frac{2\sqrt{13}}{13}i + \frac{3\sqrt{13}}{13}j$

## Chapter 11

## Try It

**11.1**. Not a solution.
**11.2**. The solution to the system is the ordered pair
$(-5,\ 3)$.




**11.3**.
$(-2, -5)$

**11.4**.
$(-6, -2)$

**11.5**.
$(10, -4)$

**11.6**. No solution. It is an inconsistent system.
**11.7**. The system is dependent so there are infinite solutions of the form
$(x, 2x + 5)$.

**11.8**. 700 children, 950 adults
**11.9**.
$(1, -1, 1)$

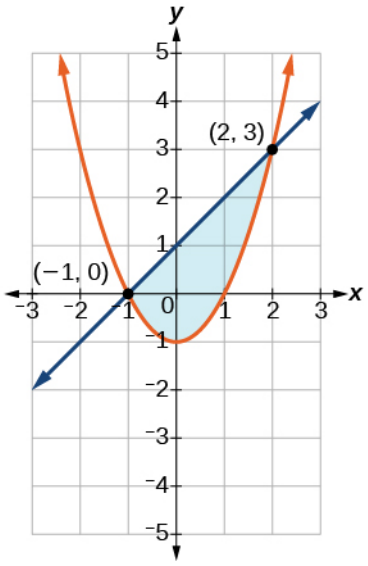
**11.10**. No solution.
**11.11**. Infinite number of solutions of the form
$(x, 4x-11, -5x + 18)$.

**11.12**.
$\left(-\frac{1}{2}, \frac{1}{2}\right)$ and
$(2, 8)$

**11.13**.
$(-1, 3)$

**11.14**.
$\{(1, 3), (1, -3), (-1, 3), (-1, -3)\}$

**11.15**. Shade the area bounded by the two curves, above the quadratic and below the line.




**11.16**.
$\frac{3}{x-3}-\frac{2}{x-2}$

**11.17**.
$\frac{6}{x-1}-\frac{5}{(x-1)^2}$

**11.18**.
$\frac{3}{x-1}+\frac{2x-4}{x^2+1}$

**11.19**.
$\frac{x-2}{x^2-2x+3}+\frac{2x+1}{\left(x^2-2x+3\right)^2}$

**11.20**. (11.192)

$$A+B=\begin{bmatrix}2 & 6\\ 1 & 0\\ 1 & -3\end{bmatrix}+\begin{bmatrix}3 & -2\\ 1 & 5\\ -4 & 3\end{bmatrix}=\begin{bmatrix}2+3 & 6+(-2)\\ 1+1 & 0+5\\ 1+(-4) & -3+3\end{bmatrix}=\begin{bmatrix}5 & 4\\ 2 & 5\\ -3 & 0\end{bmatrix}$$

**11.21**.
$-2B=\begin{bmatrix}-8 & -2\\ -6 & -4\end{bmatrix}$

**11.22**.
$\left[\begin{array}{cc|c}4 & -3 & 11\\ 3 & 2 & 4\end{array}\right]$

**11.23**.
$$\begin{aligned}x-y+z&=5\\ 2x-y+3z&=1\\ y+z&=-9\end{aligned}$$

**11.24**.
$(2,\ 1)$

**11.25**.
$\left[\begin{array}{ccc|c}1 & -\frac{5}{2} & \frac{5}{2} & \frac{17}{2}\\ 0 & 1 & 5 & 9\\ 0 & 0 & 1 & 2\end{array}\right]$

**11.26**.
$(1,\ 1,\ 1)$

**11.27**. $150,000 at 7%, $750,000 at 8%, $600,000 at 10%

**11.28**. (11.308)

$$AB = \begin{bmatrix} 1 & 4 \\ -1 & -3 \end{bmatrix}\begin{bmatrix} -3 & -4 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1(-3)+4(1) & 1(-4)+4(1) \\ -1(-3)+-3(1) & -1(-4)+-3(1) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$BA = \begin{bmatrix} -3 & -4 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 4 \\ -1 & -3 \end{bmatrix} = \begin{bmatrix} -3(1)+-4(-1) & -3(4)+-4(-3) \\ 1(1)+1(-1) & 1(4)+1(-3) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

**11.29**.

$$A^{-1} = \begin{bmatrix} \frac{3}{5} & \frac{1}{5} \\ -\frac{2}{5} & \frac{1}{5} \end{bmatrix}$$

**11.30**.

$$A^{-1} = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 4 & -3 \\ 3 & 6 & -5 \end{bmatrix}$$

**11.31**.

$$X = \begin{bmatrix} 4 \\ 38 \\ 58 \end{bmatrix}$$

**11.32**.

$(3, -7)$

**11.33**.

$-10$

**11.34**.

$\left(-2, \frac{3}{5}, \frac{12}{5}\right)$

## Section Exercises

**1**. No, you can either have zero, one, or infinitely many. Examine graphs.
**3**. This means there is no realistic break-even point. By the time the company produces one unit they are already making profit.
**5**. You can solve by substitution (isolating $x$ or $y$), graphically, or by addition.
**7**. Yes
**9**. Yes
**11**.

$(-1, 2)$

**13**.

$(-3, 1)$

**15**.

$\left(-\frac{3}{5}, 0\right)$

**17**. No solutions exist.
**19**.

$\left(\frac{72}{5}, \frac{132}{5}\right)$

**21**.

$(6, -6)$

**23**.

$\left(-\frac{1}{2}, \frac{1}{10}\right)$

**25**. No solutions exist.
**27**.

$\left(-\frac{1}{5}, \frac{2}{3}\right)$

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**29**.
$\left(x, \frac{x+3}{2}\right)$

**31**.
$(-4, 4)$

**33**.
$\left(\frac{1}{2}, \frac{1}{8}\right)$

**35**.
$\left(\frac{1}{6}, 0\right)$

**37**.
$(x, 2(7x-6))$

**39**.
$\left(-\frac{5}{6}, \frac{4}{3}\right)$

**41**. Consistent with one solution
**43**. Consistent with one solution
**45**. Dependent with infinitely many solutions
**47**.
$(-3.08, 4.91)$

**49**.
$(-1.52, 2.29)$

**51**.
$\left(\frac{A+B}{2}, \frac{A-B}{2}\right)$

**53**.
$\left(\frac{-1}{A-B}, \frac{A}{A-B}\right)$

**55**.
$\left(\frac{CE-BF}{BD-AE}, \frac{AF-CD}{BD-AE}\right)$

**57**. They never turn a profit.
**59**.
$(1, 250, 100, 000)$

**61**. The numbers are 7.5 and 20.5.
**63**. 24,000
**65**. 790 sophomores, 805 freshman
**67**. 56 men, 74 women
**69**. 10 gallons of 10% solution, 15 gallons of 60% solution
**71**. Swan Peak: $750,000, Riverside: $350,000
**73**. $12,500 in the first account, $10,500 in the second account.
**75**. High-tops: 45, Low-tops: 15
**77**. Infinitely many solutions. We need more information.
**78**. No, there can be only one, zero, or infinitely many solutions.
**80**. Not necessarily. There could be zero, one, or infinitely many solutions. For example,
$(0, 0, 0)$ is not a solution to the system below, but that does not mean that it has no solution.

$$\begin{aligned} 2x+3y-6z &= 1 \\ -4x-6y+12z &= -2 \\ x+2y+5z &= 10 \end{aligned}$$

**82**. Every system of equations can be solved graphically, by substitution, and by addition. However, systems of three equations become very complex to solve graphically so other methods are usually preferable.
**84**. No
**86**. Yes
**88**.
$(-1, 4, 2)$

**90**.
$\left(-\frac{85}{107}, \frac{312}{107}, \frac{191}{107}\right)$

**92**.
$\left(1, \frac{1}{2}, 0\right)$

**94**.
$(4, -6, 1)$

**96**.
$\left(x, \frac{1}{27}(65-16x), \frac{x+28}{27}\right)$

**98**.
$\left(-\frac{45}{13}, \frac{17}{13}, -2\right)$

**100**. No solutions exist

**102**.
$(0, 0, 0)$

**104**.
$\left(\frac{4}{7}, -\frac{1}{7}, -\frac{3}{7}\right)$

**106**.
$(7, 20, 16)$

**108**.
$(-6, 2, 1)$

**110**.
$(5, 12, 15)$

**112**.
$(-5, -5, -5)$

**114**.
$(10, 10, 10)$

**116**.
$\left(\frac{1}{2}, \frac{1}{5}, \frac{4}{5}\right)$

**118**.
$\left(\frac{1}{2}, \frac{2}{5}, \frac{4}{5}\right)$

**120**.
$(2, 0, 0)$

**122**.
$(1, 1, 1)$

**124**.
$\left(\frac{128}{557}, \frac{23}{557}, \frac{28}{557}\right)$

**126**.
$(6, -1, 0)$

**128**. 24, 36, 48

**130**. 70 grandparents, 140 parents, 190 children

**132**. Your share was \$19.95, Sarah's share was \$40, and your other roommate's share was \$22.05.

**134**. There are infinitely many solutions; we need more information

**136**. 500 students, 225 children, and 450 adults

**138**. The BMW was \$49,636, the Jeep was \$42,636, and the Toyota was \$47,727.

**140**. \$400,000 in the account that pays 3% interest, \$500,000 in the account that pays 4% interest, and \$100,000 in the account that pays 2% interest.

**142**. The United States consumed 26.3%, Japan 7.1%, and China 6.4% of the world's oil.

**144**. Saudi Arabia imported 16.8%, Canada imported 15.1%, and Mexico 15.0%

**146**. Birds were 19.3%, fish were 18.6%, and mammals were 17.1% of endangered species

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**148**. A nonlinear system could be representative of two circles that overlap and intersect in two locations, hence two solutions. A nonlinear system could be representative of a parabola and a circle, where the vertex of the parabola meets the circle and the branches also intersect the circle, hence three solutions.

**150**. No. There does not need to be a feasible region. Consider a system that is bounded by two parallel lines. One inequality represents the region above the upper line; the other represents the region below the lower line. In this case, no points in the plane are located in both regions; hence there is no feasible region.

**152**. Choose any number between each solution and plug into $C(x)$ and $R(x)$. If $C(x) < R(x)$, then there is profit.

**154**.
$(0, -3), (3, 0)$

**156**.
$\left(-\frac{3\sqrt{2}}{2}, \frac{3\sqrt{2}}{2}\right), \left(\frac{3\sqrt{2}}{2}, -\frac{3\sqrt{2}}{2}\right)$

**158**.
$(-3, 0), (3, 0)$

**160**.
$\left(\frac{1}{4}, -\frac{\sqrt{62}}{8}\right), \left(\frac{1}{4}, \frac{\sqrt{62}}{8}\right)$

**162**.
$\left(-\frac{\sqrt{398}}{4}, \frac{199}{4}\right), \left(\frac{\sqrt{398}}{4}, \frac{199}{4}\right)$

**164**.
$(0, 2), (1, 3)$

**166**.
$\left(-\sqrt{\frac{1}{2}(\sqrt{5}-1)}, \frac{1}{2}(1-\sqrt{5})\right), \left(\sqrt{\frac{1}{2}(\sqrt{5}-1)}, \frac{1}{2}(1-\sqrt{5})\right)$

**168**.
$(5, 0)$

**170**.
$(0, 0)$

**172**.
$(3, 0)$

**174**. No Solutions Exist

**176**. No Solutions Exist

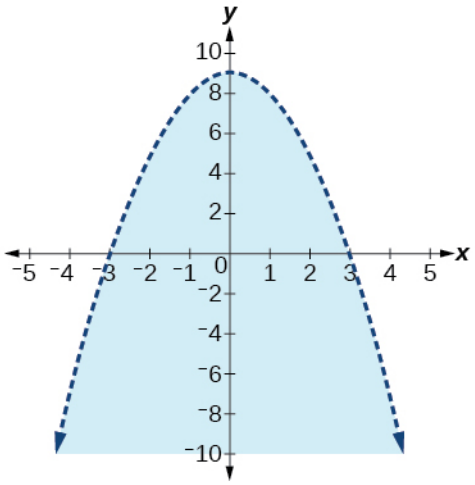
**178**.
$\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right), \left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right), \left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right), \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

**180**.
$(2, 0)$

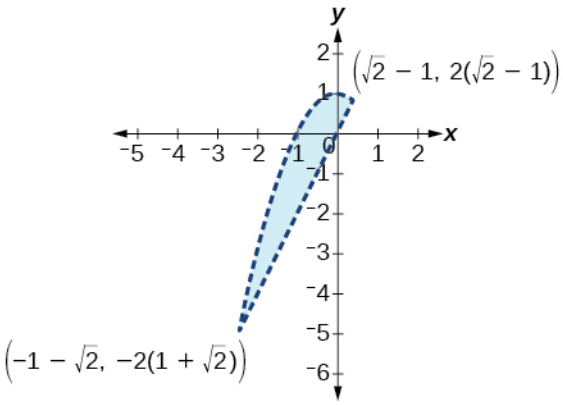
**182**.
$(-\sqrt{7}, -3), (-\sqrt{7}, 3), (\sqrt{7}, -3), (\sqrt{7}, 3)$

**184**.
$\left(-\sqrt{\frac{1}{2}(\sqrt{73}-5)}, \frac{1}{2}(7-\sqrt{73})\right), \left(\sqrt{\frac{1}{2}(\sqrt{73}-5)}, \frac{1}{2}(7-\sqrt{73})\right)$

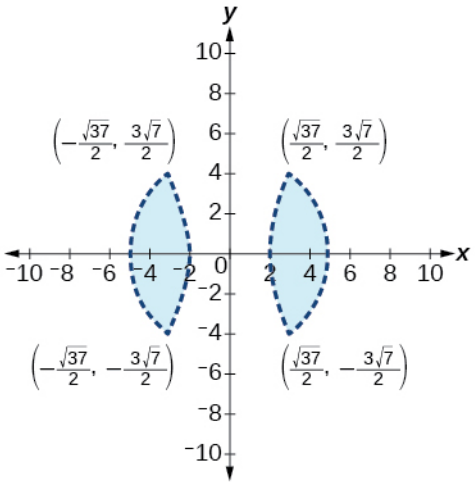
**186**.


y
x
10
8
6
4
2
-2
-4
-6
-8
-10
-5 -4 -3 -2 -1 0 1 2 3 4 5

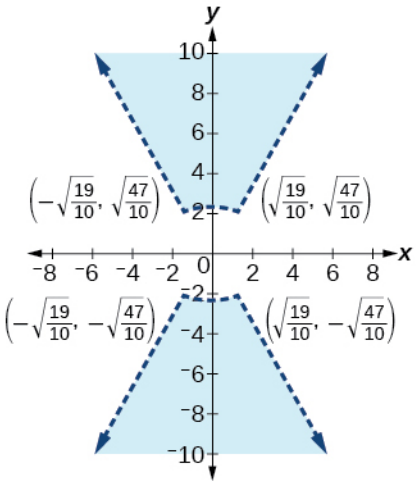
**188**.


y
x
2
1
-1
-2
-3
-4
-5
-6
-5 -4 -3 -2 -1 0 1 2
$\left(\sqrt{2}-1,\ 2(\sqrt{2}-1)\right)$
$\left(-1-\sqrt{2},\ -2(1+\sqrt{2})\right)$

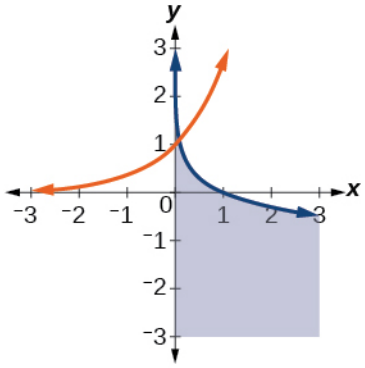
**190**.


y
x
10
8
6
4
2
-2
-4
-6
-8
-10
-10 -8 -6 -4 -2 0 2 4 6 8 10
$\left(-\frac{\sqrt{37}}{2},\ \frac{3\sqrt{7}}{2}\right)$
$\left(\frac{\sqrt{37}}{2},\ \frac{3\sqrt{7}}{2}\right)$
$\left(-\frac{\sqrt{37}}{2},\ -\frac{3\sqrt{7}}{2}\right)$
$\left(\frac{\sqrt{37}}{2},\ -\frac{3\sqrt{7}}{2}\right)$

**192**.




**194**.




**196**.

$\left(-2\sqrt{\frac{70}{383}}, -2\sqrt{\frac{35}{29}}\right), \left(-2\sqrt{\frac{70}{383}}, 2\sqrt{\frac{35}{29}}\right), \left(2\sqrt{\frac{70}{383}}, -2\sqrt{\frac{35}{29}}\right), \left(2\sqrt{\frac{70}{383}}, 2\sqrt{\frac{35}{29}}\right)$

**198**. No Solution Exists

**200**.

$x = 0,\ y > 0$ and

$0 < x < 1,\ \sqrt{x} < y < \frac{1}{x}$

**202**. 12, 288

**204**. 2–20 computers

**206**. No, a quotient of polynomials can only be decomposed if the denominator can be factored. For example, $\frac{1}{x^2 + 1}$ cannot be decomposed because the denominator cannot be factored.

**208**. Graph both sides and ensure they are equal.

**210**. If we choose $x = -1,$ then the *B*-term disappears, letting us immediately know that $A = 3.$ We could alternatively plug in $x = -\frac{5}{3},$ giving us a *B*-value of $-2.$

**212**.

$\frac{8}{x+3} - \frac{5}{x-8}$

**214**.

$\frac{1}{x+5} + \frac{9}{x+2}$

**216**.

$\frac{3}{5x-2} + \frac{4}{4x-1}$

**218**.
$\frac{5}{2(x+3)}+\frac{5}{2(x-3)}$

**220**.
$\frac{3}{x+2}+\frac{3}{x-2}$

**222**.
$\frac{9}{5(x+2)}+\frac{11}{5(x-3)}$

**224**.
$\frac{8}{x-3}-\frac{5}{x-2}$

**226**.
$\frac{1}{x-2}+\frac{2}{(x-2)^2}$

**228**.
$-\frac{6}{4x+5}+\frac{3}{(4x+5)^2}$

**230**.
$-\frac{1}{x-7}-\frac{2}{(x-7)^2}$

**232**.
$\frac{4}{x}-\frac{3}{2(x+1)}+\frac{7}{2(x+1)^2}$

**234**.
$\frac{4}{x}+\frac{2}{x^2}-\frac{3}{3x+2}+\frac{7}{2(3x+2)^2}$

**236**.
$\frac{x+1}{x^2+x+3}+\frac{3}{x+2}$

**238**.
$\frac{4-3x}{x^2+3x+8}+\frac{1}{x-1}$

**240**.
$\frac{2x-1}{x^2+6x+1}+\frac{2}{x+3}$

**242**.
$\frac{1}{x^2+x+1}+\frac{4}{x-1}$

**244**.
$\frac{2}{x^2-3x+9}+\frac{3}{x+3}$

**246**.
$-\frac{1}{4x^2+6x+9}+\frac{1}{2x-3}$

**248**.
$\frac{1}{x}+\frac{1}{x+6}-\frac{4x}{x^2-6x+36}$

**250**.
$\frac{x+6}{x^2+1}+\frac{4x+3}{\left(x^2+1\right)^2}$

**252**.
$\frac{x+1}{x+2}+\frac{2x+3}{(x+2)^2}$

**254**.

$$\frac{1}{x^2+3x+25}-\frac{3x}{\left(x^2+3x+25\right)^2}$$

**256**.

$$\frac{1}{8x}-\frac{x}{8\left(x^2+4\right)}+\frac{10-x}{2\left(x^2+4\right)^2}$$

**258**.

$$-\frac{16}{x}-\frac{9}{x^2}+\frac{16}{x-1}-\frac{7}{(x-1)^2}$$

**260**.

$$\frac{1}{x+1}-\frac{2}{(x+1)^2}+\frac{5}{(x+1)^3}$$

**262**.

$$\frac{5}{x-2}-\frac{3}{10(x+2)}+\frac{7}{x+8}-\frac{7}{10(x-8)}$$

**264**.

$$-\frac{5}{4x}-\frac{5}{2(x+2)}+\frac{11}{2(x+4)}+\frac{5}{4(x+4)}$$

**265**. No, they must have the same dimensions. An example would include two matrices of different dimensions. One cannot add the following two matrices because the first is a $2\times2$ matrix and the second is a $2\times3$ matrix.

$\begin{bmatrix}1&2\\3&4\end{bmatrix}+\begin{bmatrix}6&5&4\\3&2&1\end{bmatrix}$ has no sum.

**267**. Yes, if the dimensions of $A$ are $m\times n$ and the dimensions of $B$ are $n\times m$, both products will be defined.

**269**. Not necessarily. To find $AB$, we multiply the first row of $A$ by the first column of $B$ to get the first entry of $AB$. To find $BA$, we multiply the first row of $B$ by the first column of $A$ to get the first entry of $BA$. Thus, if those are unequal, then the matrix multiplication does not commute.

**271**.

$$\begin{bmatrix}11&19\\15&94\\17&67\end{bmatrix}$$

**273**.

$$\begin{bmatrix}-4&2\\8&1\end{bmatrix}$$

**275**. Undidentified; dimensions do not match

**277**.

$$\begin{bmatrix}9&27\\63&36\\0&192\end{bmatrix}$$

**279**.

$$\begin{bmatrix}-64&-12&-28&-72\\-360&-20&-12&-116\end{bmatrix}$$

**281**.
$$\begin{bmatrix} 1,800 & 1,200 & 1,300 \\ 800 & 1,400 & 600 \\ 700 & 400 & 2,100 \end{bmatrix}$$
**283**.
$$\begin{bmatrix} 20 & 102 \\ 28 & 28 \end{bmatrix}$$
**285**.
$$\begin{bmatrix} 60 & 41 & 2 \\ -16 & 120 & -216 \end{bmatrix}$$
**287**.
$$\begin{bmatrix} -68 & 24 & 136 \\ -54 & -12 & 64 \\ -57 & 30 & 128 \end{bmatrix}$$
**289**. Undefined; dimensions do not match.
**291**.
$$\begin{bmatrix} -8 & 41 & -3 \\ 40 & -15 & -14 \\ 4 & 27 & 42 \end{bmatrix}$$
**293**.
$$\begin{bmatrix} -840 & 650 & -530 \\ 330 & 360 & 250 \\ -10 & 900 & 110 \end{bmatrix}$$
**295**.
$$\begin{bmatrix} -350 & 1,050 \\ 350 & 350 \end{bmatrix}$$
**297**. Undefined; inner dimensions do not match.
**299**.
$$\begin{bmatrix} 1,400 & 700 \\ -1,400 & 700 \end{bmatrix}$$
**301**.
$$\begin{bmatrix} 332,500 & 927,500 \\ -227,500 & 87,500 \end{bmatrix}$$
**303**.
$$\begin{bmatrix} 490,000 & 0 \\ 0 & 490,000 \end{bmatrix}$$
**305**.
$$\begin{bmatrix} -2 & 3 & 4 \\ -7 & 9 & -7 \end{bmatrix}$$
**307**.
$$\begin{bmatrix} -4 & 29 & 21 \\ -27 & -3 & 1 \end{bmatrix}$$
**309**.
$$\begin{bmatrix} -3 & -2 & -2 \\ -28 & 59 & 46 \\ -4 & 16 & 7 \end{bmatrix}$$
**311**.
$$\begin{bmatrix} 1 & -18 & -9 \\ -198 & 505 & 369 \\ -72 & 126 & 91 \end{bmatrix}$$
**313**.
$$\begin{bmatrix} 0 & 1.6 \\ 9 & -1 \end{bmatrix}$$

**315**.

$$\begin{bmatrix} 2 & 24 & -4.5 \\ 12 & 32 & -9 \\ -8 & 64 & 61 \end{bmatrix}$$

**317**.

$$\begin{bmatrix} 0.5 & 3 & 0.5 \\ 2 & 1 & 2 \\ 10 & 7 & 10 \end{bmatrix}$$

**319**.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

**321**.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

**323**.

$$B^n = \begin{cases} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, & n \text{ even}, \\ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}, & n \text{ odd}. \end{cases}$$

**324**. Yes. For each row, the coefficients of the variables are written across the corresponding row, and a vertical bar is placed; then the constants are placed to the right of the vertical bar.

**326**. No, there are numerous correct methods of using row operations on a matrix. Two possible ways are the following: (1) Interchange rows 1 and 2. Then

$R_2 = R_2 - 9R_1$. (2)

$R_2 = R_1 - 9R_2$. Then divide row 1 by 9.

**328**. No. A matrix with 0 entries for an entire row would have either zero or infinitely many solutions.

**330**.

$$\left[\begin{array}{ccc} 0 & 16 & 4 \\ 9 & -1 & 2 \end{array}\right]$$

**332**.

$$\left[\begin{array}{cccc} 1 & 5 & 8 & 16 \\ 12 & 3 & 0 & 4 \\ 3 & 4 & 9 & -7 \end{array}\right]$$

**334**.

$$\begin{aligned} -2x + 5y &= 5 \\ 6x - 18y &= 26 \end{aligned}$$

**336**.
$3x + 2y = 13$
$-x - 9y + 4z = 53$
$8x + 5y + 7z = 80$

**338**.
$4x + 5y - 2z = 12$
$y + 58z = 2$
$8x + 7y - 3z = -5$

**340**. No solutions

**342**.
$(-1, -2)$

**344**.
$(6, 7)$

**346**.
$(3, 2)$

**348**.
$\left(\frac{1}{5}, \frac{1}{2}\right)$

**350**.
$\left(x, \frac{4}{15}(5x + 1)\right)$

**352**.
$(3, 4)$

**354**.
$\left(\frac{196}{39}, -\frac{5}{13}\right)$

**356**.
$(31, -42, 87)$

**358**.
$\left(\frac{21}{40}, \frac{1}{20}, \frac{9}{8}\right)$

**360**.
$\left(\frac{18}{13}, \frac{15}{13}, -\frac{15}{13}\right)$

**362**.
$\left(x, y, \frac{1}{2}(1 - 2x - 3y)\right)$

**364**.
$\left(x, -\frac{x}{2}, -1\right)$

**366**.
$(125, -25, 0)$

**368**.
$(8, 1, -2)$

**370**.
$(1, 2, 3)$

**372**.
$\left(x, \frac{31}{28} - \frac{3x}{4}, \frac{1}{28}(-7x - 3)\right)$

**374**. No solutions exist.

**376**. 860 red velvet, 1,340 chocolate

**378**. 4% for account 1, 6% for account 2

**380**. $126

**382**. Banana was 3%, pumpkin was 7%, and rocky road was 2%

**384**. 100 almonds, 200 cashews, 600 pistachios

**385**. If $A^{-1}$ is the inverse of $A$, then $AA^{-1} = I$, the identity matrix. Since $A$ is also the inverse of $A^{-1}$, $A^{-1}A = I$. You can also check by proving this for a $2\times2$ matrix.

**387**. No, because $ad$ and $bc$ are both 0, so $ad - bc = 0$, which requires us to divide by 0 in the formula.

**389**. Yes. Consider the matrix $\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$. The inverse is found with the following calculation:

$$A^{-1} = \frac{1}{0(0)-1(1)}\begin{bmatrix}0 & -1\\ -1 & 0\end{bmatrix} = \begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}.$$

**391**.

$$AB = BA = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} = I$$

**393**.

$$AB = BA = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} = I$$

**395**.

$$AB = BA = \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix} = I$$

**397**.

$$\frac{1}{29}\begin{bmatrix}9 & 2\\ -1 & 3\end{bmatrix}$$

**399**.

$$\frac{1}{69}\begin{bmatrix}-2 & 7\\ 9 & 3\end{bmatrix}$$

**401**. There is no inverse

**403**.

$$\frac{4}{7}\begin{bmatrix}0.5 & 1.5\\ 1 & -0.5\end{bmatrix}$$

**405**.

$$\frac{1}{17}\begin{bmatrix}-5 & 5 & -3\\ 20 & -3 & 12\\ 1 & -1 & 4\end{bmatrix}$$

**407**.

$$\frac{1}{209}\begin{bmatrix}47 & -57 & 69\\ 10 & 19 & -12\\ -24 & 38 & -13\end{bmatrix}$$

**409**.

$$\begin{bmatrix}18 & 60 & -168\\ -56 & -140 & 448\\ 40 & 80 & -280\end{bmatrix}$$

**411**.

$(-5, 6)$

**413**.

$(2, 0)$

**415**.
$\left(\frac{1}{3}, -\frac{5}{2}\right)$
**417**.
$\left(-\frac{2}{3}, -\frac{11}{6}\right)$
**419**.
$\left(7, \frac{1}{2}, \frac{1}{5}\right)$
**421**.
$(5, 0, -1)$
**423**.
$\frac{1}{34}(-35, -97, -154)$
**425**.
$\frac{1}{690}(65, -1136, -229)$
**427**.
$\left(-\frac{37}{30}, \frac{8}{15}\right)$
**429**.
$\left(\frac{10}{123}, -1, \frac{2}{5}\right)$
**431**.
$\frac{1}{2}\begin{bmatrix} 2 & 1 & -1 & -1 \\ 0 & 1 & 1 & -1 \\ 0 & -1 & 1 & 1 \\ 0 & 1 & -1 & 1 \end{bmatrix}$
**433**.
$\frac{1}{39}\begin{bmatrix} 3 & 2 & 1 & -7 \\ 18 & -53 & 32 & 10 \\ 24 & -36 & 21 & 9 \\ -9 & 46 & -16 & -5 \end{bmatrix}$
**435**.
$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ -1 & -1 & -1 & -1 & -1 & 1 \end{bmatrix}$
**437**. Infinite solutions.
**439**. 50% oranges, 25% bananas, 20% apples
**441**. 10 straw hats, 50 beanies, 40 cowboy hats
**443**. Tom ate 6, Joe ate 3, and Albert ate 3.
**445**. 124 oranges, 10 lemons, 8 pomegranates
**446**. A determinant is the sum and products of the entries in the matrix, so you can always evaluate that product—even if it does end up being 0.
**448**. The inverse does not exist.
**450**.
$-2$
**452**.
$7$
**454**.
$-4$
**456**.
$0$

**458**.
$-7,\ 990.7$

**460**.
3

**462**.
$-1$

**464**.
224

**466**.
15

**468**.
$-17.03$

**470**.
$(1,\ 1)$

**472**.
$\left(\frac{1}{2},\ \frac{1}{3}\right)$

**474**.
$(2,\ 5)$

**476**.
$\left(-1,\ -\frac{1}{3}\right)$

**478**.
$(15,\ 12)$

**480**.
$(1,\ 3,\ 2)$

**482**.
$(-1,\ 0,\ 3)$

**484**.
$\left(\frac{1}{2},\ 1,\ 2\right)$

**486**.
$(2,\ 1,\ 4)$

**488**. Infinite solutions

**490**.
24

**492**.
1

**494**. Yes; 18, 38

**496**. Yes; 33, 36, 37

**498**. $7,000 in first account, $3,000 in second account.

**500**. 120 children, 1,080 adult

**502**. 4 gal yellow, 6 gal blue

**504**. 13 green tomatoes, 17 red tomatoes

**506**. Strawberries 18%, oranges 9%, kiwi 10%

**508**. 100 for movie 1, 230 for movie 2, 312 for movie 3

**510**. 20–29: 2,100, 30–39: 2,600, 40–49: 825

**512**. 300 almonds, 400 cranberries, 300 cashews

## Review Exercises

**514**. No

**516**.
$(-2,\ 3)$

**518**.
$(4,\ -1)$

**520**. No solutions exist.

**522**.
$(300,\ 60,\ 000)$

**523**.
$(400,\ 30,\ 000)$

**524**.
$(10,\ -10,\ 10)$

**526**. No solutions exist.

**528**.
$(-1,\ -2,\ 3)$

**530**.
$\left(x,\ \frac{8x}{5},\ \frac{14x}{5}\right)$

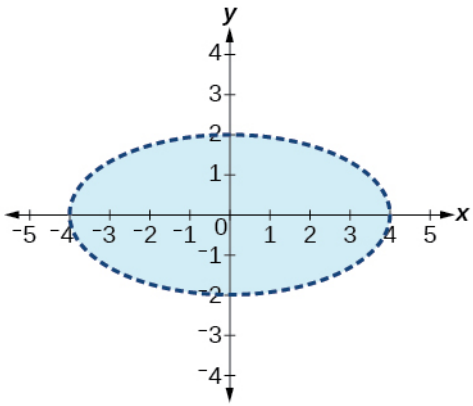
**532**. 11, 17, 33

**534**.
$(2,\ -3),\ (3,\ 2)$

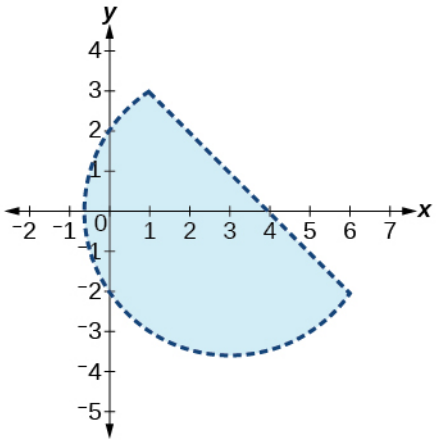
**536**. No solution

**538**. No solution

**540**.




**542**.




**544**.
$\frac{2}{x+2},\ \frac{-4}{x+1}$

**546**.
$\frac{7}{x+5},\ \frac{-15}{(x+5)^2}$

**548**.
$\frac{3}{x-5},\ \frac{-4x+1}{x^2+5x+25}$

**550**.
$$\frac{x-4}{(x^2-2)}, \frac{5x+3}{(x^2-2)^2}$$

**552**.
$$\begin{bmatrix} -16 & 8 \\ -4 & -12 \end{bmatrix}$$

**554**. undefined; dimensions do not match
**556**. undefined; inner dimensions do not match
**558**.
$$\begin{bmatrix} 113 & 28 & 10 \\ 44 & 81 & -41 \\ 84 & 98 & -42 \end{bmatrix}$$

**560**.
$$\begin{bmatrix} -127 & -74 & 176 \\ -2 & 11 & 40 \\ 28 & 77 & 38 \end{bmatrix}$$

**562**. undefined; inner dimensions do not match
**564**.
$$x - 3z = 7$$
$$y + 2z = -5$$ with infinite solutions

**566**.
$$\left[\begin{array}{ccc|c} -2 & 2 & 1 & 7 \\ 2 & -8 & 5 & 0 \\ 19 & -10 & 22 & 3 \end{array}\right]$$

**568**.
$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & 12 \\ -1 & 4 & 0 & 0 \\ 0 & 1 & 2 & -7 \end{array}\right]$$

**570**. No solutions exist.
**572**. No solutions exist.
**574**.
$$\frac{1}{8}\begin{bmatrix} 2 & 7 \\ 6 & 1 \end{bmatrix}$$

**576**. No inverse exists.
**578**.
$$(-20,\ 40)$$

**580**.
$$(-1,\ 0.2,\ 0.3)$$

**582**. 17% oranges, 34% bananas, 39% apples
**584**. 0
**586**. 6
**588**.
$$\left(6,\ \frac{1}{2}\right)$$

**590**. ($x$, $5x + 3$)
**592**.
$$\left(0,\ 0,\ -\frac{1}{2}\right)$$

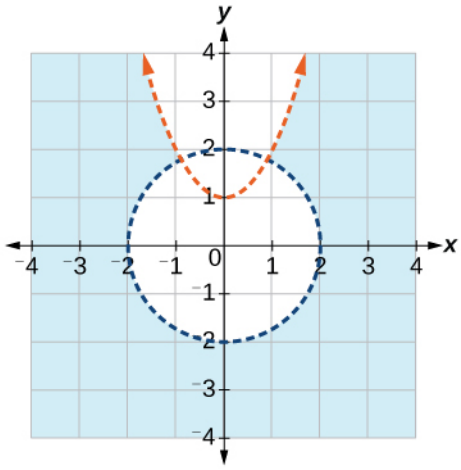
## Practice Test

**594**. Yes
**596**. No solutions exist.
**598**.
$$\frac{1}{20}(10,\ 5,\ 4)$$

**600**.
$\left(x, \frac{16x}{5} - \frac{13x}{5}\right)$

**602**.
$(-2\sqrt{2}, -\sqrt{17}), (-2\sqrt{2}, \sqrt{17}), (2\sqrt{2}, -\sqrt{17}), (2\sqrt{2}, \sqrt{17})$

**604**.




**606**.
$\frac{5}{3x+1} - \frac{2x+3}{(3x+1)^2}$

**608**.
$\begin{bmatrix} 17 & 51 \\ -8 & 11 \end{bmatrix}$

**610**.
$\begin{bmatrix} 12 & -20 \\ -15 & 30 \end{bmatrix}$

**612**.
$-\frac{1}{8}$

**614**.
$\left[\begin{array}{ccc|c} 14 & -2 & 13 & 140 \\ -2 & 3 & -6 & -1 \\ 1 & -5 & 12 & 11 \end{array}\right]$

**616**. No solutions exist.

**618**.
$(100, 90)$

**620**.
$\left(\frac{1}{100}, 0\right)$

**622**. 32 or more cell phones per day

## Chapter 12

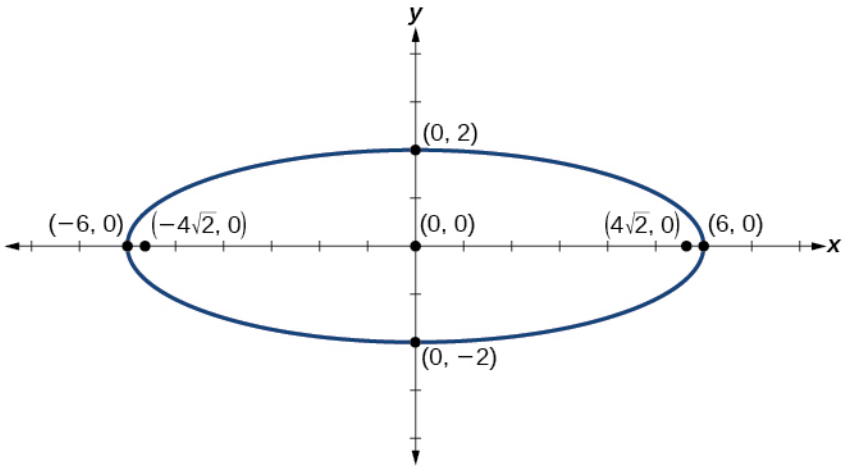
## Try It

**12.1**.
$x^2 + \frac{y^2}{16} = 1$

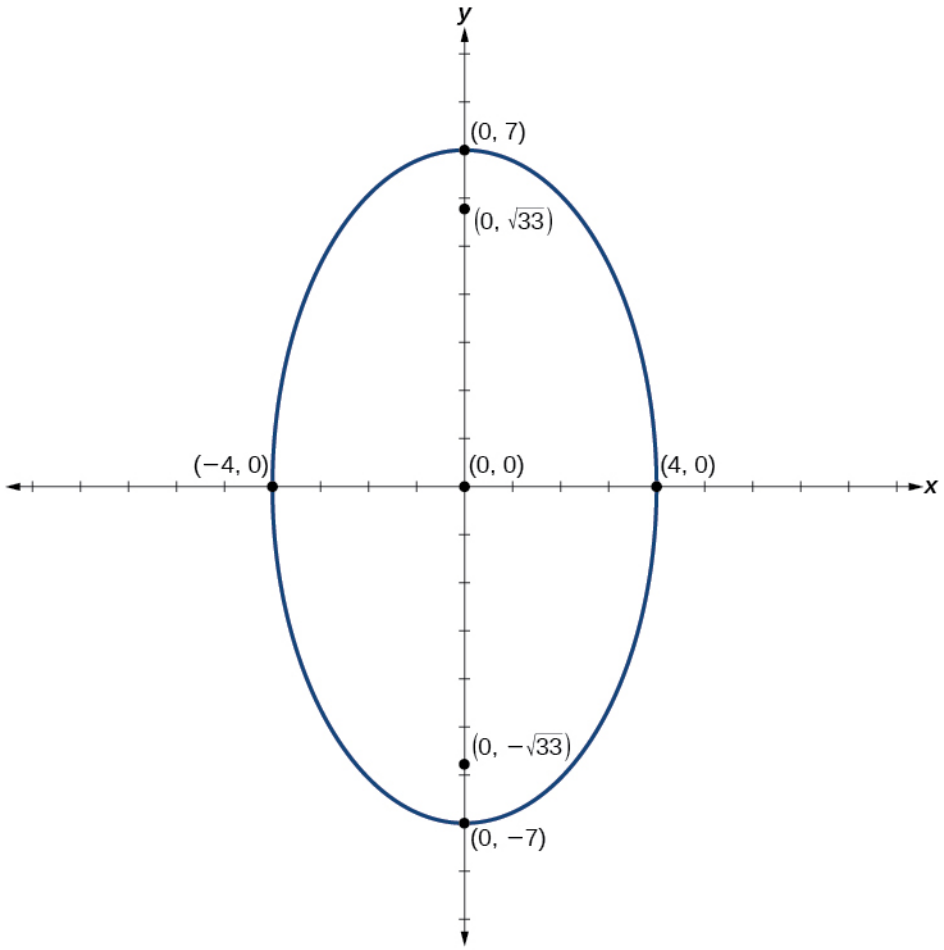
**12.2**.
$\frac{(x-1)^2}{16} + \frac{(y-3)^2}{4} = 1$

**12.3**. center:
$(0, 0)$; vertices:

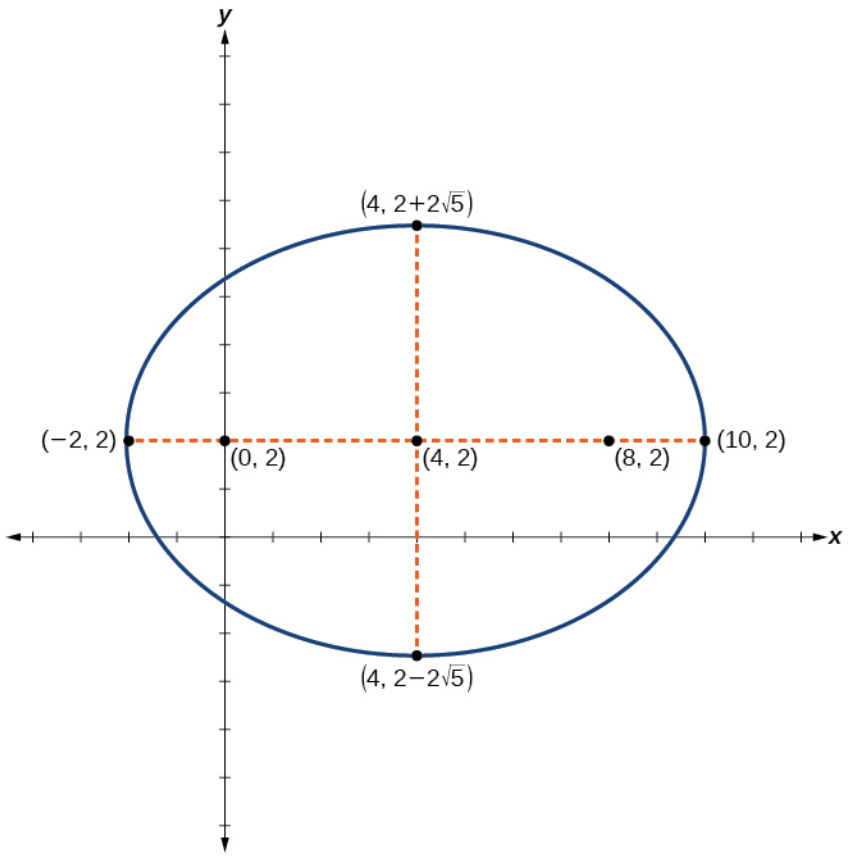
$(\pm 6,\ 0)$; co-vertices:
$(0,\ \pm 2)$; foci:
$\left(\pm 4\sqrt{2},\ 0\right)$




**12.4**. Standard form:
$\frac{x^2}{16} + \frac{y^2}{49} = 1$; center:
$(0,\ 0)$; vertices:
$(0,\ \pm 7)$; co-vertices:
$(\pm 4,\ 0)$; foci:
$\left(0,\ \pm\sqrt{33}\right)$




**12.5**. Center:
$(4,\ 2)$; vertices:
$(-2,\ 2)$ and
$(10,\ 2)$; co-vertices:
$\left(4,\ 2-2\sqrt{5}\right)$ and
$\left(4,\ 2+2\sqrt{5}\right)$; foci:
$(0,\ 2)$ and
$(8,\ 2)$




**12.6**.

$\frac{(x-3)^2}{4}+\frac{(y+1)^2}{16}=1$; center: $(3,\ -1)$; vertices: $(3,\ -5)$ and $(3,\ 3)$; co-vertices: $(1,\ -1)$ and $(5,\ -1)$; foci: $\left(3,\ -1-2\sqrt{3}\right)$ and $\left(3,\ -1+2\sqrt{3}\right)$

**12.7**.

a.

$$\frac{x^2}{57,\ 600}+\frac{y^2}{25,\ 600}=1$$

b. The people are standing 358 feet apart.

**12.8**. Vertices: $(\pm 3,\ 0)$; Foci: $\left(\pm\sqrt{34},\ 0\right)$

**12.9**.

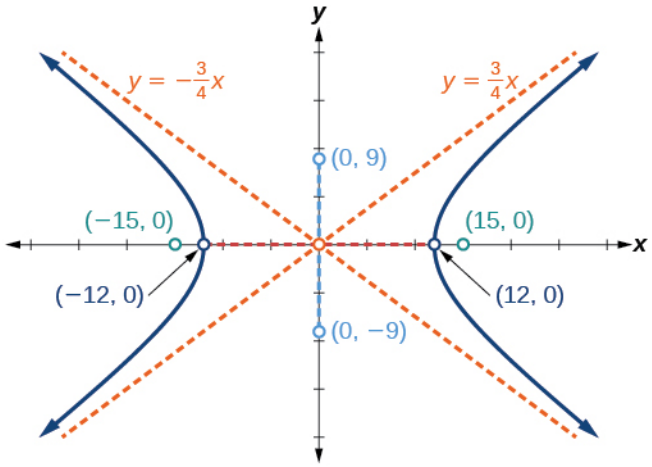
$$\frac{y^2}{4}-\frac{x^2}{16}=1$$

**12.10**.

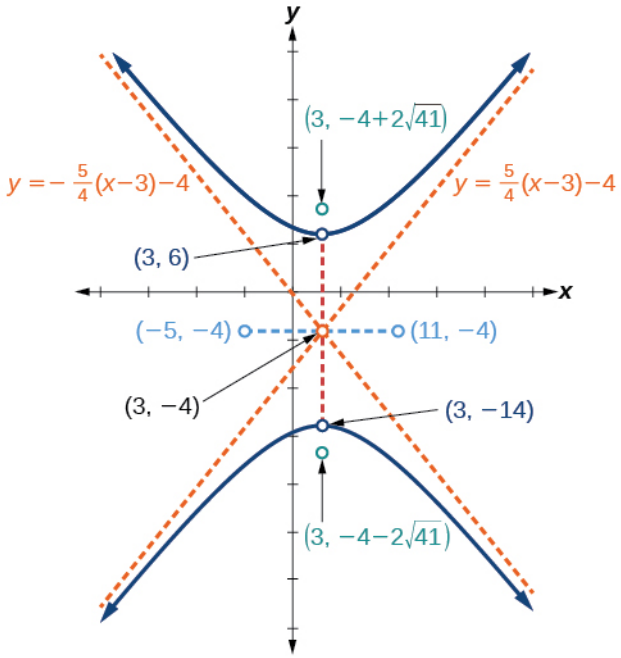
$$\frac{(y-3)^2}{25}+\frac{(x-1)^2}{144}=1$$

**12.11**. vertices: $(\pm 12,\ 0)$; co-vertices:

$(0, \pm 9)$; foci:
$(\pm 15, 0)$; asymptotes:
$y = \pm \frac{3}{4}x$;

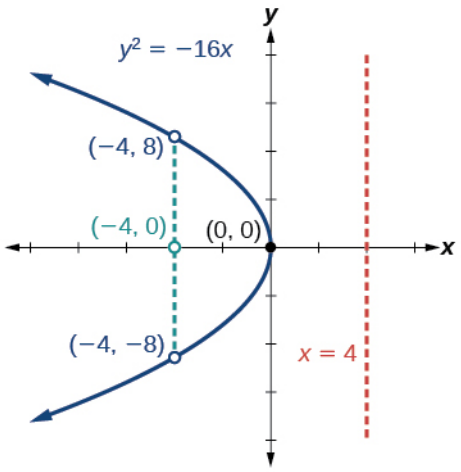



**12.12**. center:
$(3, -4)$; vertices:
$(3, -14)$ and
$(3, 6)$; co-vertices:
$(-5, -4)$; and
$(11, -4)$; foci:
$\left(3, -4 - 2\sqrt{41}\right)$ and
$\left(3, -4 + 2\sqrt{41}\right)$; asymptotes:
$y = \pm \frac{5}{4}(x - 3) - 4$

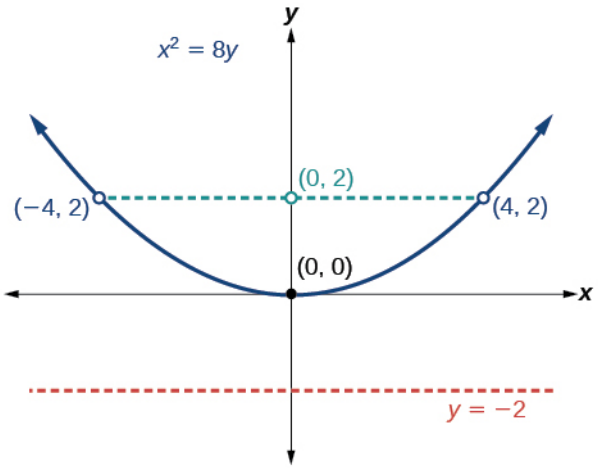



**12.13**. The sides of the tower can be modeled by the hyperbolic equation.
$\frac{x^2}{400} - \frac{y^2}{3600} = 1 \text{or } \frac{x^2}{20^2} - \frac{y^2}{60^2} = 1.$

**12.14**. Focus:
$(-4, 0)$; Directrix:
$x = 4$; Endpoints of the latus rectum:
$(-4, \pm 8)$

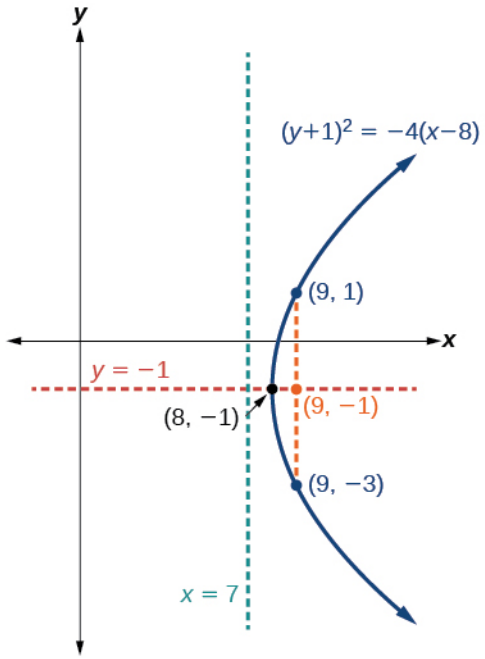



**12.15**. Focus:
$(0, 2)$; Directrix:
$y = -2$; Endpoints of the latus rectum:
$(\pm 4, 2)$.

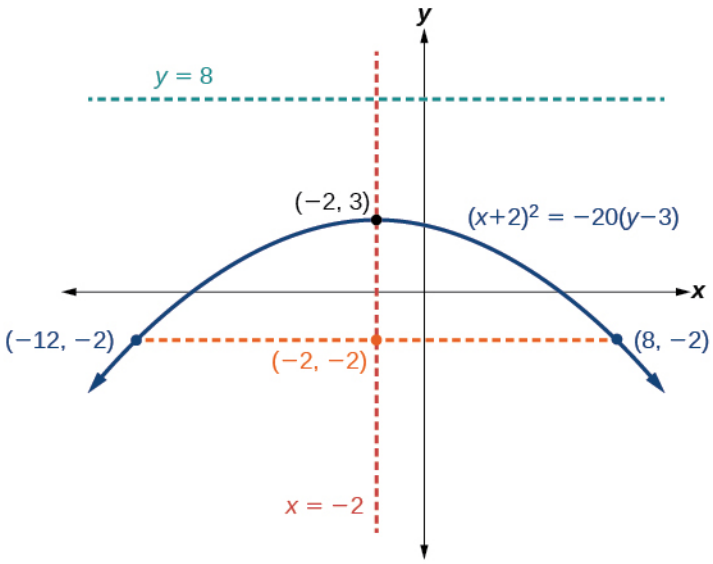



**12.16**.
$x^2 = 14y$.

**12.17**. Vertex:
$(8, -1)$; Axis of symmetry:
$y = -1$; Focus:
$(9, -1)$; Directrix:
$x = 7$; Endpoints of the latus rectum:
$(9, -3)$ and
$(9, 1)$.




**12.18**. Vertex:
$(-2,\ 3)$; Axis of symmetry:
$x = -2$; Focus:
$(-2,\ -2)$; Directrix:
$y = 8$; Endpoints of the latus rectum:
$(-12,\ -2)$ and
$(8,\ -2)$.




**12.19**.

a.

$$y^2 = 1280x$$

b. The depth of the cooker is 500 mm

**12.20**.

a. hyperbola

b. ellipse

**12.21**.

$$\frac{x'^2}{4}+\frac{y'^2}{1}=1$$

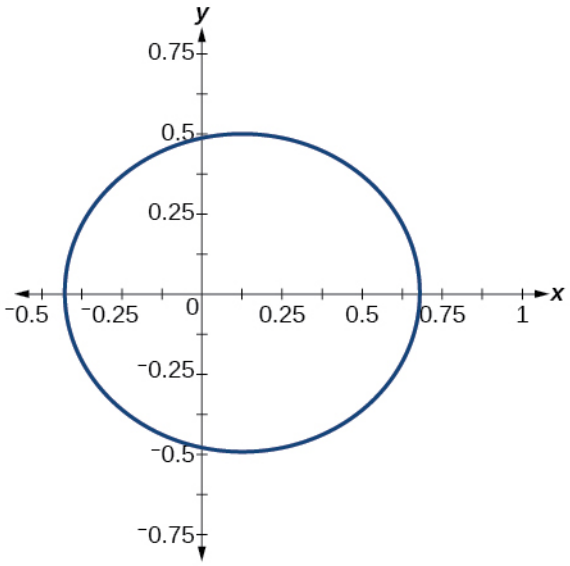
**12.22**.

a. hyperbola
b. ellipse

**12.23**. ellipse;

$e=\frac{1}{3};\ x=-2$

**12.24**.




**12.25**.

$$r=\frac{1}{1-\cos\theta}$$

**12.26**.

$4-8x+3x^2-y^2=0$

## Section Exercises

**1**. An ellipse is the set of all points in the plane the sum of whose distances from two fixed points, called the foci, is a constant.
**3**. This special case would be a circle.
**5**. It is symmetric about the *x*-axis, *y*-axis, and the origin.
**7**. yes;

$$\frac{x^2}{3^2}+\frac{y^2}{2^2}=1$$

**9**. yes;

$$\frac{x^2}{\left(\frac{1}{2}\right)^2}+\frac{y^2}{\left(\frac{1}{3}\right)^2}=1$$

**11**.

$\frac{x^2}{2^2}+\frac{y^2}{7^2}=1$; Endpoints of major axis $(0,\ 7)$ and $(0,\ -7)$. Endpoints of minor axis $(2,\ 0)$ and $(-2,\ 0)$. Foci at $\left(0,\ 3\sqrt{5}\right),\ \left(0,\ -3\sqrt{5}\right)$.

**13**.

$\frac{x^2}{(1)^2} + \frac{y^2}{\left(\frac{1}{3}\right)^2} = 1$; Endpoints of major axis $(1, 0)$ and $(-1, 0)$. Endpoints of minor axis $\left(0, \frac{1}{3}\right), \left(0, -\frac{1}{3}\right)$. Foci at $\left(\frac{2\sqrt{2}}{3}, 0\right), \left(-\frac{2\sqrt{2}}{3}, 0\right)$.

**15**.

$\frac{(x-2)^2}{7^2} + \frac{(y-4)^2}{5^2} = 1$; Endpoints of major axis $(9, 4), (-5, 4)$. Endpoints of minor axis $(2, 9), (2, -1)$. Foci at $\left(2 + 2\sqrt{6}, 4\right), \left(2 - 2\sqrt{6}, 4\right)$.

**17**.

$\frac{(x+5)^2}{2^2} + \frac{(y-7)^2}{3^2} = 1$; Endpoints of major axis $(-5, 10), (-5, 4)$. Endpoints of minor axis $(-3, 7), (-7, 7)$. Foci at $\left(-5, 7 + \sqrt{5}\right), \left(-5, 7 - \sqrt{5}\right)$.

**19**.

$\frac{(x-1)^2}{3^2} + \frac{(y-4)^2}{2^2} = 1$; Endpoints of major axis $(4, 4), (-2, 4)$. Endpoints of minor axis $(1, 6), (1, 2)$. Foci at $\left(1 + \sqrt{5}, 4\right), \left(1 - \sqrt{5}, 4\right)$.

**21**.

$\frac{(x-3)^2}{\left(3\sqrt{2}\right)^2} + \frac{(y-5)^2}{(\sqrt{2})^2} = 1$; Endpoints of major axis $\left(3 + 3\sqrt{2}, 5\right), \left(3 - 3\sqrt{2}, 5\right)$. Endpoints of minor axis $\left(3, 5 + \sqrt{2}\right), \left(3, 5 - \sqrt{2}\right)$. Foci at $(7, 5), (-1, 5)$.

**23**.

$\frac{(x+5)^2}{(5)^2} + \frac{(y-2)^2}{(2)^2} = 1$; Endpoints of major axis $(0, 2), (-10, 2)$. Endpoints of minor axis $(-5, 4), (-5, 0)$. Foci at $\left(-5 + \sqrt{21}, 2\right), \left(-5 - \sqrt{21}, 2\right)$.

**25**.

$\frac{(x+3)^2}{(5)^2} + \frac{(y+4)^2}{(2)^2} = 1$; Endpoints of major axis $(2, -4), (-8, -4)$. Endpoints of minor axis $(-3, -2), (-3, -6)$. Foci at $\left(-3 + \sqrt{21}, -4\right), \left(-3 - \sqrt{21}, -4\right)$.

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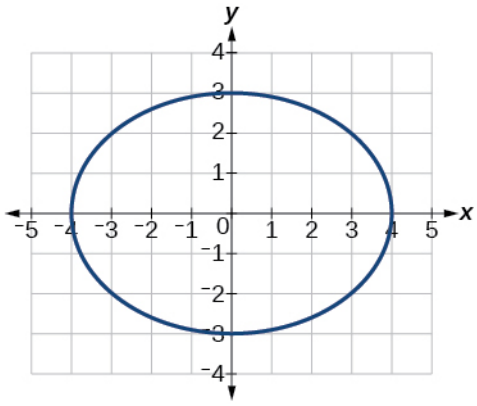
**27**. Foci
$(-3,\ -1+\sqrt{11}), (-3,\ -1-\sqrt{11})$

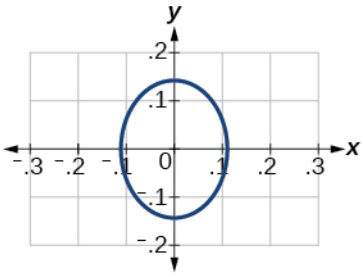
**29**. Focus
$(0,\ 0)$

**31**. Foci
$(-10,\ 30), (-10,\ -30)$

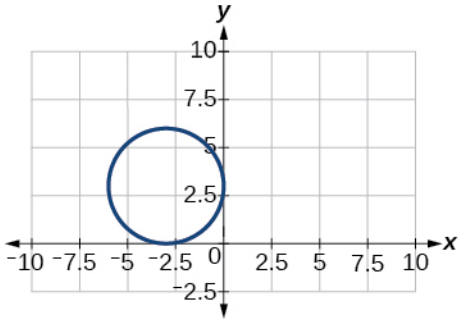
**33**. Center
$(0,\ 0),$ Vertices
$(4,\ 0), (-4,\ 0), (0,\ 3), (0,\ -3),$ Foci
$(\sqrt{7},\ 0), (-\sqrt{7},\ 0)$




**35**. Center
$(0,\ 0),$ Vertices
$\left(\frac{1}{9},\ 0\right), \left(-\frac{1}{9},\ 0\right), \left(0,\ \frac{1}{7}\right), \left(0,\ -\frac{1}{7}\right),$ Foci
$\left(0,\ \frac{4\sqrt{2}}{63}\right), \left(0,\ -\frac{4\sqrt{2}}{63}\right)$

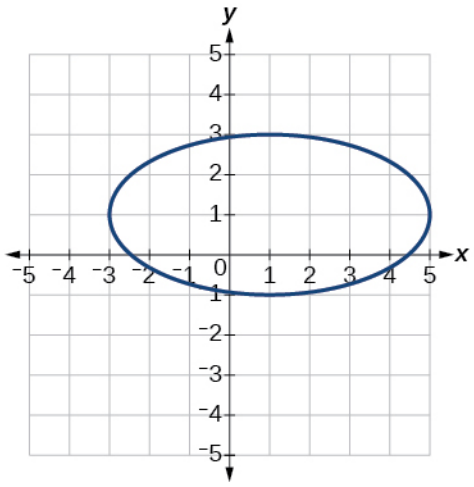



**37**. Center
$(-3,\ 3),$ Vertices
$(0,\ 3), (-6,\ 3), (-3,\ 0), (-3,\ 6),$ Focus
$(-3,\ 3)$ Note that this ellipse is a circle. The circle has only one focus, which coincides with the center.

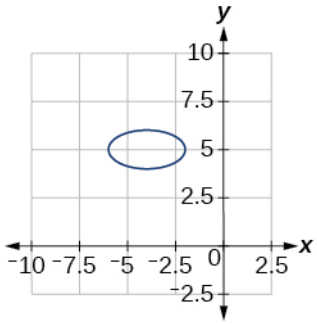



**39**. Center
$(1,\ 1),$ Vertices

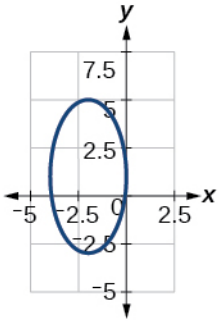
$(5,\ 1),\ (-3,\ 1),\ (1,\ 3),\ (1,\ -1),$ Foci
$(1,\ 1+4\sqrt{3}),\ (1,\ 1-4\sqrt{3})$



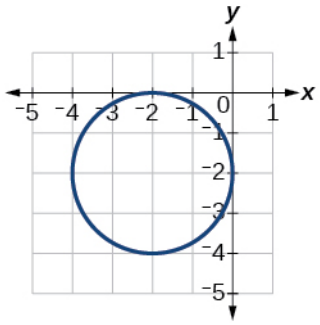

**41**. Center
$(-4,\ 5),$ Vertices
$(-2,\ 5),\ (-6,\ 4),\ (-4,\ 6),\ (-4,\ 4),$ Foci
$(-4+\sqrt{3},\ 5),\ (-4-\sqrt{3},\ 5)$




**43**. Center
$(-2,\ 1),$ Vertices
$(0,\ 1),\ (-4,\ 1),\ (-2,\ 5),\ (-2,\ -3),$ Foci
$(-2,\ 1+2\sqrt{3}),\ (-2,\ 1-2\sqrt{3})$




**45**. Center
$(-2,\ -2),$ Vertices
$(0,\ -2),\ (-4,\ -2),\ (-2,\ 0),\ (-2,\ -4),$ Focus
$(-2,\ -2)$




**47**.
$$\frac{x^2}{25}+\frac{y^2}{29}=1$$
**49**.
$$\frac{(x-4)^2}{25}+\frac{(y-2)^2}{1}=1$$
**51**.
$$\frac{(x+3)^2}{16}+\frac{(y-4)^2}{4}=1$$
**53**.
$$\frac{x^2}{81}+\frac{y^2}{9}=1$$
**55**.
$$\frac{(x+2)^2}{4}+\frac{(y-2)^2}{9}=1$$
**57**.
$\text{Area}=12\pi$ square units
**59**.
$\text{Area}=2\sqrt{5}\pi$ square units
**61**.
$\text{Area}=9\pi$ square units
**63**.
$$\frac{x^2}{4h^2}+\frac{y^2}{\frac{1}{4}h^2}=1$$
**65**.
$\frac{x^2}{400}+\frac{y^2}{144}=1$. Distance = 17.32 feet
**67**. Approximately 51.96 feet
**69**. A hyperbola is the set of points in a plane the difference of whose distances from two fixed points (foci) is a positive constant.
**71**. The foci must lie on the transverse axis and be in the interior of the hyperbola.
**73**. The center must be the midpoint of the line segment joining the foci.
**75**. yes
$$\frac{x^2}{6^2}-\frac{y^2}{3^2}=1$$
**77**. yes
$$\frac{x^2}{4^2}-\frac{y^2}{5^2}=1$$
**79**.
$\frac{x^2}{5^2}-\frac{y^2}{6^2}=1$; vertices: $(5,\,0),\,(-5,\,0)$; foci:

$(\sqrt{61}, 0), (-\sqrt{61}, 0)$; asymptotes:
$y = \frac{6}{5}x, y = -\frac{6}{5}x$

**81**.
$\frac{y^2}{2^2} - \frac{x^2}{9^2} = 1$; vertices:
$(0, 2), (0, -2)$; foci:
$(0, \sqrt{85}), (0, -\sqrt{85})$; asymptotes:
$y = \frac{2}{9}x, y = -\frac{2}{9}x$

**83**.
$\frac{(x-1)^2}{3^2} - \frac{(y-2)^2}{4^2} = 1$; vertices:
$(4, 2), (-2, 2)$; foci:
$(6, 2), (-4, 2)$; asymptotes:
$y = \frac{4}{3}(x-1) + 2, y = -\frac{4}{3}(x-1) + 2$

**85**.
$\frac{(x-2)^2}{7^2} - \frac{(y+7)^2}{7^2} = 1$; vertices:
$(9, -7), (-5, -7)$; foci:
$(2 + 7\sqrt{2}, -7), (2 - 7\sqrt{2}, -7)$; asymptotes:
$y = x - 9, y = -x - 5$

**87**.
$\frac{(x+3)^2}{3^2} - \frac{(y-3)^2}{3^2} = 1$; vertices:
$(0, 3), (-6, 3)$; foci:
$(-3 + 3\sqrt{2}, 1), (-3 - 3\sqrt{2}, 1)$; asymptotes:
$y = x + 6, y = -x$

**89**.
$\frac{(y-4)^2}{2^2} - \frac{(x-3)^2}{4^2} = 1$; vertices:
$(3, 6), (3, 2)$; foci:
$(3, 4 + 2\sqrt{5}), (3, 4 - 2\sqrt{5})$; asymptotes:
$y = \frac{1}{2}(x-3) + 4, y = -\frac{1}{2}(x-3) + 4$

**91**.
$\frac{(y+5)^2}{7^2} - \frac{(x+1)^2}{70^2} = 1$; vertices:
$(-1, 2), (-1, -12)$; foci:
$(-1, -5 + 7\sqrt{101}), (-1, -5 - 7\sqrt{101})$; asymptotes:
$y = \frac{1}{10}(x+1) - 5, y = -\frac{1}{10}(x+1) - 5$

**93**.
$\frac{(x+3)^2}{5^2} - \frac{(y-4)^2}{2^2} = 1$; vertices:
$(2, 4), (-8, 4)$; foci:

$(-3+\sqrt{29},\ 4),\ (-3-\sqrt{29},\ 4)$; asymptotes:

$y=\frac{2}{5}(x+3)+4,\ y=-\frac{2}{5}(x+3)+4$

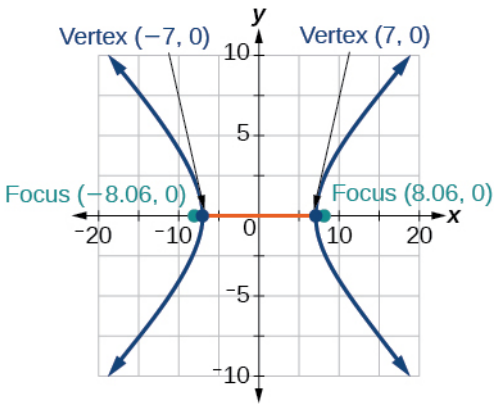
**95**.

$y=\frac{2}{5}(x-3)-4,\ y=-\frac{2}{5}(x-3)-4$

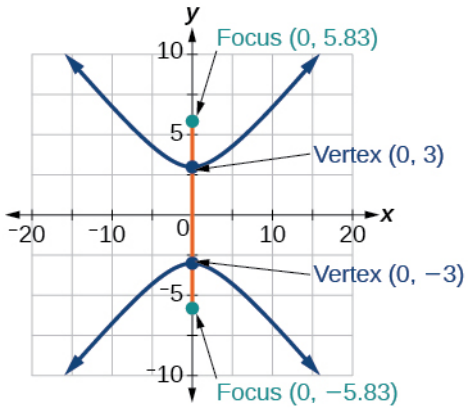
**97**.

$y=\frac{3}{4}(x-1)+1,\ y=-\frac{3}{4}(x-1)+1$

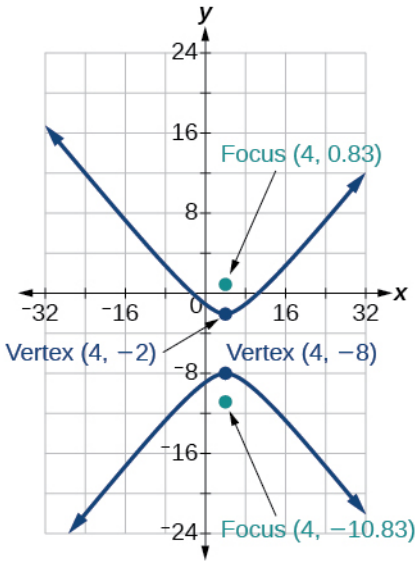
**99**.




**101**.

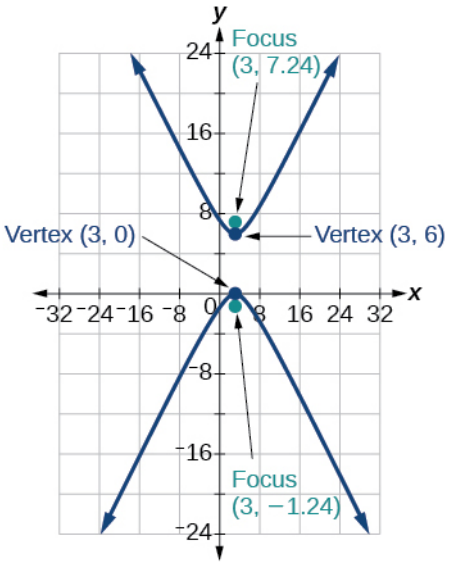



**103**.

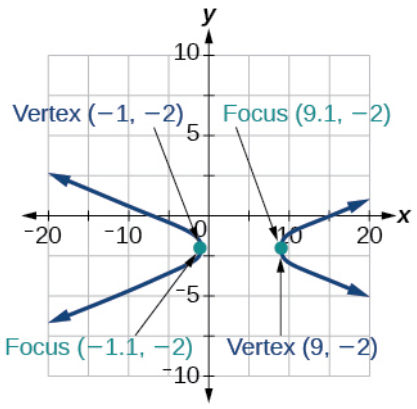



**105**.

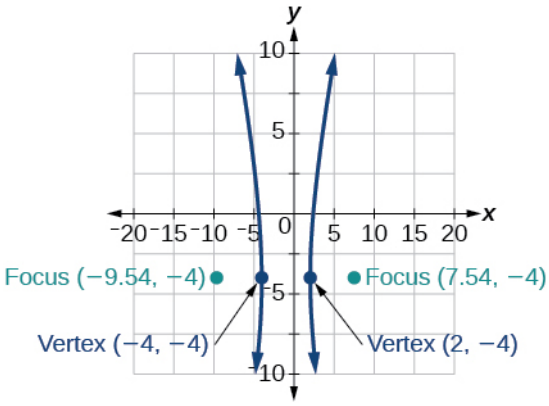
**107**.

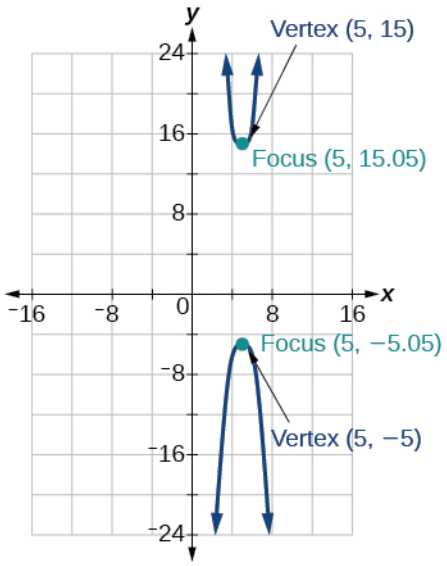

y
x
Focus
(3, 7.24)
Vertex (3, 0)
Vertex (3, 6)
Focus
(3, −1.24)
24
16
8
−8
−16
−24
−32
−24
−16
−8
0
8
16
24
32

**109**.


y
x
Vertex (−1, −2)
Focus (9.1, −2)
Focus (−1.1, −2)
Vertex (9, −2)
10
5
−5
−10
−20
−10
0
10
20

**111**.


y
x
Focus (−9.54, −4)
Focus (7.54, −4)
Vertex (−4, −4)
Vertex (2, −4)
10
5
−5
−10
−20
−15
−10
−5
0
5
10
15
20




**113**.

$$\frac{x^2}{9} - \frac{y^2}{16} = 1$$

**115**.

$$\frac{(x-6)^2}{25} - \frac{(y-1)^2}{11} = 1$$

**117**.

$$\frac{(x-4)^2}{25} - \frac{(y-2)^2}{1} = 1$$

**119**.

$$\frac{y^2}{16} - \frac{x^2}{25} = 1$$

**121**.

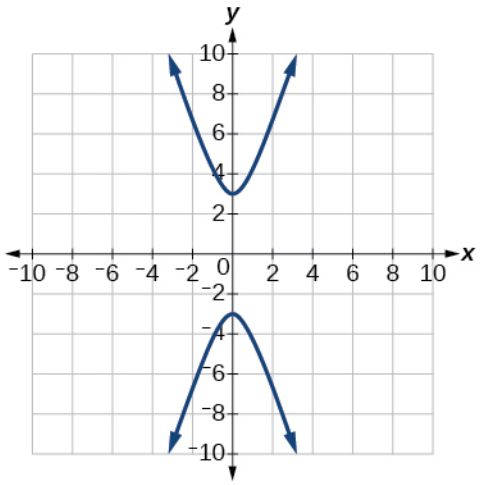
$$\frac{y^2}{9} - \frac{(x+1)^2}{9} = 1$$

**123**.

$$\frac{(x+3)^2}{25} - \frac{(y+3)^2}{25} = 1$$

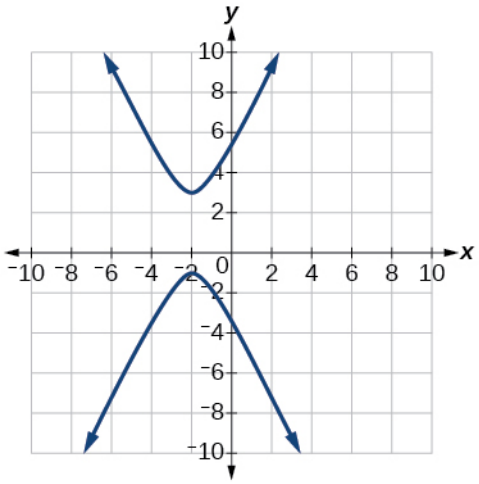
**125**.

$$y(x) = 3\sqrt{x^2+1},\ y(x) = -3\sqrt{x^2+1}$$



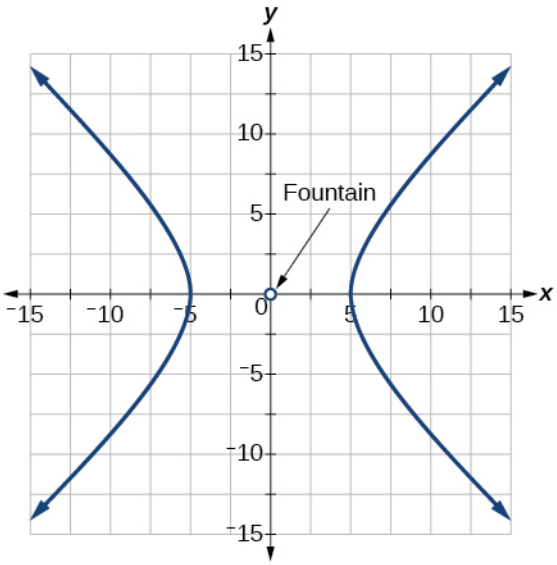
**127**.

$y(x) = 1 + 2\sqrt{x^2 + 4x + 5},\ y(x) = 1 - 2\sqrt{x^2 + 4x + 5}$



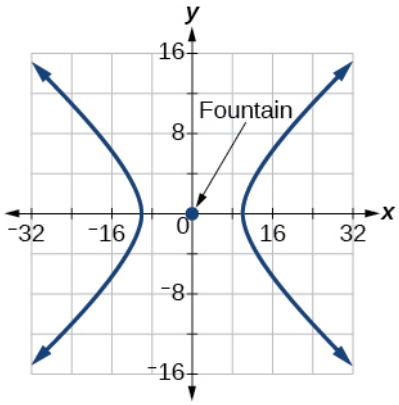

**129**.

$\frac{x^2}{25} - \frac{y^2}{25} = 1$



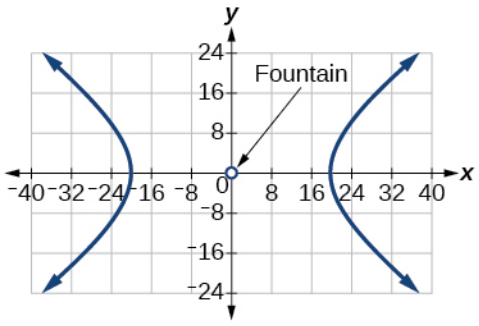

**131**.

$\frac{x^2}{100} - \frac{y^2}{25} = 1$



**133**.

$$\frac{x^2}{400} - \frac{y^2}{225} = 1$$




**135**.

$$\frac{(x-1)^2}{0.25} - \frac{y^2}{0.75} = 1$$

**137**.

$$\frac{(x-3)^2}{4} - \frac{y^2}{5} = 1$$

**139**. A parabola is the set of points in the plane that lie equidistant from a fixed point, the focus, and a fixed line, the directrix.

**141**. The graph will open down.

**143**. The distance between the focus and directrix will increase.

**145**. yes

$$y = 4(1)x^2$$

**147**. yes

$$(y-3)^2 = 4(2)(x-2)$$

**149**.

$$y^2 = \frac{1}{8}x,\ V:(0,\ 0);\ F:\left(\frac{1}{32},\ 0\right);\ d:x = -\frac{1}{32}$$

**151**.

$$x^2 = -\frac{1}{4}y,\ V:(0,\ 0);\ F:\left(0,\ -\frac{1}{16}\right);\ d:y = \frac{1}{16}$$

**153**.

$$y^2 = \frac{1}{36}x,\ V:(0,\ 0);\ F:\left(\frac{1}{144},\ 0\right);\ d:x = -\frac{1}{144}$$

**155**.

$$(x-1)^2 = 4(y-1),\ V:(1,\ 1);\ F:(1,\ 2);\ d:y = 0$$

**157**.

$$(y-4)^2 = 2(x+3),\ V:(-3,\ 4);\ F:\left(-\frac{5}{2},\ 4\right);\ d:x = -\frac{7}{2}$$

**159**.

$$(x+4)^2 = 24(y+1),\ V:(-4,\ -1);\ F:(-4,\ 5);\ d:y = -7$$

**161**.

$$(y-3)^2 = -12(x+1),\ V:(-1,\ 3);\ F:(-4,\ 3);\ d:x = 2$$

**163**.

$$(x-5)^2 = \frac{4}{5}(y+3),\ V:(5,\ -3);\ F:\left(5,\ -\frac{14}{5}\right);\ d:y = -\frac{16}{5}$$

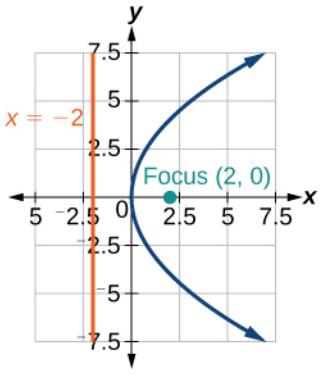
**165**.

$$(x-2)^2 = -2(y-5),\ V:(2,\ 5);\ F:\left(2,\ \frac{9}{2}\right);\ d:y = \frac{11}{2}$$

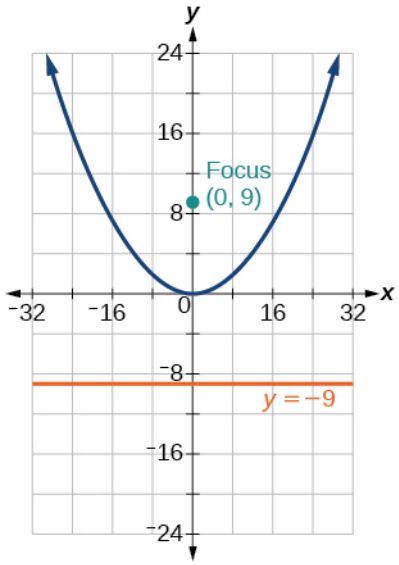
**167**.

$$(y-1)^2 = \frac{4}{3}(x-5),\ V:(5,\ 1);\ F:\left(\frac{16}{3},\ 1\right);\ d:x = \frac{14}{3}$$

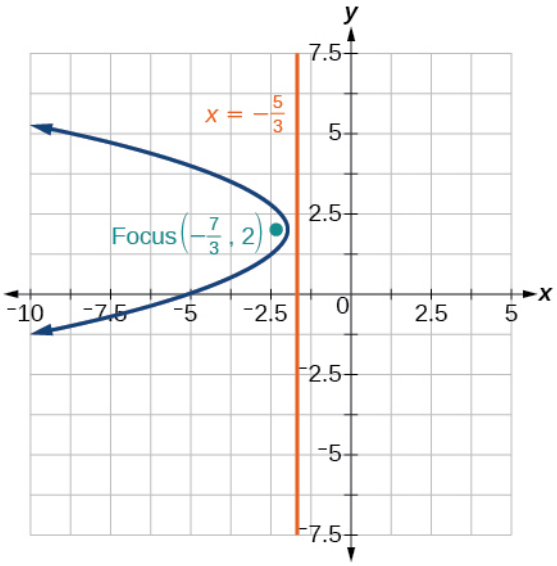
**169**.


y
x
x = −2
Focus (2, 0)
7.5
5
2.5
−2.5
−5
−7.5
−5
2.5
5
7.5

**171**.

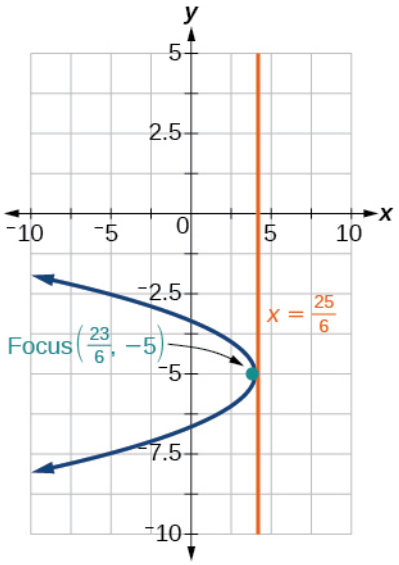

y
x
Focus
(0, 9)
y = −9
24
16
8
−8
−16
−24
−32
−16
16
32

**173**.

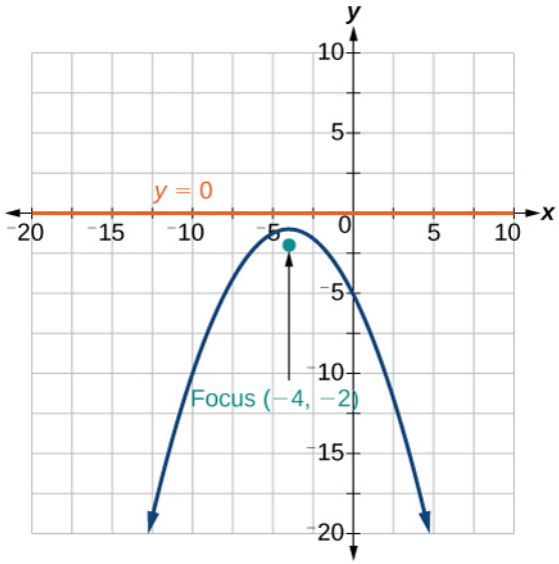

y
x
x = −5/3
Focus (−7/3, 2)
7.5
5
2.5
−2.5
−5
−7.5
−10
−7.5
−5
−2.5
2.5
5

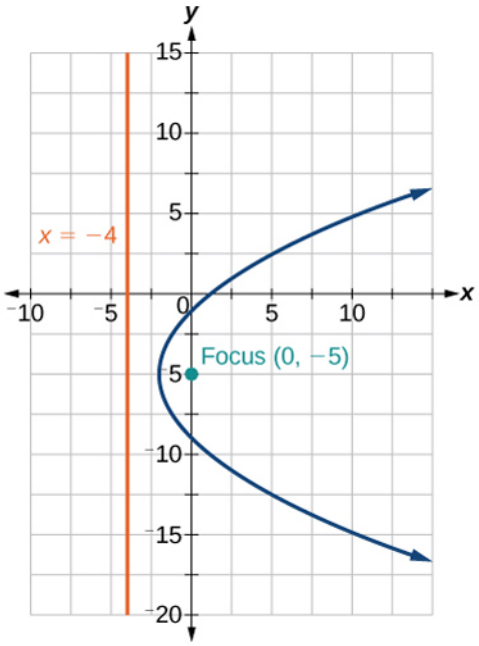
**175**.

**177**.

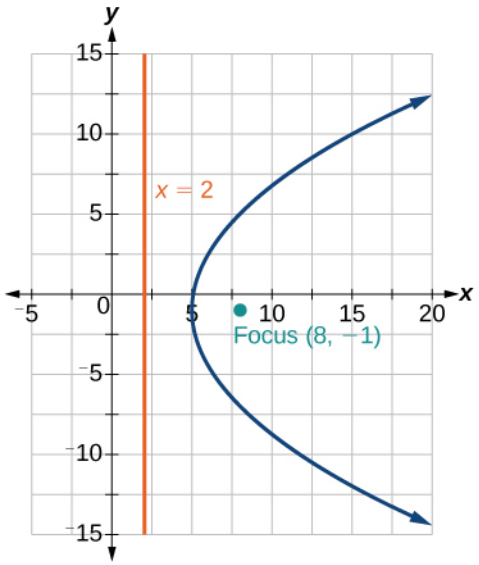

y
x
5
2.5
0
−10
−5
5
10
−2.5
−5
−7.5
−10
$x = \frac{25}{6}$
Focus $\left(\frac{23}{6}, -5\right)$

**179**.


y
x
10
5
0
−20
−15
−10
−5
5
10
−5
−10
−15
−20
$y = 0$
Focus $(-4, -2)$




**181**.




**183**.

$x^2 = -16y$

**185**.

$(y-2)^2 = 4\sqrt{2}(x-2)$

**187**.

$\left(y+\sqrt{3}\right)^2 = -4\sqrt{2}\left(x-\sqrt{2}\right)$

**189**.

$x^2 = y$

**191**.

$(y-2)^2 = \frac{1}{4}(x+2)$

**193**.

$\left(y-\sqrt{3}\right)^2 = 4\sqrt{5}\left(x+\sqrt{2}\right)$

**195**.

$y^2 = -8x$

**197**.
$(y+1)^2 = 12(x+3)$

**199**.
$(0,\ 1)$

**201**. At the point 2.25 feet above the vertex.
**203**. 0.5625 feet
**205**.
$x^2 = -125(y-20),$ height is 7.2 feet

**207**. 2304 feet
**209**. The
$xy$ term causes a rotation of the graph to occur.
**211**. The conic section is a hyperbola.
**213**. It gives the angle of rotation of the axes in order to eliminate the
$xy$ term.
**215**.
$AB = 0,$ parabola

**217**.
$AB = -4 < 0,$ hyperbola

**219**.
$AB = 6 > 0,$ ellipse

**221**.
$B^2 - 4AC = 0,$ parabola

**223**.
$B^2 - 4AC = 0,$ parabola

**225**.
$B^2 - 4AC = -96 < 0,$ ellipse

**227**.
$7x'^2 + 9y'^2 - 4 = 0$

**229**.
$3x'^2 + 2x'y' - 5y'^2 + 1 = 0$

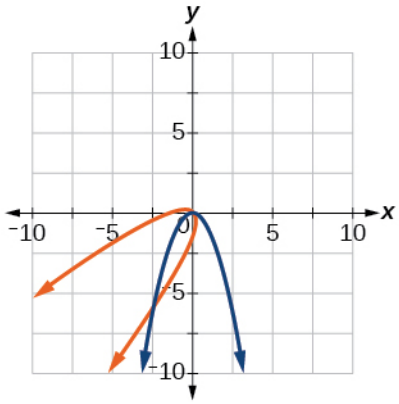
**231**.
$\theta = 60^\circ,\ 11x'^2 - y'^2 + \sqrt{3}x' + y' - 4 = 0$

**233**.
$\theta = 150^\circ,\ 21x'^2 + 9y'^2 + 4x' - 4\sqrt{3}y' - 6 = 0$

**235**.
$\theta \approx 36.9^\circ,\ 125x'^2 + 6x' - 42y' + 10 = 0$

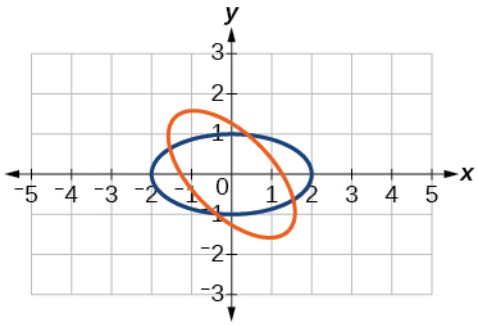
**237**.
$\theta = 45^\circ,\ 3x'^2 - y'^2 - \sqrt{2}x' + \sqrt{2}y' + 1 = 0$

**239**.
$\frac{\sqrt{2}}{2}(x' + y') = \frac{1}{2}(x' - y')^2$



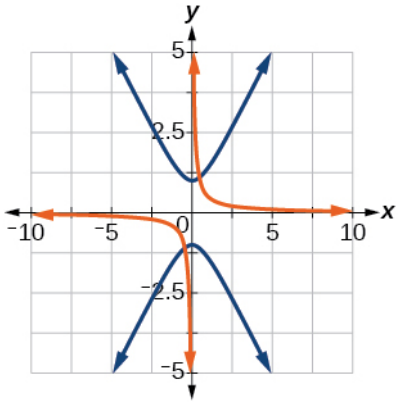

**241.**

$$\frac{(x'-y')^2}{8}+\frac{(x'+y')^2}{2}=1$$



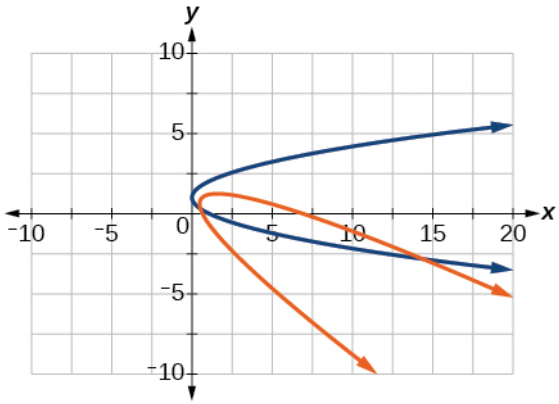

**243.**

$$\frac{(x'+y')^2}{2}-\frac{(x'-y')^2}{2}=1$$

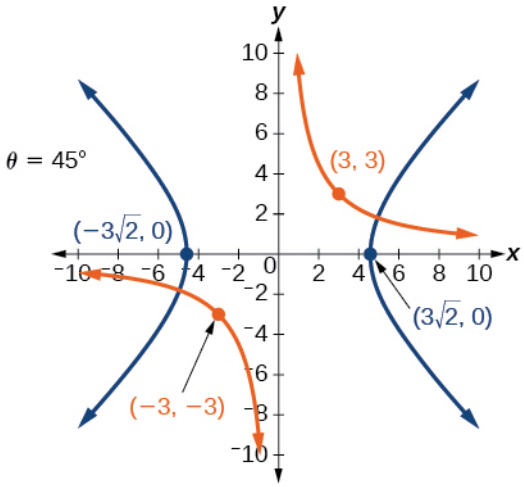



**245.**

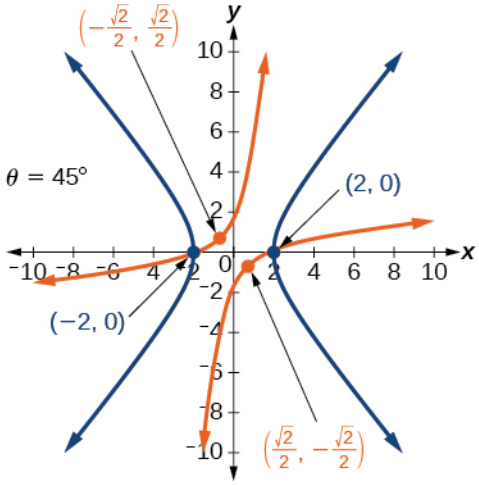
$$\frac{\sqrt{3}}{2}x'-\frac{1}{2}y'=\left(\frac{1}{2}x'+\frac{\sqrt{3}}{2}y'-1\right)^2$$




**247**.

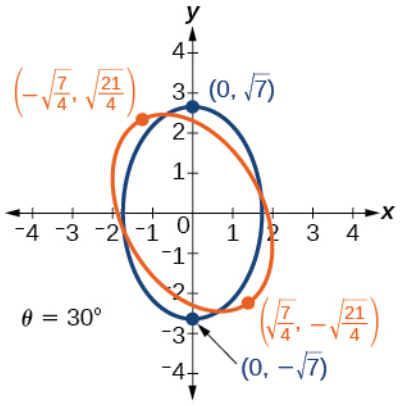



**249**.

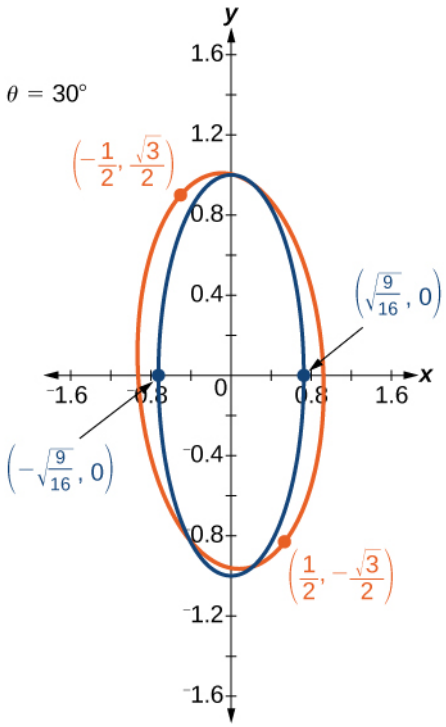



**251**.

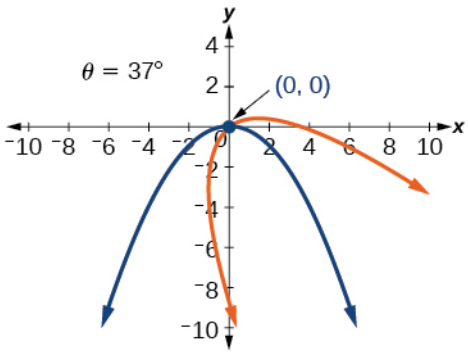
**253**.

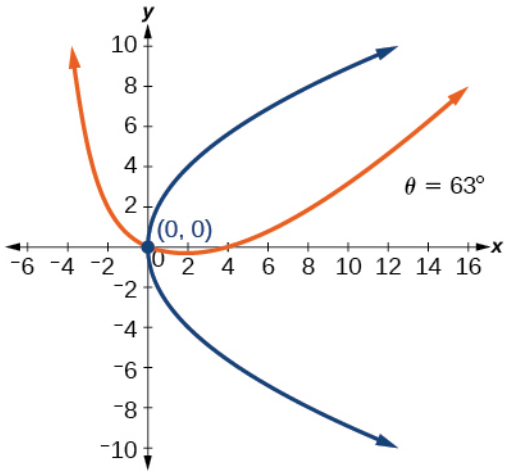

y
x
$\left(-\sqrt{\frac{7}{4}}, \sqrt{\frac{21}{4}}\right)$
$(0, \sqrt{7})$
$\theta = 30°$
$\left(\sqrt{\frac{7}{4}}, -\sqrt{\frac{21}{4}}\right)$
$(0, -\sqrt{7})$

**255**.

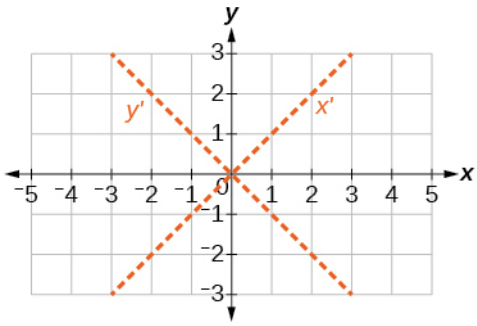

y
x
$\theta = 30°$
$\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$
$\left(\sqrt{\frac{9}{16}}, 0\right)$
$\left(-\sqrt{\frac{9}{16}}, 0\right)$
$\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

**257**.

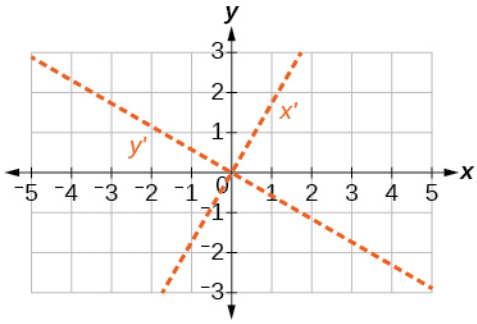

y
x
$\theta = 37°$
$(0, 0)$



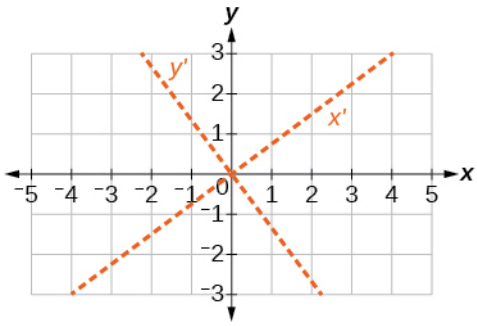

**259**.
$\theta = 45^\circ$




**261**.
$\theta = 60^\circ$




**263**.
$\theta \approx 36.9^\circ$




**265**.
$-4\sqrt{6} < k < 4\sqrt{6}$

**267**.
$k = 2$

**269**. If eccentricity is less than 1, it is an ellipse. If eccentricity is equal to 1, it is a parabola. If eccentricity is greater than 1, it is a hyperbola.
**271**. The directrix will be parallel to the polar axis.
**273**. One of the foci will be located at the origin.
**275**. Parabola with
$e = 1$ and directrix
$\frac{3}{4}$ units below the pole.
**277**. Hyperbola with
$e = 2$ and directrix
$\frac{5}{2}$ units above the pole.
**279**. Parabola with
$e = 1$ and directrix
$\frac{3}{10}$ units to the right of the pole.
**281**. Ellipse with
$e = \frac{2}{7}$ and directrix
$2$ units to the right of the pole.
**283**. Hyperbola with
$e = \frac{5}{3}$ and directrix
$\frac{11}{5}$ units above the pole.
**285**. Hyperbola with
$e = \frac{8}{7}$ and directrix
$\frac{7}{8}$ units to the right of the pole.
**287**.
$25x^2 + 16y^2 - 12y - 4 = 0$
**289**.
$21x^2 - 4y^2 - 30x + 9 = 0$
**291**.
$64y^2 = 48x + 9$
**293**.
$96y^2 - 25x^2 + 110y + 25 = 0$
**295**.
$3x^2 + 4y^2 - 2x - 1 = 0$
**297**.
$5x^2 + 9y^2 - 24x - 36 = 0$
**299**.

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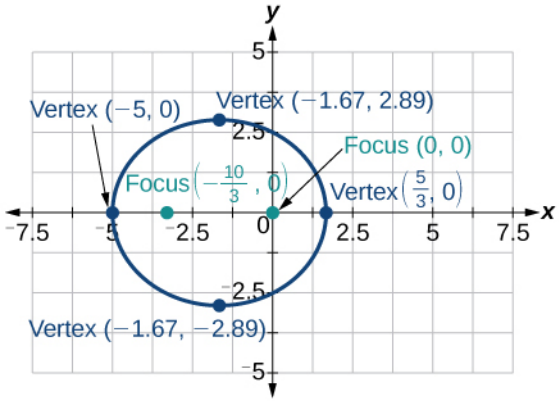
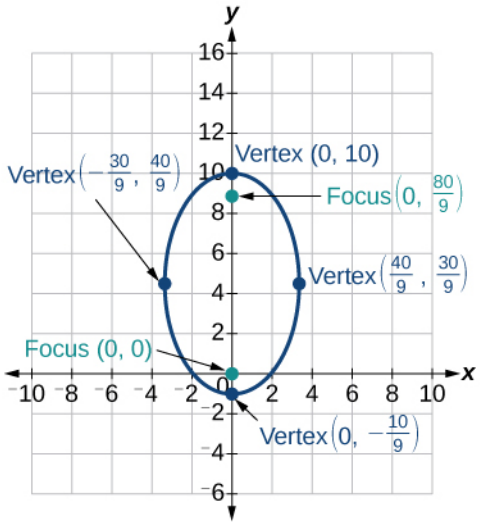
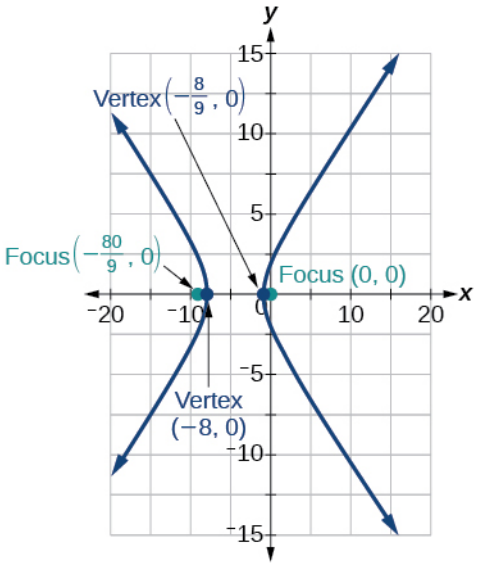

y
x
Vertex (−5, 0)
Vertex (−1.67, 2.89)
Focus (0, 0)
Focus $\left(-\frac{10}{3}, 0\right)$
Vertex $\left(\frac{5}{3}, 0\right)$
Vertex (−1.67, −2.89)

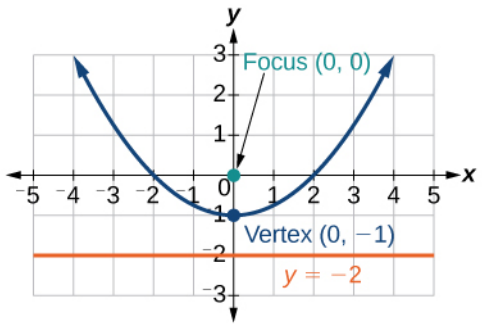
**301**.


y
x
Vertex (0, 10)
Vertex $\left(-\frac{30}{9}, \frac{40}{9}\right)$
Focus $\left(0, \frac{80}{9}\right)$
Vertex $\left(\frac{40}{9}, \frac{30}{9}\right)$
Focus (0, 0)
Vertex $\left(0, -\frac{10}{9}\right)$

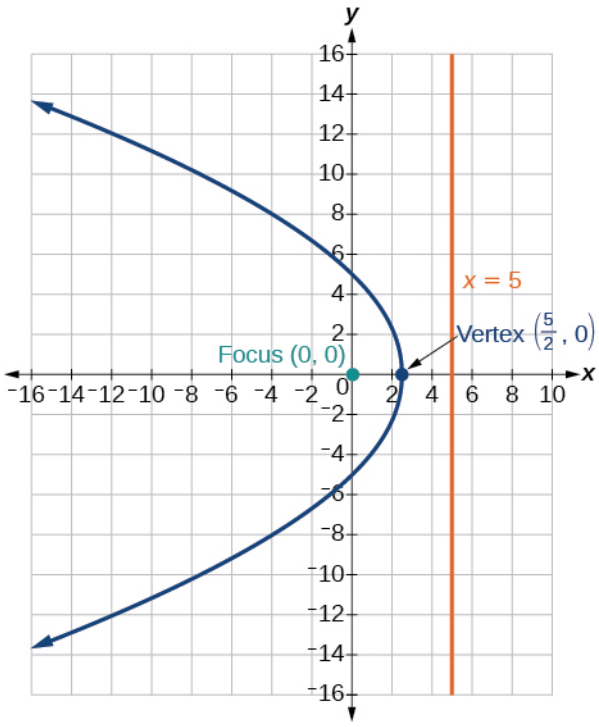
**303**.


y
x
Vertex $\left(-\frac{8}{9}, 0\right)$
Focus $\left(-\frac{80}{9}, 0\right)$
Focus (0, 0)
Vertex
(−8, 0)

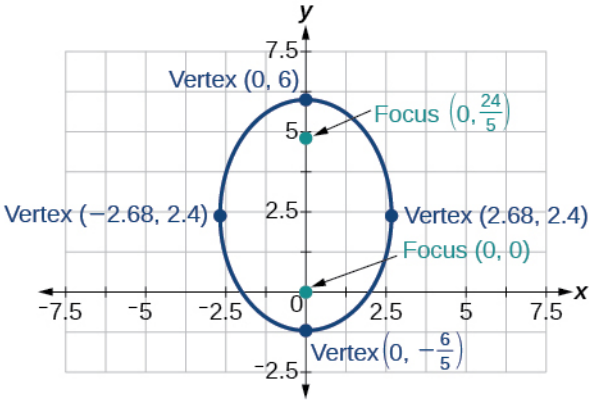
**305**.




**307**.




**309**.




**311**.

$$r = \frac{4}{5 + \cos\theta}$$

**313**.

$$r = \frac{4}{1 + 2\sin\theta}$$

**315**.

$$r = \frac{1}{1 + \cos\theta}$$

**317**.

$$r = \frac{7}{8 - 28\cos\theta}$$

**319**.

$$r = \frac{12}{2 + 3\sin\theta}$$

**321**.

$$r = \frac{15}{4 - 3\cos\theta}$$

**323**.

$$r = \frac{3}{3 - 3\cos\theta}$$

**325**.

$$r = \pm\frac{2}{\sqrt{1 + \sin\theta\cos\theta}}$$

**327**.

$$r = \pm\frac{2}{4\cos\theta + 3\sin\theta}$$

## Review Exercises

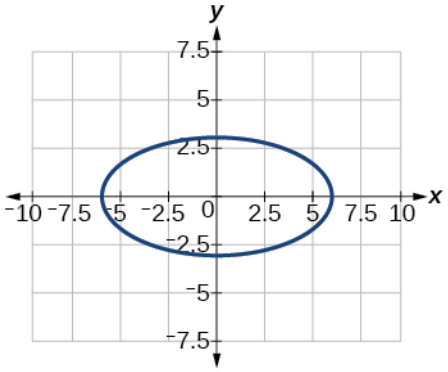
**329**.

$\frac{x^2}{5^2} + \frac{y^2}{8^2} = 1$; center: $(0, 0)$; vertices: $(5, 0), (-5, 0), (0, 8), (0, -8)$; foci: $\left(0, \sqrt{39}\right), \left(0, -\sqrt{39}\right)$

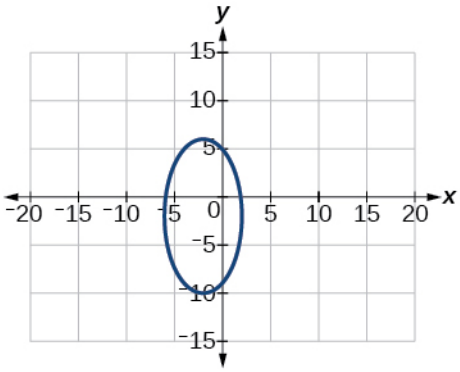
**331**.

$\frac{(x+3)^2}{1^2} + \frac{(y-2)^2}{3^2} = 1$ $(-3, 2)$; $(-2, 2), (-4, 2), (-3, 5), (-3, -1)$; $\left(-3, 2 + 2\sqrt{2}\right), \left(-3, 2 - 2\sqrt{2}\right)$

**333**. center: $(0, 0)$; vertices: $(6, 0), (-6, 0), (0, 3), (0, -3)$; foci: $\left(3\sqrt{3}, 0\right), \left(-3\sqrt{3}, 0\right)$




**335**. center: $(-2, -2)$; vertices: $(2, -2), (-6, -2), (-2, 6), (-2, -10)$; foci: $\left(-2, -2 + 4\sqrt{3}, \right), \left(-2, -2-4\sqrt{3}\right)$




**337**.

$$\frac{x^2}{25}+\frac{y^2}{16}=1$$

**339**. Approximately 35.71 feet

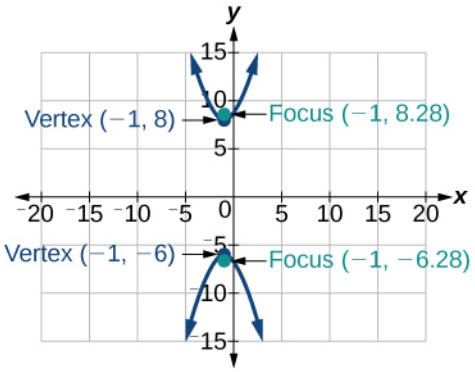
**341**.

$\frac{(y+1)^2}{4^2}-\frac{(x-4)^2}{6^2}=1$; center: $(4,\ -1)$; vertices: $(4,\ 3),\ (4,\ -5)$; foci: $\left(4,\ -1+2\sqrt{13}\right),\ \left(4,\ -1-2\sqrt{13}\right)$

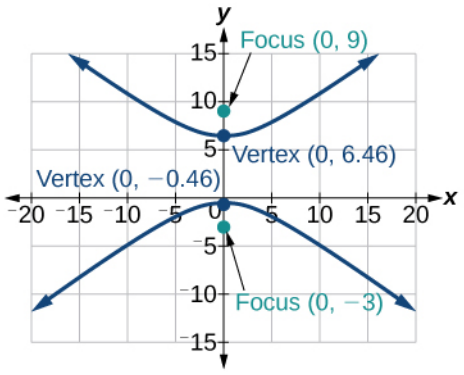
**343**.

$\frac{(x-2)^2}{2^2}-\frac{(y+3)^2}{\left(2\sqrt{3}\right)^2}=1$; center: $(2,\ -3)$; vertices: $(4,\ -3),\ (0,\ -3)$; foci: $(6,\ -3),\ (-2,\ -3)$

**345**.



**347**.




**349**.

$$\frac{(x-5)^2}{1}-\frac{(y-7)^2}{3}=1$$

**351**.

$(x+2)^2=\frac{1}{2}(y-1)$; vertex:

$(-2,\ 1)$; focus:

$\left(-2,\ \frac{9}{8}\right)$; directrix:

$y=\frac{7}{8}$

**353**.

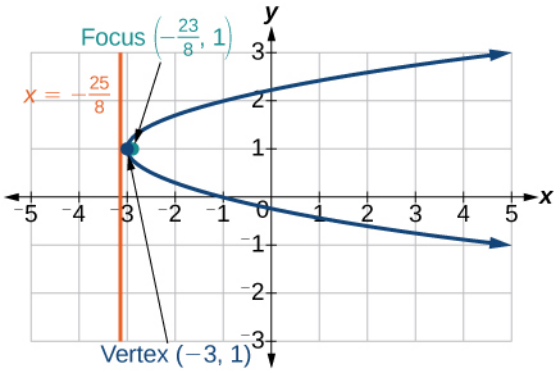
$(x+5)^2=(y+2)$; vertex:

$(-5,\ -2)$; focus:

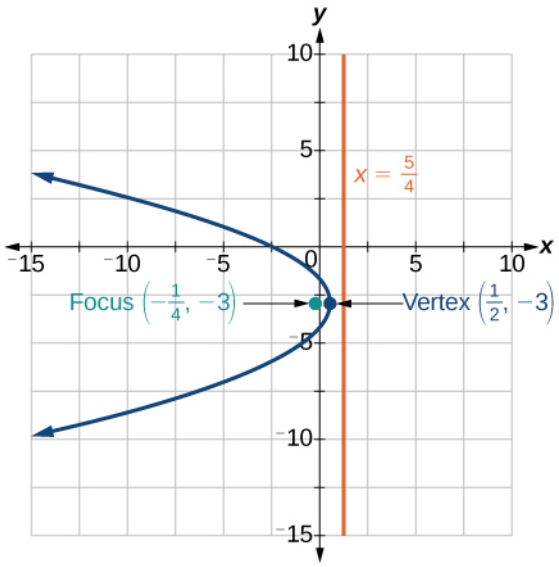
$\left(-5,\ -\frac{7}{4}\right)$; directrix:

$y=-\frac{9}{4}$

**355**.



**357**.




**359**.

$(x-2)^2 = \left(\frac{1}{2}\right)(y-1)$

**361**.

$B^2 - 4AC = 0,$ parabola

**363**.

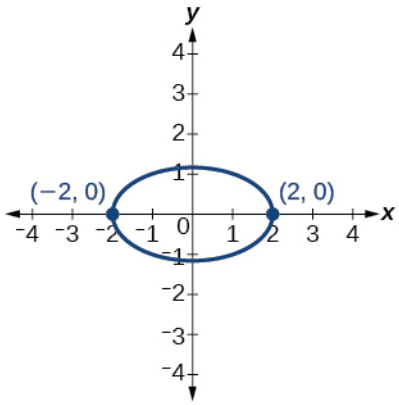
$B^2 - 4AC = -31 < 0,$ ellipse

**365**.

$\theta = 45°,\ x'^2 + 3y'^2 - 12 = 0$

**367**.

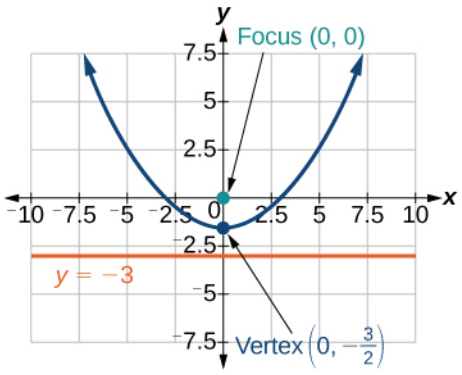
$\theta = 45°$



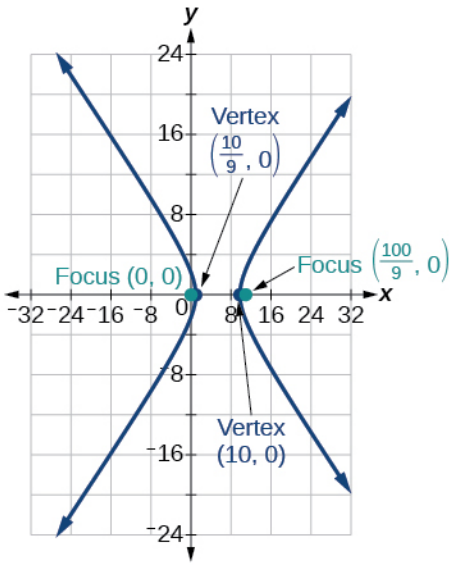

**369**. Hyperbola with $e = 5$ and directrix $2$ units to the left of the pole.

**371**. Ellipse with $e = \frac{3}{4}$ and directrix $\frac{1}{3}$ unit above the pole.

**373**.




**375**.




**377**.

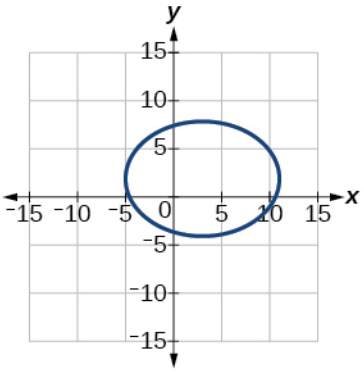
$$r = \frac{3}{1 + \cos\ \theta}$$

## Practice Test

**379**.

$\frac{x^2}{3^2} + \frac{y^2}{2^2} = 1$; center: $(0, 0)$; vertices: $(3, 0), (-3, 0), (0, 2), (0, -2)$; foci: $(\sqrt{5}, 0), (-\sqrt{5}, 0)$

**381**. center: $(3, 2)$; vertices: $(11, 2), (-5, 2), (3, 8), (3, -4)$; foci: $(3 + 2\sqrt{7}, 2), (3 - 2\sqrt{7}, 2)$



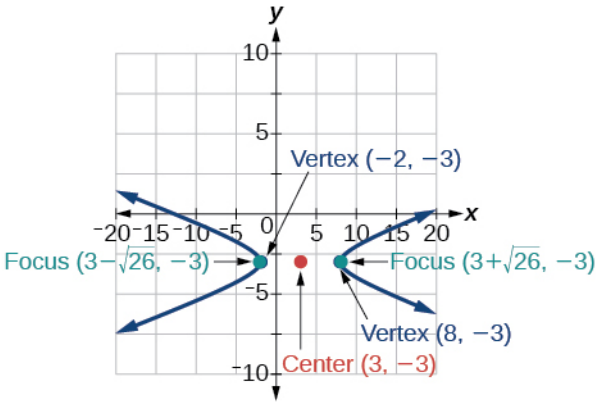

**383**.

$$\frac{(x-1)^2}{36}+\frac{(y-2)^2}{27}=1$$

**385**.

$\frac{x^2}{7^2}-\frac{y^2}{9^2}=1$; center: $(0,\ 0)$; vertices $(7,\ 0),\ (-7,\ 0)$; foci: $\left(\sqrt{130},\ 0\right),\ \left(-\sqrt{130},\ 0\right)$; asymptotes: $y=\pm\frac{9}{7}x$

**387**. center: $(3,\ -3)$; vertices: $(8,\ -3),\ (-2,\ -3)$; foci: $\left(3+\sqrt{26},\ -3\right),\ \left(3-\sqrt{26},\ -3\right)$; asymptotes: $y=\pm\frac{1}{5}(x-3)-3$



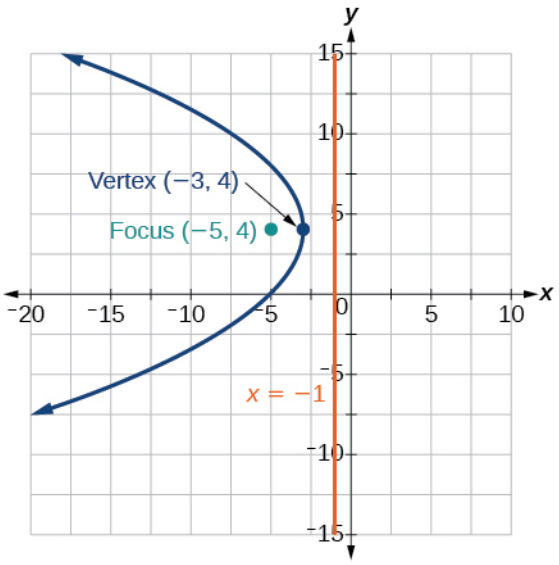

**389**.

$$\frac{(y-3)^2}{1}-\frac{(x-1)^2}{8}=1$$

**391**.

$(x-2)^2=\frac{1}{3}(y+1)$; vertex: $(2,\ -1)$; focus: $\left(2,\ -\frac{11}{12}\right)$; directrix: $y=-\frac{13}{12}$

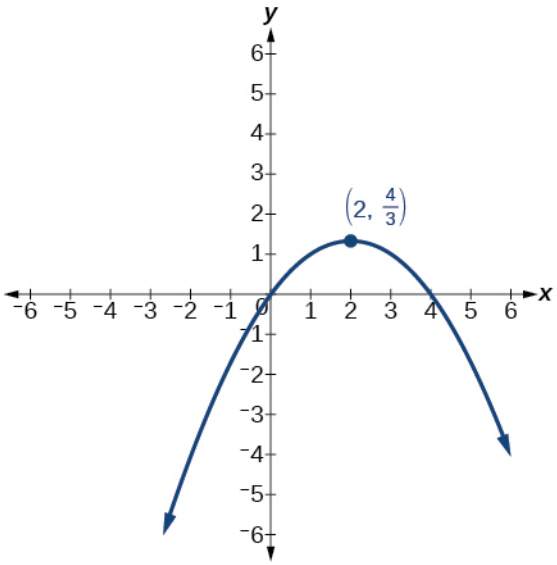
**393**.




**395**. Approximately
$8.49$ feet

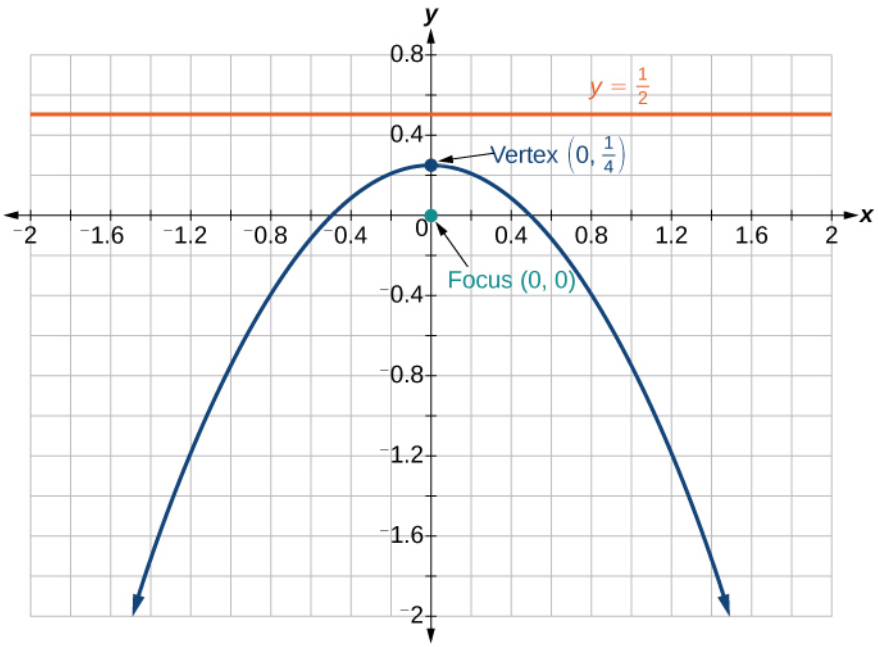
**397**. parabola;
$\theta \approx 63.4^\circ$

**399**.
$x'^2 - 4x' + 3y' = 0$




**401**. Hyperbola with
$e = \frac{3}{2}$, and directrix
$\frac{5}{6}$ units to the right of the pole.

**403**.




# Chapter 13

## Try It

**13.1**. The first five terms are
$\{1, 6, \quad 11, \quad 16, \quad 21\}$.

**13.2**. The first five terms are
$\left\{-2, \quad 2, \quad -\frac{3}{2}, \quad 1, \quad -\frac{5}{8}\right\}$.

**13.3**. The first six terms are
$\{2, \ 5, \ 54, \ 10, \ 250, \ 15\}$.

**13.4**.
$a_n = (-1)^{n+1} 9^n$

**13.5**.
$a_n = -\frac{3^n}{4n}$

**13.6**.
$a_n = e^{n-3}$

**13.7**.
$\{2, \quad 5, \quad 11, \quad 23, \quad 47\}$

**13.8**.
$\left\{0, \quad 1, \quad 1, \quad 1, \quad 2, \quad 3, \quad \frac{5}{2}, \ \frac{17}{6}\right\}$.

**13.9**. The first five terms are
$\left\{1, \quad \frac{3}{2}, \quad 4, \ 15, \ 72\right\}$.

**13.10**. The sequence is arithmetic. The common difference is
$-2$.

**13.11**. The sequence is not arithmetic because
$3 - 1 \neq 6 - 3$.

**13.12**.
$\{1, \quad 6, \quad 11, \quad 16, \quad 21\}$

**13.13**.
$a_2 = 2$

**13.14**.
$a_1 = 25$
$a_n = a_{n-1} + 12, \text{ for } n \geq 2$

**13.15**.
$a_n = 53 - 3n$

**13.16**. There are 11 terms in the sequence.

**13.17**. The formula is
$T_n = 10 + 4n,$ and it will take her 42 minutes.

**13.18**. The sequence is not geometric because
$\frac{10}{5} \neq \frac{15}{10}$.

**13.19**. The sequence is geometric. The common ratio is
$\frac{1}{5}$.

**13.20**.
$\left\{18, 6, 2, \frac{2}{3}, \frac{2}{9}\right\}$

**13.21**.
$a_1 = 2$
$a_n = \frac{2}{3}a_{n-1} \text{ for } n \geq 2$

**13.22**.
$a_6 = 16,384$

**13.23**.
$a_n = -(-3)^{n-1}$

**13.24**.

a.
$$P_n = 293 \cdot 1.026a^n$$

b. The number of hits will be about 333.

**13.25**. 38

**13.26**.
26.4

**13.27**.
328

**13.28**.
$-280$

**13.29**. $2,025

**13.30**.
$\approx 2,000.00$

**13.31**. 9,840

**13.32**. $275,513.31

**13.33**. The sum is defined. It is geometric.

**13.34**. The sum of the infinite series is defined.

**13.35**. The sum of the infinite series is defined.

**13.36**. 3

**13.37**. The series is not geometric.

**13.38**.
$-\frac{3}{11}$

**13.39**. $92,408.18

**13.40**. 7

**13.41**. There are 60 possible breakfast specials.

**13.42**. 120

**13.43**. 60

**13.44**. 12

**13.45**.
$P(7,\ 7) = 5,\ 040$

**13.46**.
$P(7,\ 5) = 2,\ 520$

**13.47**.
$C(10,\ 3) = 120$

**13.48**. 64 sundaes

**13.49**. 840

**13.50**.

a. 35

b. 330

**13.51**.

a.

$$x^5 - 5x^4 y + 10x^3 y^2 - 10x^2 y^3 + 5xy^4 - y^5$$

b.

$$8x^3 + 60x^2 y + 150xy^2 + 125y^3$$

**13.52**.
$-10,\ 206x^4 y^5$

**13.53**.

| Outcome | Probability |
|---|---|
| Roll of 1 | |
| Roll of 2 | |
| Roll of 3 | |
| Roll of 4 | |
| Roll of 5 | |
| Roll of 6 | |

**13.54**.
$\frac{2}{3}$

**13.55**.
$\frac{7}{13}$

**13.56**.
$\frac{2}{13}$

**13.57**.
$\frac{5}{6}$

**13.58**.
a. $\frac{1}{91}$; b. $\frac{5}{91}$; c. $\frac{86}{91}$

## Section Exercises

**1**. A sequence is an ordered list of numbers that can be either finite or infinite in number. When a finite sequence is defined by a formula, its domain is a subset of the non-negative integers. When an infinite sequence is defined by a formula, its domain is all positive or all non-negative integers.

**3**. Yes, both sets go on indefinitely, so they are both infinite sequences.

**5**. A factorial is the product of a positive integer and all the positive integers below it. An exclamation point is used to indicate the operation. Answers may vary. An example of the benefit of using factorial notation is when indicating the product It is much easier to write than it is to write out

$13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1.$

**7**. First four terms:

$-8, \ -\frac{16}{3}, \ -4, \ -\frac{16}{5}$

**9**. First four terms:

$2, \ \frac{1}{2}, \ \frac{8}{27}, \ \frac{1}{4}$.

**11**. First four terms:

$1.25, \ -5, \ 20, \ -80$.

**13**. First four terms:

$\frac{1}{3}, \ \frac{4}{5}, \ \frac{9}{7}, \ \frac{16}{9}$.

**15**. First four terms:

$-\frac{4}{5}, \ 4, \ -20, \ 100$

**17**.

$\frac{1}{3}, \ \frac{4}{5}, \ \frac{9}{7}, \ \frac{16}{9}, \ \frac{25}{11}, \ 31, \ 44, \ 59$

**19**.

$-0.6, \ -3, \ -15, \ -20, \ -375, \ -80, \ -9375, \ -320$

**21**.

$a_n = n^2 + 3$

**23**.

$a_n = \frac{2^n}{2n}$ or $\frac{2^{n-1}}{n}$

**25**.

$a_n = \left(-\frac{1}{2}\right)^{n-1}$

**27**. First five terms:

$3, \ -9, \ 27, \ -81, \ 243$

**29**. First five terms:

$-1, \ 1, \ -9, \ \frac{27}{11}, \ \frac{891}{5}$

**31**.

$\frac{1}{24}, \ 1, \frac{1}{4}, \ \frac{3}{2}, \ \frac{9}{4}, \ \frac{81}{4}, \ \frac{2187}{8}, \ \frac{531,441}{16}$

**33**.

$2, \ 10, \ 12, \ \frac{14}{5}, \ \frac{4}{5}, \ 2, \ 10, \ 12$

**35**.

$a_1 = -8, a_n = a_{n-1} + n$

**37**.

$a_1 = 35, a_n = a_{n-1} + 3$

**39**.

720

**41**.

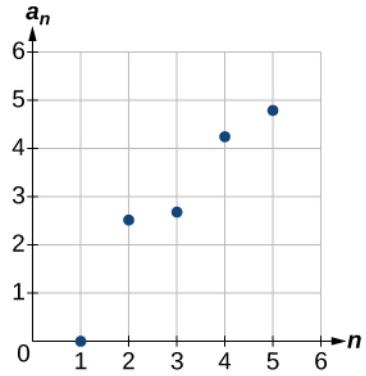
665, 280

**43**. First four terms:

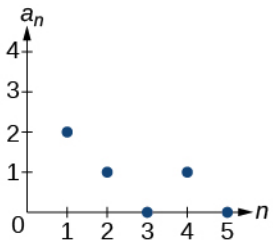
$1, \frac{1}{2}, \frac{2}{3}, \frac{3}{2}$

**45**. First four terms:
$-1,\ 2,\ \frac{6}{5},\ \frac{24}{11}$

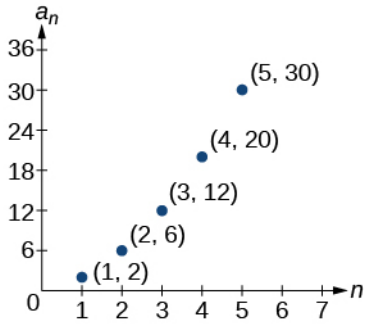
**47**.




**49**.




**51**.




**53**.
$a_n = 2^{n-2}$

**55**.
$a_1 = 6,\ \ a_n = 2a_{n-1} - 5$

**57**. First five terms:
$\frac{29}{37},\ \frac{152}{111},\ \frac{716}{333},\ \frac{3188}{999},\ \frac{13724}{2997}$

**59**. First five terms:
$2,\ 3,\ 5,\ 17,\ 65537$

**61**.
$a_{10} = 7,\ 257,\ 600$

**63**. First six terms:
$0.042,\ 0.146,\ 0.875,\ 2.385,\ 4.708$

**65**. First four terms:
$5.975,\ 32.765,\ 185.743,\ 1057.25,\ 6023.521$

**67**. If
$a_n = -421$ is a term in the sequence, then solving the equation
$-421 = -6 - 8n$ for
$n$ will yield a non-negative integer. However, if

$-421 = -6 - 8n,$ then

$n = 51.875$ so

$a_n = -421$ is not a term in the sequence.

**69**.

$a_1 = 1,\ a_2 = 0,\ a_n = a_{n-1} - a_{n-2}$

**71**.

$$\frac{(n+2)!}{(n-1)!} = \frac{(n+2)\cdot(n+1)\cdot(n)\cdot(n-1)\cdot\ldots\cdot 3\cdot 2\cdot 1}{(n-1)\cdot\ldots\cdot 3\cdot 2\cdot 1} = n(n+1)(n+2) = n^3 + 3n^2 + 2n$$

**72**. A sequence where each successive term of the sequence increases (or decreases) by a constant value.

**74**. We find whether the difference between all consecutive terms is the same. This is the same as saying that the sequence has a common difference.

**76**. Both arithmetic sequences and linear functions have a constant rate of change. They are different because their domains are not the same; linear functions are defined for all real numbers, and arithmetic sequences are defined for natural numbers or a subset of the natural numbers.

**78**. The common difference is

$\frac{1}{2}$

**80**. The sequence is not arithmetic because

$16 - 4 \neq 64 - 16.$

**82**.

$0,\ \frac{2}{3},\ \frac{4}{3},\ 2,\ \frac{8}{3}$

**84**.

$0,\ -5,\ -10,\ -15,\ -20$

**86**.

$a_4 = 19$

**88**.

$a_6 = 41$

**90**.

$a_1 = 2$

**92**.

$a_1 = 5$

**94**.

$a_1 = 6$

**96**.

$a_{21} = -13.5$

**98**.

$-19,\ -20.4,\ -21.8,\ -23.2,\ -24.6$

**100**.

$a_1 = 17; \quad a_n = a_{n-1} + 9 \ \ n \geq 2$

**102**.

$a_1 = 12; \quad a_n = a_{n-1} + 5 \ \ n \geq 2$

**104**.

$a_1 = 8.9; \quad a_n = a_{n-1} + 1.4 \ \ n \geq 2$

**106**.

$a_1 = \frac{1}{5}; \quad a_n = a_{n-1} + \frac{1}{4} \ \ n \geq 2$

**108**.

$a_1 = \frac{1}{6}; \quad a_n = a_{n-1} - \frac{13}{12} \ \ n \geq 2$

**110**.

$a_1 = 4;\ a_n = a_{n-1} + 7;\ a_{14} = 95$

**112**. First five terms:

$20,\ 16,\ 12,\ 8,\ 4.$

**114**.
$a_n = 1 + 2n$

**116**.
$a_n = -105 + 100n$

**118**.
$a_n = 1.8n$

**120**.
$a_n = 13.1 + 2.7n$

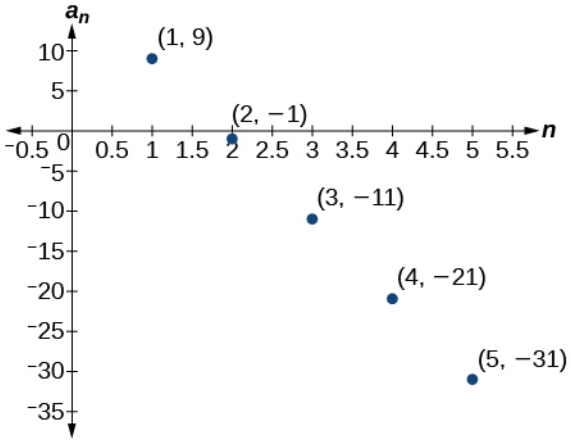
**122**.
$a_n = \frac{1}{3}n - \frac{1}{3}$

**124**. There are 10 terms in the sequence.

**126**. There are 6 terms in the sequence.

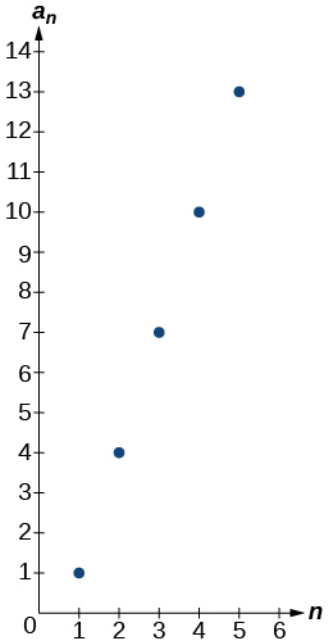
**128**. The graph does not represent an arithmetic sequence.

**130**.

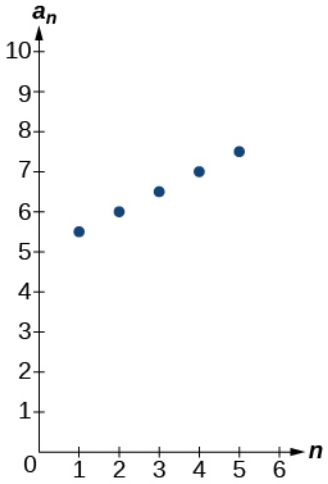



**132**.
1, 4, 7, 10, 13, 16, 19

**134**.




**136**.




**138**. Answers will vary. Examples:
$a_n = 20.6n$ and
$a_n = 2 + 20.4\text{n}.$

**140**.
$a_{11} = -17a + 38b$

**142**. The sequence begins to have negative values at the $13^{\text{th}}$ term,
$a_{13} = -\frac{1}{3}$

**144**. Answers will vary. Check to see that the sequence is arithmetic. Example: Recursive formula:
$a_1 = 3,\ a_n = a_{n-1} - 3.$ First 4 terms:
$3,\ 0,\ -3,\ -6\ \ a_{31} = -87$

**146**. A sequence in which the ratio between any two consecutive terms is constant.
**148**. Divide each term in a sequence by the preceding term. If the resulting quotients are equal, then the sequence is geometric.
**150**. Both geometric sequences and exponential functions have a constant ratio. However, their domains are not the same. Exponential functions are defined for all real numbers, and geometric sequences are defined only for positive integers. Another difference is that the base of a geometric sequence (the common ratio) can be negative, but the base of an exponential function must be positive.
**152**. The common ratio is
$-2$

**154**. The sequence is geometric. The common ratio is 2.
**156**. The sequence is geometric. The common ratio is
$-\frac{1}{2}.$

**158**. The sequence is geometric. The common ratio is
5.

**160**.
$5,\ 1,\ \frac{1}{5},\ \frac{1}{25},\ \frac{1}{125}$

**162**.
$800,\ 400,\ 200,\ 100,\ 50$

**164**.
$a_4 = -\frac{16}{27}$

**166**.
$a_7 = -\frac{2}{729}$

**168**.
$7,\ 1.4,\ 0.28,\ 0.056,\ 0.0112$

**170**.
$a = -32, \quad a_n = \frac{1}{2}a_{n-1}$

**172**.
$a_1 = 10, \quad a_n = -0.3a_{n-1}$

**174**.
$a_1 = \frac{3}{5}, \quad a_n = \frac{1}{6}a_{n-1}$

**176**.
$a_1 = \frac{1}{512}, \; a_n = -4a_{n-1}$

**178**.
$12, \; -6, \; 3, \; -\frac{3}{2}, \; \frac{3}{4}$

**180**.
$a_n = 3^{n-1}$

**182**.
$a_n = 0.8 \cdot (-5)^{n-1}$

**184**.
$a_n = -\left(\frac{4}{5}\right)^{n-1}$

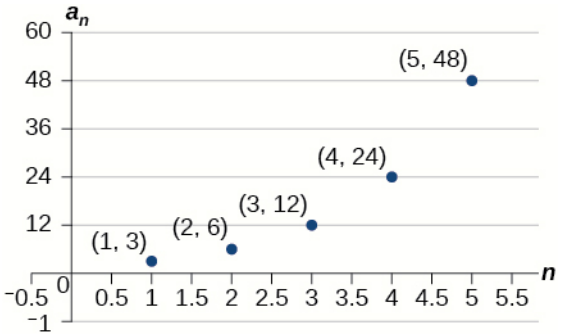
**186**.
$a_n = 3 \cdot \left(-\frac{1}{3}\right)^{n-1}$

**188**.
$a_{12} = \frac{1}{177,\ 147}$

**190**. There are
$12$ terms in the sequence.

**192**. The graph does not represent a geometric sequence.

**194**.




**196**. Answers will vary. Examples:
$a_1 = 800, \quad a_n = 0.5a_{n-1}$ and
$a_1 = 12.5, \quad a_n = 4a_{n-1}$

**198**.
$a_5 = 256b$

**200**. The sequence exceeds
$100$ at the $14^{\text{th}}$ term,
$a_{14} \approx 107$.

**202**.
$a_4 = -\frac{32}{3}$ is the first non-integer value

**204**. Answers will vary. Example: Explicit formula with a decimal common ratio:
$a_n = 400 \cdot 0.5^{n-1}$; First 4 terms:
$400, \; 200, \; 100, \; 50; \; a_8 = 3.125$

**206**. An $n$th partial sum is the sum of the first $n$ terms of a sequence.

**208**. A geometric series is the sum of the terms in a geometric sequence.

**210**. An annuity is a series of regular equal payments that earn a constant compounded interest.

**212**.

$$\sum_{n=0}^{4} 5n$$

**214**.

$$\sum_{k=1}^{5} 4$$

**216**.

$$\sum_{k=1}^{20} 8k+2$$

**218**.

$$S_5 = \frac{5\left(\frac{3}{2}+\frac{7}{2}\right)}{2}$$

**220**.

$$S_{13} = \frac{13(3.2+5.6)}{2}$$

**222**.

$$\sum_{k=1}^{7} 8 \cdot 0.5^{k-1}$$

**224**.

$$S_5 = \frac{9\left(1-\left(\frac{1}{3}\right)^5\right)}{1-\frac{1}{3}} = \frac{121}{9} \approx 13.44$$

**226**.

$$S_{11} = \frac{64\left(1-0.2^{11}\right)}{1-0.2} = \frac{781,249,984}{9,765,625} \approx 80$$

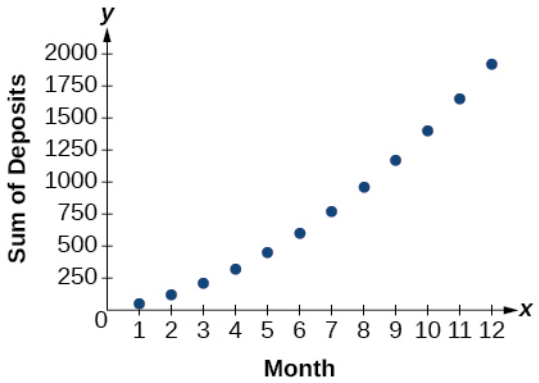
**228**. The series is defined.

$$S = \frac{2}{1-0.8}$$

**230**. The series is defined.

$$S = \frac{-1}{1-\left(-\frac{1}{2}\right)}$$

**232**.




**234**. Sample answer: The graph of $S_n$ seems to be approaching 1. This makes sense because

$\sum_{k=1}^{\infty} \left(\frac{1}{2}\right)^k$ is a defined infinite geometric series with

$S = \frac{\frac{1}{2}}{1 - \left(\frac{1}{2}\right)} = 1.$

**236**. 49

**238**. 254

**240**.

$S_7 = \frac{147}{2}$

**242**.

$S_{11} = \frac{55}{2}$

**244**.

$S_7 = 5208.4$

**246**.

$S_{10} = -\frac{1023}{256}$

**248**.

$S = -\frac{4}{3}$

**250**.

$S = 9.2$

**252**. $3,705.42

**254**. $695,823.97

**256**.

$a_k = 30 - k$

**258**. 9 terms

**260**.

$r = \frac{4}{5}$

**262**. $400 per month

**264**. 420 feet

**266**. 12 feet

**268**. There are

$m + n$ ways for either event

$A$ or event

$B$ to occur.

**270**. The addition principle is applied when determining the total possible of outcomes of either event occurring. The multiplication principle is applied when determining the total possible outcomes of both events occurring. The word “or” usually implies an addition problem. The word “and” usually implies a multiplication problem.

**272**. A combination;

$C(n, r) = \frac{n!}{(n-r)!r!}$

**274**.

$4 + 2 = 6$

**276**.

$5 + 4 + 7 = 16$

**278**.

$2 \times 6 = 12$

**280**.

$10^3 = 1000$

**282**.

$P(5, 2) = 20$

**284**.

$P(3, 3) = 6$

**286**.
$P(11, 5) = 55, 440$
**288**.
$C(12, 4) = 495$
**290**.
$C(7, 6) = 7$
**292**.
$2^{10} = 1024$
**294**.
$2^{12} = 4096$
**296**.
$2^9 = 512$
**298**.
$\frac{8!}{3!} = 6720$
**300**.
$\frac{12!}{3!2!3!4!}$
**302**. 9
**304**. Yes, for the trivial cases
$r = 0$ and
$r = 1.$ If
$r = 0,$ then
$C(n, r) = P(n, r) = 1.$ If
$r = 1,$ then
$r = 1,$
$C(n, r) = P(n, r) = n.$
**306**.
$\frac{6!}{2!} \times 4! = 8640$
**308**.
$6 - 3 + 8 - 3 = 8$
**310**.
$4 \times 2 \times 5 = 40$
**312**.
$4 \times 12 \times 3 = 144$
**314**.
$P(15, 9) = 1, 816, 214, 400$
**316**.
$C(10, 3) \times C(6, 5) \times C(5, 2) = 7, 200$
**318**.
$2^{11} = 2048$
**320**.
$\frac{20!}{6!6!8!} = 116, 396, 280$
**322**. A binomial coefficient is an alternative way of denoting the combination
$C(n, r).$ It is defined as
$\binom{n}{r} = C(n, r) = \frac{n!}{r!(n-r)!}.$
**324**. The Binomial Theorem is defined as
$(x + y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k$ and can be used to expand any binomial.
**326**. 15

**328**. 35
**330**. 10
**332**. 12,376
**334**.
$64a^3 - 48a^2b + 12ab^2 - b^3$
**336**.
$27a^3 + 54a^2b + 36ab^2 + 8b^3$
**338**.
$1024x^5 + 2560x^4y + 2560x^3y^2 + 1280x^2y^3 + 320xy^4 + 32y^5$
**340**.
$1024x^5 - 3840x^4y + 5760x^3y^2 - 4320x^2y^3 + 1620xy^4 - 243y^5$
**342**.
$\frac{1}{x^4} + \frac{8}{x^3y} + \frac{24}{x^2y^2} + \frac{32}{xy^3} + \frac{16}{y^4}$
**344**.
$a^{17} + 17a^{16}b + 136a^{15}b^2$
**346**.
$a^{15} - 30a^{14}b + 420a^{13}b^2$
**348**.
$3,486,784,401a^{20} + 23,245,229,340a^{19}b + 73,609,892,910a^{18}b^2$
**350**.
$x^{24} - 8x^{21}\sqrt{y} + 28x^{18}y$
**352**.
$-720x^2y^3$
**354**.
$220,812,466,875,000y^7$
**356**.
$35x^3y^4$
**358**.
$1,082,565a^3b^{16}$
**360**.
$\frac{1152y^2}{x^7}$
**362**.
$f_2(x) = x^4 + 12x^3$

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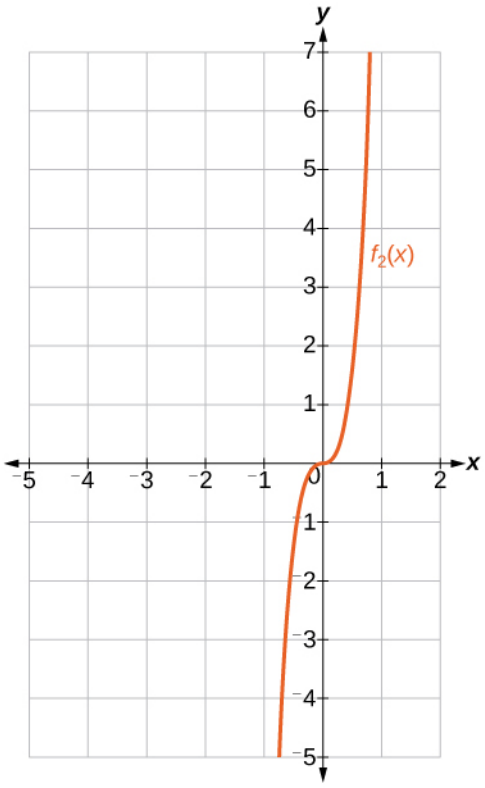
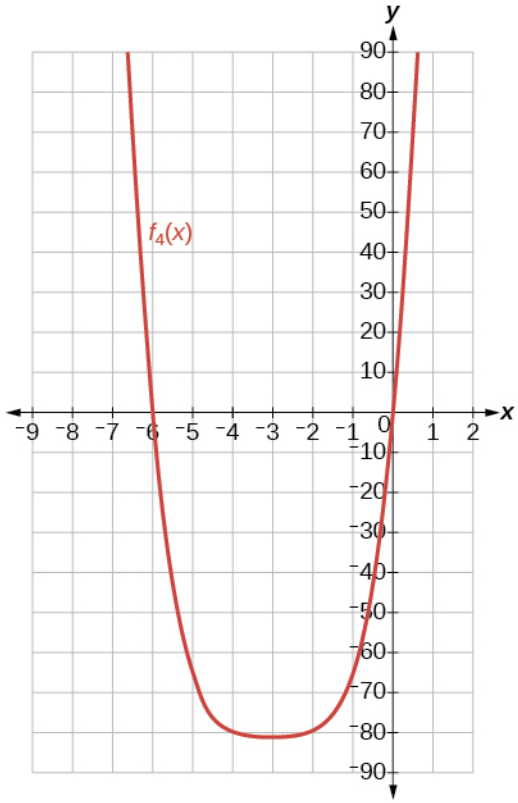



**364**.

$f_4(x) = x^4 + 12x^3 + 54x^2 + 108x$




**366**.

590, $625x^5y^2$

**368**.
$k-1$
**369**.
$\binom{n}{k-1}+\binom{n}{k}=\binom{n+1}{k}$; Proof:

$$\begin{aligned}
&\binom{n}{k-1}+\binom{n}{k}\\
&=\frac{n!}{k!(n-k)!}+\frac{n!}{(k-1)!(n-(k-1))!}\\
&=\frac{n!}{k!(n-k)!}+\frac{n!}{(k-1)!(n-k+1)!}\\
&=\frac{(n-k+1)n!}{(n-k+1)k!(n-k)!}+\frac{kn!}{k(k-1)!(n-k+1)!}\\
&=\frac{(n-k+1)n!+kn!}{k!(n-k+1)!}\\
&=\frac{(n+1)n!}{k!((n+1)-k)!}\\
&=\frac{(n+1)!}{k!((n+1)-k)!}\\
&=\binom{n+1}{k}
\end{aligned}$$

**370**. The expression
$(x^3+2y^2-z)^5$ cannot be expanded using the Binomial Theorem because it cannot be rewritten as a binomial.
**371**. probability; The probability of an event is restricted to values between
0 and
1, inclusive of
0 and
1.
**373**. An experiment is an activity with an observable result.
**375**. The probability of the *union of two events* occurring is a number that describes the likelihood that at least one of the events from a probability model occurs. In both a union of sets
$A$ and $B$ and a union of events
$A$ and $B$, the union includes either
$A$ or $B$ or both. The difference is that a union of sets results in another set, while the union of events is a probability, so it is always a numerical value between
0 and
1.
**377**.
$\frac{1}{2}$.
**379**.
$\frac{5}{8}$.
**381**.
$\frac{1}{2}$.
**383**.
$\frac{3}{8}$.
**385**.
$\frac{1}{4}$.
**387**.
$\frac{3}{4}$.

**389**.
$\frac{3}{8}$.

**391**.
$\frac{1}{8}$.

**393**.
$\frac{15}{16}$.

**395**.
$\frac{5}{8}$.

**397**.
$\frac{1}{13}$.

**399**.
$\frac{1}{26}$.

**401**.
$\frac{12}{13}$.

**403**.

| | **1** | **2** | **3** | **4** | **5** | **6** |
|---|---|---|---|---|---|---|
| **1** | (1, 1) 2 | (1, 2) 3 | (1, 3) 4 | (1, 4) 5 | (1, 5) 6 | (1, 6) 7 |
| **2** | (2, 1) 3 | (2, 2) 4 | (2, 3) 5 | (2, 4) 6 | (2, 5) 7 | (2, 6) 8 |
| **3** | (3, 1) 4 | (3, 2) 5 | (3, 3) 6 | (3, 4) 7 | (3, 5) 8 | (3, 6) 9 |
| **4** | (4, 1) 5 | (4, 2) 6 | (4, 3) 7 | (4, 4) 8 | (4, 5) 9 | (4, 6) 10 |
| **5** | (5, 1) 6 | (5, 2) 7 | (5, 3) 8 | (5, 4) 9 | (5, 5) 10 | (5, 6) 11 |
| **6** | (6, 1) 7 | (6, 2) 8 | (6, 3) 9 | (6, 4) 10 | (6, 5) 11 | (6, 6) 12 |

**405**.
$\frac{5}{12}$.

**407**.
0.

**409**.
$\frac{4}{9}$.

**411**.
$\frac{1}{4}$.

**413**.
$\frac{3}{4}$

**415**.
$\frac{21}{26}$

**417**.
$\frac{C(12,\ 5)}{C(48,\ 5)} = \frac{1}{2162}$

**419**.
$\frac{C(12,\ 3)C(36,\ 2)}{C(48,\ 5)} = \frac{175}{2162}$

**421**.
$\frac{C(20,\ 3)C(60,\ 17)}{C(80,\ 20)} \approx 12.49\%$

**423**.
$\frac{C(20,\ 5)C(60,\ 15)}{C(80,\ 20)} \approx 23.33\%$

**425**.
$20.50 + 23.33 - 12.49 = 31.34\%$

**427**.
$\frac{C(40000000,\ 1)C(277000000,\ 4)}{C(317000000,\ 5)} = 36.78\%$

**429**.
$\frac{C(40000000,\ 4)C(277000000,\ 1)}{C(317000000,\ 5)} = 0.11\%$

## Review Exercises

**431**.
$2,\ 4,\ 7,\ 11$

**433**.
$13,\ 103,\ 1003,\ 10003$

**435**. The sequence is arithmetic. The common difference is $d = \frac{5}{3}$.

**437**.
$18,\ 10,\ 2,\ -6,\ -14$

**439**.
$a_1 = -20,\ a_n = a_{n-1} + 10$

**441**.
$a_n = \frac{1}{3}n + \frac{13}{24}$

**443**.
$r = 2$

**445**.
$4,\ 16,\ 64,\ 256,\ 1024$

**447**.
$3,\ 12,\ 48,\ 192,\ 768$

**449**.
$a_n = -\frac{1}{5} \cdot \left(\frac{1}{3}\right)^{n-1}$

**451**.
$\sum_{m=0}^{5} \left(\frac{1}{2}m + 5\right)$.

**453**.
$S_{11} = 110$

**455**.
$S_9 \approx 23.95$

**457**.
$S = \frac{135}{4}$

**459**. $5,617.61

**461**. 6

**463**.
$10^4 = 10{,}000$

**465**.
$P(18,\ 4) = 73{,}440$

**467**.
$C(15,\ 6) = 5005$

**469**.
$2^{50} = 1.13 \times 10^{15}$

**471**.
$\frac{8!}{3!2!} = 3360$

**473**.
490,314

**475**.
$131{,}072a^{17} + 1{,}114{,}112a^{16}b + 4{,}456{,}448a^{15}b^2$

**477**.

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 1, 1 | 1, 2 | 1, 3 | 1, 4 | 1, 5 | 1, 6 |
| 2 | 2, 1 | 2, 2 | 2, 3 | 2, 4 | 2, 5 | 2, 6 |
| 3 | 3, 1 | 3, 2 | 3, 3 | 3, 4 | 3, 5 | 3, 6 |
| 4 | 4, 1 | 4, 2 | 4, 3 | 4, 4 | 4, 5 | 4, 6 |
| 5 | 5, 1 | 5, 2 | 5, 3 | 5, 4 | 5, 5 | 5, 6 |
| 6 | 6, 1 | 6, 2 | 6, 3 | 6, 4 | 6, 5 | 6, 6 |

**479**.
$\frac{1}{6}$

**481**.
$\frac{5}{9}$

**483**.
$\frac{4}{9}$

**485**.
$1 - \frac{C(350,\ 8)}{C(500,\ 8)} \approx 94.4\%$

**487**.
$\frac{C(150,\ 3)C(350,\ 5)}{C(500,\ 8)} \approx 25.6\%$

## Practice Test

**488**.
$-14,\ -6,\ -2,\ 0$

**490**. The sequence is arithmetic. The common difference is $d = 0.9$.

**492**.
$a_1 = -2,\ a_n = a_{n-1} - \frac{3}{2};\ a_{22} = -\frac{67}{2}$

**494**. The sequence is geometric. The common ratio is $r = \frac{1}{2}$.

**496**.
$a_1 = 1,\ a_n = -\frac{1}{2} \cdot a_{n-1}$

**498**.
$\sum_{k=-3}^{15} \left(3k^2 - \frac{5}{6}k\right)$

**500**.
$S_7 = -2604.2$

**502**. Total in account:
$\$140,\ 355.75;$ Interest earned:
$\$14,\ 355.75$

**504**.
$5 \times 3 \times 2 \times 3 \times 2 = 180$

**506**.
$C(15,\ 3) = 455$

**508**.
$\frac{10!}{2!3!2!} = 151{,}200$

**510**.
$\frac{429x^{14}}{16}$

**512**.
$\frac{4}{7}$

**514**.
$\frac{5}{7}$

**516**.
$\frac{C(14,\ 3)C(26,\ 4)}{C(40,\ 7)} \approx 29.2\%$